A Scheme for Transformation of Tolerance Specifications to Generalized Deviation Space for Use in Tolerance Synthesis and Analysis

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ABSTRACT

Traditionally tolerances for manufactured parts are specified using symbolic schemes as per AMSE or ISO standards. For using these tolerance specifications in computerized tolerance synthesis and analysis, we need information model to represent the tolerances. Tolerance specifications could be modeled as a class with its attributes and methods [ROY01]. All tolerances are essentially small quantities that impose some kind of restriction on the possible deviation of features from its nominal size/shape. These variations of shape/size of a feature could be modeled as deviation of a set of generalized coordinates defined at some convenient point on the feature [BAL98]. In this paper, we have presented a method for converting tolerance specifications as per MMC / LMC / RFS material conditions for standard mating features (planar, cylindrical, and spherical) into a set of inequalities in a deviation space for representation of deviation of a feature from it’s nominal shape. We have used the virtual condition boundaries (VCB) as well as tolerance zones (as the case may be) for these mappings. For the planar feature, these relations are linear and the bounded space is diamond shaped. For the other cases, the mapping is a set of nonlinear inequalities. This mapping is a transformation of the tolerance specifications into a generalized coordinate frame in the form of a set of inequalities. These inequalities could be treated as constraints for use in tolerance synthesis, and analysis as well as in assemblability analysis. We have elaborated the procedure with an example and also indicated where/how these mapped results are applicable.

INTRODUCTION AND SCOPE

Tolerancing a part, as we use in engineering practice, has two aspects: i) to assign tolerance values to a part so that the variation of its size/dimensions during manufacturing could be specified (tolerance synthesis), and ii) to assess how the part will behave in the final assembly (tolerance analysis) after a part has been designed with specified tolerance values. Apart from the traditional size tolerance (which could be described by specifying a plus-minus type value on a nominal dimension like length, diameter, etc.), there are different types of tolerances that come into play when a part is placed in relation to other parts in an assembly. In general, we have four different types of tolerances: Size, Form, Position and Orientation. While size and form tolerance control the shapes of the features themselves, orientation and positional tolerances control the relative orientation and location between features. In addition, three material conditions are defined to these tolerances. They are MMC (Maximum Material Condition), LMC (Least Material Condition) and RFS (Regardless of Feature’s Size). While MMC and LMC will allow a bonus tolerance to the geometric tolerance specified if feature size departs from its MMC, RFS will not. The modification to MMC or LMC will result in a virtual condition boundary to control the surface of the feature of size1 while the modification to RFS will result in

1 Feature of size: Feature of size is one cylindrical or spherical surface, or a set of opposed elements or opposed parallel surfaces, associated with a size dimension.
a tolerance zone that control the derived element, such as center plane, axis of a cylinder, center of a ball, etc. MMC is always used when the tolerated feature or feature pattern mates with the feature or feature pattern on the other part. LMC is used in the situation when material reservation is of the main consideration. RFS is used when balance is important (for example, for parts that rotate).

In our earlier work [ROY01] we have used an abstract tolerance class for using in an assembly. The need for converting these tolerance specifications to a generalized coordinate frame arises from tolerance synthesis and analysis requirements. In order to take into account the effect of all components in an assembly in a global coordinate frame in a systematic manner, we convert them into a generalized coordinate system so that each tolerance could be treated on the same footing. There have been different approaches by different researchers to identify/use suitable coordinate systems. In the present case we have adopted the small displacement torsor (SDT), a well-developed [BAL98] system suitable for representing the deviations of features from its nominal shape/size, for representing the deviations of features.

