Achieving Full Rate Network Coding with Constellation Compatible Modulation and Coding

Suhua TANG\(^1\), Hiroyuki YOMO\(^1,2\), Tetsuro UEDA\(^1\), Ryu MIURA\(^1\), Sadao OBANA\(^1\)
\(^1\)ATR Adaptive Communications Research Laboratories, Japan
\(^2\)Faculty of Engineering Science, Kansai University, Japan

Email: \{shtang, yomo, teueda, miurar, obana\}@atr.jp

Abstract—Network coding is an effective method to improving relay efficiency by reducing the number of transmissions. However, its performance is limited by several factors such as packet length mismatch and rate mismatch. Although the former may be solved by re-framing, the latter remains a challenge and is likely to greatly degrade the efficiency of network coding. In this paper, we re-interpret network coding as a mapping of modulation constellation. On this basis, we extend such mapping to enable simultaneous use of different modulations by nesting the low-level constellation as a subset of the high level constellation. When relay links have different qualities, the messages of different flows are combined together in such a way that for each relay link its desired message is transmitted at its own highest rate. Compared with previous solutions to rate mismatch, the proposed scheme achieves the full rate of all relay links on the broadcast channel.

I. INTRODUCTION

Wireless communications suffer greatly from multipath fading where outages may occur and degrade communication quality. Different schemes have been exploited to mitigate this problem and among them adaptive modulation and coding is an effective method. It has been widely used in 3G cellular networks and wireless LANs. In the latter, the rate adaptation runs on top of the media access control function and selects a suitable transmit rate, statistically or based on instantaneous signal to noise ratio (SNR). Each transmit rate corresponds to a channel coding policy and modulation method. Transmissions at a low rate over the direct links may be inefficient. Instead, path diversity can be used, which is also known as cooperative communication or relay [1].

When relay is used, the number of transmissions is increased, although the total time could get shorter compared with the direct transmission. If the traffic pattern and the a priori information are exploited, the relay efficiency can be further improved by reducing the number of transmissions via network coding [2]. Typical transmission patterns suitable for applying network coding include two-way relay [3], [4], multiple access channel [5], multicast channel, etc.

In the two-way relay model [3], [4], two nodes \(M_1\) and \(M_2\) exchange packets via the common relay \(R\). First \(P_1\) is transferred from \(M_1\) to \(R\) and then \(P_2\) from \(M_2\) to \(R\), in two successive slots. In the third slot, \(R\) forwards \(P_2 = P_1 \oplus P_2\), and \(M_1\) recovers \(P_2\) by \(P_1 \oplus P_2\), \(M_2\) recovers \(P_1\) by \(P_2 \oplus P_2\). In this way, 2 packets are exchanged in 3 slots. In the multiple access up-link, the direct link is also considered and the relay works in the ARQ (Automatic Repeat-reQuest) mode [5]. In either case, joint network and channel coding can further improve spectral efficiency [5], [6].

In general, when network coding is used, packets are first transferred from nodes to relay in a multiple access channel. Then in the broadcast channel from relay to nodes, packets are network coded together to reduce the number of transmissions. However, the performance of network coding is limited by several factors such as packet length mismatch (the short packets are zero padded), traffic rate mismatch (some packets cannot be network coded due to lack of pairing packets) and transmit rate mismatch. The last factor is neglected in most previous works, with a few exceptions where rate adaptation was exploited in network coding-based relay selection [7].

In the two-way relay scenarios, the rate mismatch may be formed due to two factors. One is the relay position (which leads to relatively stable differences in link qualities) and the other is fading. Even though the relay lies exactly in the middle of two nodes, the two relay links may have different instantaneous quality. Since the network coded packet is intended to be received by two nodes, the minimal rate is chosen so that the packet can be correctly decoded at both receivers. In this way, transmission at a low rate on the link supporting a high rate wastes channel bandwidth.

When the bidirectional transmission in the two-way relay model is extended to multiple flows, the effect of rate mismatch becomes more serious, since the minimal rate over more links only gets lower. One solution to this problem is to exploit opportunistic scheduling. Instead of transmitting network coded packets to all potential nodes, only some of them are selected by taking the tradeoff between the number of links and the actual rate [8]. This opportunistic scheduling, however, cannot exploit the full power of network coding.