Since, in general the tolerance values are small compared to the dimensions of the part, in the first order approximation of rotation, the deviations of a surface/feature could be accurately described by two vectors: one 3 component displacement vector and one 3 component rotation vector defined at a convenient point on the feature [BOU96]. These two components in a collective form are called a small displacement torsor (SDT). Assuming the two vectors are given by: small displacement \( d = (d_x, d_y, d_z)^T \) and small rotation \( r = (r_x, r_y, r_z)^T \), the SDT can be written as: \( T = (r, d)^T = (r_x, r_y, r_z, d_x, d_y, d_z)^T \). Even though we start with six components for each SDT, we can eliminate some of the components depending upon the type/nature of the feature it represents. This is possible due to the fact that most features used in engineering are regular and in general, they may have some invariants along some of their six degrees of freedom (three linear, three rotational). For example, for a cylindrical feature, there are two such parameters: axial rotation and axial movement, both keep the cylindrical feature invariant and thus these two parameters could be eliminated. This leads to a four component deviation for a cylindrical feature with axis along the x-axis: \((0, r_y, r_z, d_x, d_y, d_z)^T\). Each deviation torsor for a feature of size will consist of two torsors: one for the deviation of the feature from the nominal position (called displacement torsor) and the other to represent the intrinsic variation of size of the feature (called intrinsic torsor). For non-size features, we will need only the displacement torsor to represent the variation of the feature from its nominal position.

In this work, we present a method for mapping tolerance specifications as per ASME Y14.5 into a deviation space for the feature. We have used SDT for representing deviations and the mapping is carried out by considering the VCB (or tolerance zone) generated by the tolerance specification.

**STEPS FOR MAPPING A TOLERANCE INTO THE DEVIATION SPACE**

Followings are the basic steps that we have used to convert the tolerance specification into a set of inequalities in the deviation parameters:

a. Generate the intrinsic torsor and the deviation torsor for the feature by eliminating the deviation parameters that are invariant for the feature to reduce the degrees of freedom. This reduces the number of deviation parameters needed to represent variation of the feature. For example, for a cylindrical feature, axial movement and axial rotation are the two invariants and we do not need these two parameters.

b. Generate a VCB\(^4\) (or a tolerance zone) based on the tolerance specification. In this step we compute the size of the VCB or the tolerance zone for restricting the variation of the feature or derived element respectively.

c. In case of VCB, take an arbitrary point on the nominal surface of the feature in parametric form and transform it to a new position by applying the effect of the deviation torsors. For example, for a cylindrical feature a point \( P \) on it’s nominal surface is represented by two parameters \((\theta, z)\), as \( P = (r \cos \theta, r \sin \theta, z) \), where \( \theta \in (0, 2\pi) \) and \( z \in (0, L) \), \( L=\text{Length of the cylinder} \). In case of tolerance zones, transform the whole derived element, such as center plane, center axis etc., to a new position by applying the effect of the deviation torsor.

d. Eliminate the free parameters (like the \((\theta, z)\) mentioned in step c above) by applying the condition that the extreme points of the transformed position should remain within the VCB or the derived element should remain within the tolerance zone. This step generates a set of inequalities connecting the deviation parameters with the tolerance specification.

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2 **Derived Element**: Elements derived from a feature of size like: derived median line (from a cylindrical feature), derived median plane (from a width-type feature), feature center point (from a spherical feature), feature axis (from a cylindrical feature), and feature center plane (from width-type feature)

3 **Non-size feature**: Non-size feature is a surface having no unique or intrinsic size (diameter or width) dimension to measure, such as a nominally flat planar surface, a revolute- a surface, such as a cone, generated by revolving a spine about an axis, etc
Procedure for applying the above four steps will be clear in subsequent sections where we generate the mappings for three basic features under various material conditions.

The category of relations arising from the mapping of the deviation parameters of the features to corresponding tolerance values are of the generic non-linear (“less than or equal to”) type and takes the form: \( F_{DP}^{k}(x) \leq F_{Tol}^{k}(t) \), where \( F_{DP} \) is some function of the deviation parameters \( x \in \mathbb{R}^{n} \), \( F_{Tol} \) is some function of the tolerance parameters \( t \), and \( k \in \mathbb{N} \) is an index. Exact forms of \( F_{DP}^{k}(x) \) and \( F_{Tol}^{k}(t) \), depend on the nature of the feature/surface. For a planar surface the above relationship becomes a set of linear constraints. The interesting feature in the above relationship is the separable nature of the equation in terms of the variables \( t \) and \( x \).

**MAPPING PROCEDURE**

The tolerances are specified on features or derived elements from feature of size. In this paper we present mapping for one non-size feature (planar feature), and two features of size (cylindrical feature and spherical feature).

There are four types of tolerances used in the industry: size tolerance, form tolerance, orientation tolerance and location/position tolerance. These tolerances are sometimes specified with material conditions as modifiers. There are three such material conditions: MMC (Maximum Material Condition), LMC (Least Material Condition) and RFS (Regardless of Feature’s Size). Since we assume that the shape of a tolerated feature remains similar to the nominal feature (for example, a plane remains a plane, a cylinder remains a cylinder, etc.), the variation of position and orientation due to form tolerance is very small [Gil91] and in this study, we do consider only the size tolerance, orientation tolerance and positional tolerance.

A geometric tolerance applied to a feature of size and modified to MMC will establish a virtual condition boundary (VCB) outside of the material space adjacent to the feature. The feature shall not cross this VCB. Likewise, a geometric tolerance applied to a feature of size and modified to LMC establishes a VCB inside the material and the feature shall not cross the VCB [DRA99]:

\[ \Delta \theta_{x}, \Delta \theta_{y}, \Delta \theta_{z}, \Delta x, \Delta y, \Delta z \] = six components of the deviation torsor.

If a geometric tolerance applied to a feature of size is neither modified to MMC nor LMC, by definition of standards such as ASME Y14.5, it is modified to RFS. In this case, instead of VCB, the tolerance specification will generate a tolerance zone into which derived element will not interfere.

Since a VCB or tolerance zone represents the basic intention of the designer that each point on the tolerated feature/derived element should remain within (or outside) this boundary/zone, the VCB or tolerance zone would be ideally suitable for representing the relational limits on the deviation parameters associated with the deviation of the feature from it’s nominal position. Since the deviation parameters for a feature could be used to define the deviation of all points on a feature (by employing one or more independent parameters), the tolerance specifications could be thought of a set of limits for the deviation parameters and vice-versa. These relationships could be used for tolerance synthesis as well as for tolerance analysis including checking the assemblability in worst-case scenario.

In the following sections of this paper, we consider the mapping for size tolerance, positional tolerance, and orientation tolerance for three types of features: planar, cylindrical, and spherical. For the planar case, there is no material condition as it is a non-size feature. For the cylindrical feature we have two cases: MMC and RFS and for the spherical feature we have MMC.

The symbols used in the following sections, unless otherwise specified, are defined as below:
- \( TU = \text{Upper limit, size tolerance} \)
- \( TL = \text{Lower limit, size tolerance} \)
- \( T_{p} = \text{Positional tolerance} \)
- \( T_{v} = \text{Perpendicularity tolerance} \)

\( (\Delta \theta_{x}, \Delta \theta_{y}, \Delta \theta_{z}, \Delta x, \Delta y, \Delta z) \) = six components of the deviation torsor.

**PLANAR FEATURES**

For each surface/feature, a local coordinate system (LCS) is defined and the deviation parameters are defined in that LCS. For a planar surface, the LCS is: \( z \)-axis outward normal (emanating from the material side of the feature) and \( (x, y) \) are local orthogonal coordinates on the plane, so that the equation for the nominal surface (plane) is given by \( z = 0 \). For this plane the deviation parameters of \( SDT \) are: \( (\Delta \theta_{x}, \Delta \theta_{y}, 0, 0, 0, \Delta z)T \).

**SizeTolerance1**: Size tolerance with respect to a datum.

**Case-1**: Rectangular (2a x 2b) planar surface. (Figure 1)

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**Virtual condition boundary**: A constant boundary generated by the collective effects of a size feature’s specified MMC or LMC material condition and the geometric tolerance for that material condition.