Recently, some initial efforts were made to fully exploit rate adaptations. Multiplicative network coding was proposed in [9] to better adapt code and modulation rates to different nodes. However, it only applies to constant modulus signals. Unconventional 5-ary modulation was introduced to work together with QPSK in [10]. This makes modulation complex and it was only studied under the two-way relay model. Compress and forward was studied in [11], with the assumption that the relay is much closer to the sink than nodes and has a much higher rate. Its application is limited to multiple access up-link. XOR-based network coding and physical layer...
superposition coding were combined together to better exploit rate adaptation in [12], however, at the cost of power loss. Moreover, its performance is degraded and approaches that of network coding when relay links have similar quality. Despite all these efforts, there is still no complete solution to the rate mismatch problem.

In this paper, we propose a simple but practical scheme, constellation compatible modulation and coding, to achieve full rate network coding on the broadcast channel. The principle of dirty paper coding [13] indicates that a signal known at the receiver is not interference at all and with suitable coding the full capacity is achievable. As an analogy for network coding, when the a priori information is available, network coding should also achieve full rate over all links. This is our start point. The basic idea is as follows: (i) At the relay node, in order to combine packets together and transmit them over links supporting different rates (modulations), the low-level constellation points are nested in the high level constellation. In other words, a subset of the high level constellation is used as the low level constellation, and this subset depends on the design of network coding. (ii) At the receiver side, nodes supporting low level modulation first find their constellation according to the a priori information and then perform demodulation and decoding. In this way, the highest rate of each link is used and the sum rate is achieved over the broadcast channel.

The rest of the paper is organized as follows: The relay model is presented in Sec. II and the re-interpretation of network coding as constellation mapping is addressed in Sec. III. In Sec. IV, the detailed procedures for achieving full rate in network coding based transmissions are discussed, and the simulation results are analyzed in Sec. V. Finally, we conclude the paper with Sec. VI.

II. SYSTEM MODEL

Figure 1 shows an example network with 4 nodes and 1 relay. $M_1$ talks with $M_3$; $M_2$ talks with $M_4$, both bidirectionally. The actual transfer of packets is via the relay $R$. At first, $M_1$ sends $P_1$ to $R$ ($M_2$ and $M_4$ overhear $P_1$), $M_2$ sends $P_2$ to $R$ ($M_1$ and $M_3$ overhear $P_2$), $M_3$ sends $P_3$ to $R$ ($M_2$ and $M_4$ overhear $P_3$), and $M_4$ sends $P_4$ to $R$ ($M_1$ and $M_3$ overhear $P_3$). After this, $M_1$ knows $P_1$, $P_2$, $P_4$; $M_2$ knows $P_1$, $P_2$, $P_3$; $M_3$ knows $P_2$, $P_3$, $P_4$; $M_4$ knows $P_1$, $P_3$, $P_4$. Then $R$ transmits $P_2 = P_1 \oplus P_2 \oplus P_3 \oplus P_4$ to all four nodes, and $M_1$ recovers $P_3$ by $P_1 \oplus P_2 \oplus P_4$; in a similar way, $M_2$, $M_3$ and $M_4$ recover $P_4$, $P_1$ and $P_2$, respectively. Hence, four packets are exchanged in 5 slots.

In general, we consider a network with $n$ nodes $M_i$, $i = 1, \cdots, n$, and a relay node $R$. Communications take place between node pairs $\langle M_i, M_j \rangle$ via $R$ and there are $n$ flows, each starting from and ending at a different node. All nodes and $R$ are synchronized and the transmissions are done in terms of slots. Packets of each flow go through a two-hop path via $R$.

Over each link, the rate can be adjusted by modulation and coding. With coding rate $c$ and modulation level $m$ (constellation size = $2^m$), on average $c \cdot m$ bits can be transmitted by each symbol. Generally, the modulation level determines a rate range, within which the coding scheme further fine adjusts the rate. Different modulation levels have constellations with different sizes. But these constellations all have the same normalized energy.

The packet transfer is divided into two stages:

(i) Multiple access channel. Each node transmits small packets to $R$ using its optimal rate (modulation and coding). The number of bits that can be transmitted within a slot depends on the actual rate and the length of the slot. Opportunistic multi-user scheduling may be used at this stage. It is assumed that each packet can be received by $R$ and overheard by all other nodes except the final destination. This stage finishes when $R$ receives enough data for each flow, during which a node may transmit in several slots. Packets belonging to the same flow are re-organized and stored in the buffer.

(ii) Broadcast channel. For each flow, $R$ chooses the optimal rate (modulation and coding) for each relay link and calculates the number of bits for the frame; the frames of all flows, network coded together, are transmitted to all nodes after modulation. The net rate for each flow is exactly the highest rate its link quality supports.