![Figure 1. Planar feature (rectangular) with size tolerance](image)
For the above feature, deviation parameters are \((\Delta \theta x, \Delta \theta y, 0, 0, 0, \Delta z)\), and the tolerance parameters are \((TU, TL)\) and the SDT is: \(Dd = [\Delta \theta Dd]T = [\Delta \theta x \ \Delta \theta y \ 0 \ 0 \ 0 \ \Delta z]T\)

Following constraints are established by considering the deviation at the four extreme points (corners) of the plane which are nominally at \((+a, +b, 0), (+a, -b, 0), (-a, +b, 0)\) and \((-a, -b, 0)\).

\[-\min (\Delta z + b \Delta \theta x, \Delta z + b \Delta \theta y, \Delta z - b \Delta \theta x, \Delta z - b \Delta \theta y) \leq TL\]

\[\max (\Delta z + b \Delta \theta x, \Delta z + b \Delta \theta y, \Delta z - b \Delta \theta x, \Delta z - b \Delta \theta y) \leq TU\]

\[\max (-\min (2b \Delta \theta x, 2a \Delta \theta y), \max (2b \Delta \theta x, 2a \Delta \theta y), -\min (b \Delta \theta x + a \Delta \theta y, b \Delta \theta y - a \Delta \theta x, -b \Delta \theta x + a \Delta \theta y, -b \Delta \theta y - a \Delta \theta x), \max (b \Delta \theta x + a \Delta \theta y, b \Delta \theta y - a \Delta \theta x, -b \Delta \theta x + a \Delta \theta y, -b \Delta \theta y - a \Delta \theta x)) \leq (TU + TL)\]

**Case-2:** Circular (radius \(r\)) planar surface. (Figure 2)

![Figure 2. Planar feature (circle) with size tolerance](image)

Deviation parameters = \((0, 0, \Delta z, \Delta \theta x, \Delta \theta y, 0)\), tolerance parameters = \((TU, TL)\). For arbitrary point \(P\), at an angle \(\beta\) with the \(y\)-axis, on the circumference, we have,

\[\delta z = \Delta z + R \Delta \theta y \sin(\beta) + R \Delta \theta x \cos(\beta)\]

Where \(\delta z\) is the pure position variation of a point in \(Z\) direction where size tolerance control. The tolerance zone in this case is made up of two planes, \(z = +TU\) and \(z = -TL\).

We need to ensure that \(\delta z\) is within the above boundary for \(\forall \beta \in (0, 2\pi)\). This can be done by treating the \(\delta z\) as a function of \(\beta\) and the finding the extreme points by putting \(\partial(\delta z) / \partial \beta = 0\). This gives, \(\delta z\) max/min = \(\Delta z \pm R \sqrt{(\Delta \theta x^2 + \Delta \theta y^2)}\)

This leads to the following mapping (constraints):

\[\Delta \theta x^2 + \Delta \theta y^2) \leq (TU + TL)^2/(4R^2)\]

We can also write in the parametric form as

\[TL \leq \Delta z + R \Delta \theta y \sin(\beta) + R \Delta \theta x \cos(\beta) \leq TU, \ \forall \beta \in (0, 2\pi)\]

The last inequality, using the value of \(\delta z\) max/min could be written, independent of \(\beta\), as:

\[TL \leq (\Delta z - R \sqrt{(\Delta \theta x^2 + \Delta \theta y^2)}) \quad \text{and} \quad (\Delta z + R \sqrt{(\Delta \theta x^2 + \Delta \theta y^2)}) \leq TU\]

**CYLINDRICAL FEATURE - MODIFIED TO MMC**

Let us assume a cylindrical feature with a given tolerance specification as shown in Figure 4. A local coordinate system (LCS), as shown in Figure 3, is defined for cylindrical surface and the deviation parameters are defined in that LCS. For a cylindrical surface, the LCS is: \(z\)-axis along the axis of the cylinder and \((x, y)\) are local orthogonal co-ordinates on the middle of the axis (Figures 3, 4). For this surface, the deviation parameters of the SDT are: \((\Delta x, \Delta y, 0, \Delta \theta x, \Delta \theta y, 0)\) and \(dr\). Based on this notation, constraints could be derived connecting these parameters with the specified tolerance values as detailed below.