Network coding is used at the second stage, where joint design of modulation and coding is necessary in order that transmission over each link reaches its highest rate. In this paper, we focus on the second stage and assume that (i) there are enough data for each flow to transfer, (ii) the a priori information is available at each node, and, (iii) the relay node knows the channel state information of all links. The key problem is how to realize full-rate on all links simultaneously.

III. REINTERPRETATION OF NETWORK CODING

In conventional network coding schemes, bits from different flows are XORed together, channel coded, and then transmitted after modulation. The receiver works in the reverse way, performing demodulation and channel decoding, and finally network decoding with the a priori information. Due to the linearity of both channel coding and network coding, their order can be exchanged [14]. In this paper, the network coding is done after channel coding.

Network coding can be re-interpreted as a function of constellation mapping. This is explained by an example shown in Fig. 2 using QPSK constellation. Assume $R$ forwards $a_1a_0$ (‘01’) from $M_2$ to $M_1$, and $b_1b_0$ (‘11’) from $M_1$ to $M_2$, respectively. When forwarding these bits, $R$ combines them
Post-COD: map (S1, S2) to (S′1, S′2), modulated to signal x′QPSK.

(ii) At the receiver side, F−1(ap1) provides the constellation for demodulating P1,c, where ap1 (ap1 = ⊕j̸=i Pj,c in conventional network coding) is the a priori information at the i-th receiver.

In conventional network coding schemes, the constellations for different flows have the same size. Since the transmit rate mainly depends on the constellation size, this forces the relay node to transmit the XORed packet with the min rate of all links so that all nodes can correctly recover the XORed packet. In this paper, we explore new network coding schemes so that different modulations (with different constellation size) can be used simultaneously and each link transmits with its own optimal coding/modulation.

IV. FULL RATE NETWORK CODING

To fully exploit network coding, modulations of different levels should be used simultaneously so that rate mismatch is mitigated. More specifically, we use constellation compatible modulations and nest the low level constellation inside the high level constellations. For example, a subset of four 16QAM constellation points can be used as QPSK constellation so that over the broadcast channel QPSK is used for one link while 16QAM is used for the other link. In other words, among the links with different modulations, the highest modulation level is always used. Other low level modulations use a subset of the high level constellation as their constellations. To meet this requirement, at the relay node, the post coding (Post-COD) stage is added before XORing all coded packets together, as shown in Fig. 3. At the receiver side, network decoding is done to perform constellation conversions—extracting the desired subset from the high level constellation, as shown in Fig. 4.

A. Encoding/Modulation at the Relay

Figure 3 shows the transmit procedure at relay R. For each flow f1, according to the SNR of its relay link, R finds the transmit rate r1 from an empirical SNR-rate table. Assume, without loss of generality, that rates, r1, i = 1, 2, · · ·, n, over links from R to M1, are in the increasing order, i.e., r1 ≤ r2 ≤ · · · ≤ rN. Each ri = ci · mi corresponds to a coding rate ci and a modulation level mi. mi, i = 1, 2, · · ·, n, are also in the increasing order; mi ≤ m2 ≤ · · · ≤ mn. In the following, mi also means a constellation with the same modulation level. Its meaning is clear from the context.

Transmission at R is done by the following steps.

(i) In order to mitigate the packet length mismatch, every time R transmits a fixed number of symbols, N. For each flow f1, the number of information bits that can be transmitted is N · r1. These information bits form a frame P1,u. On P1,u channel coding with rate c1 is performed, which generates P1,c. P1,c, i = 1, 2, · · ·, n, have different length in bits.

(ii) In order to transmit P1,c, which requires a constellation m1 ≤ mn, via the constellation mn, there should be a mapping from constellation m1 to mn. This is done by Post-COD as follows, which generates Pi of the same length in bits.

Post-COD: map (m1, m2, · · ·, mn) to (m1, m1, · · ·, mn).

(iii) P1, i = 1, 2, · · ·, n are XORed together as Py = ⊕i P1, y. Py, modulated to signal xQPSK with constellation mn, is transmitted to all nodes (together with out-of-band rate information r1, i = 1, · · ·, n).