Deviation parameters = \((\Delta x, \Delta y, 0, \Delta \theta x, \Delta \theta y, 0)\), tolerance parameters = \((TU, TL, TS)\), SDT is:

\[Dd = [\Delta \theta Dd] = [\Delta \theta x \ \Delta \theta y \ 0 \ \Delta x \ \Delta y \ 0]T\]

\[Di = [\Delta \theta Di] = [0 \ 0 \ dr \cos \theta \ dr \sin \theta \ 0]T\]

where \(Di\) is a displacement torsor and \(Di\) is intrinsic torsor.\(^6\)

Let us assume that a point \(P (r \cos \theta, r \sin \theta, z)\) (Figure 3) on the nominal cylindrical surface, after transformation due to the effect of the two small displacement torsors, takes the new position \(P'\) given by:

![Figure 3. Cylindrical Feature with LCS](image)

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\(^5\) Displacement torsor: torsor for the deviation of the feature from the nominal position.

\(^6\) Intrinsic torsor: torsor to represent the intrinsic variation of size of the feature.
The 4 by 4 matrix on left hand side of the equation is the transformation matrix for point P following the assumed transformation sequence \( \Delta x \rightarrow \Delta y \rightarrow \Delta \theta_x \rightarrow \Delta \theta_y \) due to displacement torsor. The 4 by 1 matrix (vector) on left hand side of the equation represent the changed position of point P due to intrinsic torsor.

So, the new position \( P' \) (Figure 3) is given by

\[
(r + dr) \cos \theta + \Delta \theta_y z + \Delta x \\
(r + dr) \sin \theta - \Delta \theta_x z + \Delta y \\
z - \Delta \theta_y (r + dr) \cos \theta + \Delta \theta_x (r + dr) \sin \theta
\]

The VCB of the cylindrical surface has properties as follows:
- It has the perfect shape as that of the nominal cylindrical feature
- It has the size (diameter) of \( 2r + TU + Tp \), because it’s external feature and the positional tolerance is modified to MMC
- It has the perfect orientation as that of the nominal cylindrical feature (vertical to C)
- It has the perfect location as that of the nominal cylindrical feature

So, the constraint equation will be:

\[
((r + dr) \cos \theta + \Delta \theta_y z + \Delta x)^2 + \\
((r + dr) \sin \theta - \Delta \theta_x z + \Delta y)^2 \\
\leq (2r + TU + Tp)^2 / 4
\]  

where \( \theta \in (0, 2\pi) \). The inequality should be valid for \( \forall \theta \in (0, 2\pi) \) and \( \forall z \in (0, L) \). We thus need to eliminate \( \theta \) and \( z \) from the LHS of (4) by finding the maximum of the above equation. LHS of (4) could be written as:

\[
f(\theta, z) = (a \cos \theta + bz + c)^2 + (a \sin \theta + dz + e)^2
\]

where \( a = r + dr, \ b = \Delta \theta_y, \ c = \Delta x, \ d = -\Delta \theta_x, \ e = \Delta y \)

By equating the partial derivative of \( f \) to zero,

\[
\frac{\partial f}{\partial \theta} = 0, \ \text{and} \ \frac{\partial f}{\partial z} = 0,
\]

we get,

\[
f_{\text{max}} = \max \left\{ \frac{1}{d^2 + b^2} (ad^2 + ab^2 - be + cd)^2 \right\} - 0
\]

\[
f_{\text{min}} = \min \left\{ \frac{1}{d^2 + b^2} (ad^2 + ab^2 - be + cd)^2 \right\} - 0
\]

The final set of constraints are:

\[
f_{\text{max}} \leq \left( \frac{2r + TU + Tp}{2} \right)^2, \quad -TL \leq dr \leq TU
\]

When the tolerated feature is internal, as for example, for a hole, the constraints are:

\[
f_{\text{min}} \geq \left( \frac{2r + TU - Tp}{2} \right)^2, \quad -TL \leq dr \leq TU
\]