B. Post-COD

There are many ways to realize the mapping in Eq. (1). One way is to choose a subset of a high level constellation (e.g., 16QAM) as the low level constellation (e.g., QPSK). Fig. 5 shows an example, where the 16QAM constellation is divided into 4 non-overlapping subsets, each corresponding to one possible QPSK constellation. This is equivalent to use the
repetition codes to map a bit vector of length \( m_1 \) to another bit vector of length \( m_n \). When \( m_n \) is not integral times of \( m_1 \), a different mapping method other than repetition codes should be used but the division of the high level constellation is similar. By exhaustive search, it can be verified that such a mapping is optimal in the sense that the min distance of the embedded constellation is maximal.

Table I shows the constellation conversion from QPSK to BPSK for a single symbol. It is assumed that \( a_1a_0 \) represents the \( a \ priori \) information \((a_1 \oplus a_0) = P_1 \) at \( M_1 \) and \( b_0 \) represents the bit to be received from \( R \). With the repetition codes, \( M_1 \) actually receives \( a_1a_0 \oplus b_0 \) under QPSK modulation. With \( a_1a_0 \) known in advance, by changing \( b_0 \), \( a_1a_0 \oplus b_0 \) only switches between a subset of two QPSK constellation points and this subset depends on \( a_1a_0 \). As an example, with \( a_1a_0 = '11' \), \( a_1a_0 \oplus b_0 \) switches between \((S_1, S_2)_{QPSK}\), which is equivalent to \((S_0, S_1)_{BPSK}\). This procedure is shown in Fig. 6.

Table II shows the conversion from 16QAM to QPSK. It is assumed that \( a_3a_2a_1a_0 \) represents the \( a \ priori \) information \((a_1 \oplus a_2 \oplus a_3) = P_3 \) at \( M_1 \) and \( b_1b_0 \) represents the bits to be received from \( R \). With the repetition codes, \( M_1 \) actually receives \( a_3a_2a_1a_0 \oplus b_1b_0 \) under QPSK modulation. With \( a_3a_2a_1a_0 \) known in advance, by changing \( b_1b_0 \), \( a_3a_2a_1a_0 \oplus b_1b_0 \) only switches between a subset of two QPSK constellation points and this subset depends on \( a_3a_2a_1a_0 \). As an example, with \( a_3a_2a_1a_0 = '111' \), \( a_3a_2a_1a_0 \oplus b_1b_0 \) switches between \((S_4, S_5, S_6, S_7)_{QPSK}\), which is equivalent to \((S_0, S_1)_{BPSK}\). This procedure is shown in Fig. 5. In a similar way, the conversion between other constellations is also possible.

The constellation conversion may have SNR loss since the min distance of the derived constellation may be a little less than that of the standard one. Table III shows the SNR loss for the typical conversions. Although the conversion from 16QAM to QPSK has SNR loss of about 0.97dB, other conversions have little SNR loss (0.05dB for 256QAM → 64QAM) or no SNR loss (0dB for QPSK → BPSK) at all. In the evaluation in Sec. V, the SNR loss is taken into account when choosing rate (modulation and coding) according to SNR.

### C. Demodulation/Decoding at the Receiver

Figure 4 shows the demodulation and decoding procedure at all nodes. At the \( s^{th} \) node, the signal received from \( R \) is

\[
s_i(t) = h_i * x \Sigma(t) + n_i(t),
\]

where \( h_i \) is the channel gain and \( n_i(t) \) is zero mean additive white Gaussian noise.

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**Table I**

<table>
<thead>
<tr>
<th>( a \ priori ) info ( a_1a_0 )</th>
<th>BPSK constellation ((a_1a_0 \oplus b_0))</th>
</tr>
</thead>
<tbody>
<tr>
<td>00</td>
<td>((S_0, S_1))_{QPSK}</td>
</tr>
<tr>
<td>01</td>
<td>((S_1, S_2))_{QPSK}</td>
</tr>
<tr>
<td>10</td>
<td>((S_2, S_3))_{QPSK}</td>
</tr>
<tr>
<td>11</td>
<td>((S_3, S_0))_{QPSK}</td>
</tr>
</tbody>
</table>