If the external feature is modified to LMC, the constraint will be:

\[
f_{\text{min}} \geq \left( \frac{2r - TL - Tp}{2} \right)^2, \quad -TL \leq dr \leq TU
\]

If the internal feature is modified to LMC, the constraint will be:

\[
f_{\text{max}} \leq \left( \frac{2r - TL + Tp}{2} \right)^2, \quad -TL \leq dr \leq TU
\]

**CYLINDRICAL FEATURE MODIFIED TO RFS**

Fig 5. Cylindrical tolerance zone
Figure 5 shows a cylindrical tolerance zone (of center axis of the cylindrical feature) which could be due to positional tolerance, concentricity tolerance and/or run-out tolerance for cylindrical surface. A LCS (local coordinate system) is defined as shown in Figure 5. The displacement torsor and intrinsic torsor are:

\[ D_d = \{\Delta \theta, \Delta d\} = \{\Delta \theta_x, \Delta \theta_y, 0, \Delta d_x, \Delta d_y\} \]

\[ D_i = \{\Delta \theta, \Delta d\} = \{0, 0, \Delta r \cos \theta, \Delta r \sin \theta, 0\} \]

The basic difference of this case from those modified to MMC/LMC is that \(D_d\) is independent from \(D_i\). It means that the size of tolerance zone, \(r\) will not change due to the variation of \(\Delta r\) which is the departure from nominal radius. Because it is the derived element, axis of the feature of size, that is tolerated but not the surface of the feature of size, different methods are used to generate the constraints on mapping deviation parameters to the tolerance zone.

Because \(D_d\) and \(D_i\) are independent, we need to treat them separately. For \(D_d\) we have 4 deviation parameters: \(\Delta \theta x, \Delta \theta y, \Delta x, \Delta y\). For ease of computation, let’s assume that the deviations of the axis takes place in the sequence of \(\Delta x \rightarrow \Delta y \rightarrow \Delta \theta x \rightarrow \Delta \theta y\). We think this assumption will not affect the generality because it doesn’t matter what sequence we choose. (In fact the matrix multiplication in former section is based on the same assumption.). The constraints on \(\Delta x\) and \(\Delta y\) will be:

\[-r \leq \Delta x \leq r \quad \text{and} \quad -\sqrt{r^2 - \Delta x^2} \leq \Delta y \leq \sqrt{r^2 - \Delta x^2} \quad \text{(1)}\]

Now we want to know constraint on \(\Delta \theta x\) given those of \(\Delta x, \Delta y\). We can get it from Figure 7.

\[-\frac{\sqrt{r^2 - \Delta x^2 - |\Delta y|}}{L/2} \leq \Delta \theta_x \leq \frac{\sqrt{r^2 - \Delta x^2 - |\Delta y|}}{L/2} \quad \text{(2)}\]

From Figure 8, we get

\[-\frac{\sqrt{r^2 - (\Delta \theta_x \cdot L/2 + |\Delta y|)^2 - |\Delta x|}}{(L/2) \cos \Delta \theta_x} \leq \Delta \theta_x \leq \frac{\sqrt{r^2 - (\Delta \theta_x \cdot L/2 + |\Delta y|)^2 - |\Delta x|}}{(L/2) \cos \Delta \theta_x} \quad \text{(3)}\]

Because \(\Delta \theta_x\) is very small, \(\cos \Delta \theta_x \approx 1\), so we have

\[-\frac{\sqrt{r^2 - (\Delta \theta_x \cdot L/2 + |\Delta y|)^2 - |\Delta x|}}{(L/2)} \leq \Delta \theta_x \leq \frac{\sqrt{r^2 - (\Delta \theta_x \cdot L/2 + |\Delta y|)^2 - |\Delta x|}}{(L/2)} \quad \text{(4)}\]
Inequalities (1,2,3) are the mapping results for the positional tolerance modified by RFS specified on cylindrical feature of size.