**Table II**

<table>
<thead>
<tr>
<th>( a \ priori ) info ( a_3a_2a_1a_0 )</th>
<th>QPSK constellation ((a_3a_2a_1a_0 \oplus b_1b_0))</th>
</tr>
</thead>
<tbody>
<tr>
<td>000</td>
<td>((S_0, S_3, S_1, S_2)_{16QAM})</td>
</tr>
<tr>
<td>001</td>
<td>((S_1, S_2, S_3, S_4)_{16QAM})</td>
</tr>
<tr>
<td>010</td>
<td>((S_2, S_3, S_4, S_5)_{16QAM})</td>
</tr>
<tr>
<td>011</td>
<td>((S_3, S_4, S_5, S_6)_{16QAM})</td>
</tr>
<tr>
<td>100</td>
<td>((S_4, S_5, S_6, S_7)_{16QAM})</td>
</tr>
<tr>
<td>101</td>
<td>((S_5, S_6, S_7, S_8)_{16QAM})</td>
</tr>
<tr>
<td>100</td>
<td>((S_6, S_7, S_8, S_9)_{16QAM})</td>
</tr>
<tr>
<td>101</td>
<td>((S_7, S_8, S_9, S_{10})_{16QAM})</td>
</tr>
<tr>
<td>110</td>
<td>((S_{10}, S_{11}, S_{12}, S_{13})_{16QAM})</td>
</tr>
<tr>
<td>111</td>
<td>((S_{11}, S_{12}, S_{13}, S_{14})_{16QAM})</td>
</tr>
</tbody>
</table>

**Table III**

<table>
<thead>
<tr>
<th>Modulation</th>
<th>16QAM</th>
<th>256QAM</th>
<th>64QAM</th>
</tr>
</thead>
<tbody>
<tr>
<td>SNR loss (dB)</td>
<td>0.97</td>
<td>0.05</td>
<td>0</td>
</tr>
</tbody>
</table>
TABLE III

<table>
<thead>
<tr>
<th>Conversion</th>
<th>Min dist (derived /std)</th>
<th>SNR loss</th>
</tr>
</thead>
<tbody>
<tr>
<td>QPSK → BPSK</td>
<td>2/2</td>
<td>0dB</td>
</tr>
<tr>
<td>16QAM → QPSK</td>
<td>(2 - 0.6325)/1.414</td>
<td>-0.97dB</td>
</tr>
<tr>
<td>64QAM → 16QAM</td>
<td>(2 - 0.3080)/0.6325</td>
<td>-0.21dB</td>
</tr>
<tr>
<td>256QAM → 64QAM</td>
<td>(2 - 0.1354)/0.3080</td>
<td>-0.05dB</td>
</tr>
</tbody>
</table>

A node $M_i$, supporting the used constellation ($m_i = m_n$), performs soft demodulation and calculates symbol log-likelihood ratio (LLR) [15] and then convert to bit LLR. The LLR of desired bits can be recovered and then channel decoding is performed. The whole procedure is shown in the right side of Fig. 4.

For a receiver $M_i$ requiring a lower constellation ($m_i < m_n$), at first the low-level constellation is derived by exploiting the a priori information, as described in previous section. This conversion of constellation is actually network decoding. Then the signal is demodulated with the derived constellation and later channel decoded to recover the bits, as shown in the left side of Fig. 4.

D. A Complete Example

Next, with the two-node ($n=2$) scenario in Fig. 7, we show how the whole procedure works. Links between $R$ and $M_1/M_2$ support rates $c_1=1/2$, $m_1=2$(QPSK), $c_2=1/2$, $m_2=4$ (16QAM), respectively. On average, $R$ can transmit $r_1 = c_1 \cdot m_1 = 1$ bit/symbol to $M_1$ and $r_2 = c_2 \cdot m_2 = 2$ bit/symbol to $M_2$. Assume the slot length is 2 symbols. Then 2 bits to $M_1$ or 4 bits to $M_2$ can be transmitted in a single slot.

Assume that the two bits from $R$ to $M_1$ are $P_{1,a}=10$ and 4 bits from $R$ to $M_2$ are $P_{2,a}=1101$. The transmit procedure is shown in Fig. 3. After channel coding, $P_{1,c}=1101$ and $P_{2,c}=11100010$. After post-coding, $P_1='11110011'$ and $P_2='11100010'$. Then the XORed sum is $P_{3}=00010001$. After mapping to 16QAM constellation, $R$ transmits $x_2=(S_1S_1)_{16QAM}$.

The receive procedure is shown in Fig. 4. Ignoring noise and channel fading, at $M_1$, at first $P_{3}=00010001$ is recovered from $x_2$. Then with $P_3='11100011'$ known a priori, $P_{2,c}=11100010$ is recovered and then $P_{2,a}=1101$ is obtained. At $M_1$, with $P_2=11100010$ known a priori, for the first symbol in $x_2$, $a_3a_2a_1a_0='1110'$, $(S_1)_{16QAM}$ is converted to $(S_3)_{QPSK}$ (refer to Table II); for the second symbol in $x_2$, $a_3a_2a_1a_0='0010'$, $(S_1)_{16QAM}$ is converted to $(S_1)_{QPSK}$. In this way, $x_2=(S_1S_1)_{16QAM}$ is converted to $(S_3S_3)_{QPSK}$. Next demodulation and channel decoding are done and finally $P_{1,a}=10$ is recovered.

A simple comparison of the broadcast channel, among network coding with min rate (NetCod), network coding with full rate (FRNC) and decode-and-forward (DecFwd), is summarized in Table IV. With FRNC, $R$ transmits $\sum_i r_i \cdot 2 = 6$ bits using two symbols; with NetCod, $R$ transmits $(\min(r_1) \cdot 2) = 4$ bits; with DecFwd, $R$ uses one symbol for each node and thus transmits 3 bits in total.

V. NUMERICAL RESULTS

In this section we evaluate the proposed FRNC scheme using Monte-Carlo simulations. Each slot consists of 4800 symbols. Messages are coded by a 4-state recursive systematic convolutional (RSC) code with the generator matrix (1, 5/7).

We use coding rate $c=1/2$, 3/4 for BPSK ($m=1$), QPSK ($m=2$), 16QAM ($m=4$), and coding rate $c=2/3$, 3/4 for 64QAM ($m=6$), 256QAM ($m=8$). In combination, 10 different transmit rates can be supported. The number of information bits in a message varies from 2400 bits to 28800 bits. Messages are transmitted from $R$ to nodes via different schemes and decoded accordingly. In the evaluation, we will compare FRNC with DecFwd, NetCod [3], network coding with opportunistic scheduling (NCSched) [8], and the combination of network coding with superposition coding (iPack) [12]. It is assumed that each link experiences independent block Rayleigh fading. Modulation constellations are adopted from IEEE 802.11a and the related parameters (symbol period, number of sub-carrier) are used in calculating throughput [16].

With a two-node-and-one-relay scenario similar to the one shown in Fig. 7, we first demonstrate how the position of the relay node affects the system performance. Adjusting the position of $R$ between $M_1$ and $M_2$ changes the normalized distance $d_{M_1R}/d_{M_2M_2}$. Average SNR of links $M_1R$ and $M_2R$ is calculated from the normalized distance $d_{M_1R}/d_{M_2M_2}$ according to the two-ray model [17] with the path loss exponent (equaling 3 in the simulation). When $R$ lies in the middle of $M_1$ and $M_2$, the SNR of both relay links equals 20dB.

Figure 8 shows the total throughput on the broadcast channel. It is interesting to see that these curves have different trends. Because the min rate is used in NetCod, its total throughput reaches the maximum when $R$ lies in the middle point and decreases as $R$ moves away from the middle point. On the other hand, in other schemes, as $R$ moves towards either node, the high rate on that link can be effectively exploited to improve the total throughput. When $R$ lies exactly in the middle of $M_1$ and $M_2$, iPack has a similar throughput as NetCod because the superposition coding can hardly be used and iPack degenerates to NetCod. Contrary to iPack and
NetCod, FRNC uses the full rates of both links, works well under all scenarios, and always achieves the highest throughput. The throughput gain of FRNC against iPack reaches the max value 24% when the normalized distance equals 0.3. Due to the effect of fading, two links with the same average SNR have different instantaneous SNR. Therefore, FRNC outperforms iPack and NetCod even when the normalized distance equals 0.5.

Next the effect of $n$, the number of flows, is evaluated. Average SNR of all relay links is fixed at 20dB. Fig. 9 shows the throughput on the broadcast channel. DecFwd transmits in a TDMA manner. Therefore, it cannot benefit from the increase in flows and its throughput is almost a constant value. On the other hand, NetCod, NC_sched and FRNC all benefit from the increase in flows more or less. Due to the different capability in handling rate mismatch, the slopes of three curves differ greatly. FRNC always has the highest throughput because the rate mismatch problem is completely solved and full rate is achieved.

VI. CONCLUSION AND FUTURE WORK

Recently, network coding is widely studied for improving the relay efficiency in wireless networks. Its performance, however, is greatly limited by factors such as rate mismatch. In this paper, we re-interpreted network coding as a mapping between modulation constellations and extended such mapping to enable simultaneous use of different modulations for network coding. In this way, the highest rate over each link can be used and the sum rate can be achieved over the broadcast channel. As a result, the rate mismatch problem is completely solved. In the future, we will also study the multiple access channel and evaluate the total effect of full rate network coding.

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