SPHERICAL SURFACE

A local coordinate system (LCS) is defined for spherical surface and the deviation parameters are defined in that LCS. For a spherical surface, the LCS is: z-axis along radius of the sphere and (x, y) are local orthogonal co-ordinates at the center of the sphere (Figures 9, 10). For this surface the deviation parameters of the SDT are: (∆x, ∆y, ∆z, 0, 0, 0) and dr. Based on this notation, constraints could be derived connecting these parameters with the specified tolerance values as detailed below.

We need the above inequality to be valid for ∀Φ ∈ (0, 2π) and ∀Ψ ∈ (0, 2π). So a maxima of the following function is desired.

\[ f(\phi, \psi) = a \sin^2 \phi + a \cos^2 \psi + 2\Delta x \sin \phi \cos \psi + 2\Delta y \sin \phi \sin \psi + 2\Delta z \cos \psi \]

where \(a = r + dr\)

\[
\frac{\partial f}{\partial \phi} = 0 \quad \Rightarrow \quad 2 \cos \phi (a \sin \phi + \Delta x \cos \psi + \Delta y \sin \psi) = 0
\]

\[
\frac{\partial f}{\partial \psi} = 0 \quad \Rightarrow \quad -2a \cos \psi \sin \psi - 2\Delta x \sin \phi \sin \psi + 2\Delta y \sin \phi \cos \psi - 2\Delta z \sin \psi = 0
\]

Here, we have two sets of solutions (Maple V has been used to get the solutions in symbolic form):

The final set of constraints are:

\[
f_{\text{max}} \leq \left( \frac{2r + TU + TP}{2} \right)^2, \quad -TL \leq dr \leq TU
\]
When tolerated feature is internal, a hole e.g., the final constraint will be:
\[
 f_{min} \geq \left( \frac{2r + TU - Tp}{2} \right)^2 , \quad -TL \leq dr \leq TU
\]

If the external feature is modified to LMC, the constraint will be:
\[
 f_{min} \geq \left( \frac{2r - TL - Tp}{2} \right)^2 , \quad -TL \leq dr \leq TU
\]

If the internal feature is modified to LMC, the constraint will be:
\[
 f_{max} \leq \left( \frac{2r - TL + Tp}{2} \right)^2 , \quad -TL \leq dr \leq TU
\]

EXAMPLE OF SOME MAPPINGS

CASE 1: PLANAR FEATURE

Above is a simple size tolerance to a planar surface with LCS specified. Let’s assume left hand side surface is implicit datum and the other side is controlled by the tolerance specification.

From above, we know the deviation parameters for the right hand side surface which is a non-size feature are \( \Delta \theta_x, \Delta \theta_y, \Delta z \).

Following constraints are established by considering the deviation at the four extreme points (corners) of the plane which are nominally at (1.5, 2, 0), (1.5, -2, 0), (-1.5, 2, 0) and (-1.5, -2, 0).

\[
\begin{align*}
- \min (\Delta z + 2\Delta \theta_x + 1.5\Delta \theta_y, \Delta z + 1.5\Delta \theta_x - 2\Delta \theta_y, \Delta z - 1.5\Delta \theta_x + 2\Delta \theta_y, \Delta z - 1.5\Delta \theta_x - 2\Delta \theta_y) & \leq T_{SL} \\
\max (\Delta z + 2\Delta \theta_x + 1.5\Delta \theta_y, \Delta z + 2\Delta \theta_x - 1.5\Delta \theta_y, \Delta z - 1.5\Delta \theta_x + 2\Delta \theta_y, \Delta z - 1.5\Delta \theta_x - 2\Delta \theta_y) & \leq T_{SU}
\end{align*}
\]

The above constraints on deviation parameters result in a diamond in deviation space as follows

CASE 2: CYLINDRICAL MATING

From the above specification, we know that the VCB of the external feature of above part is equal to that of the internal feature of the bottom part. The size of the VCB for the external cylindrical feature is \( 5.25 + 0.05 = 5.3 \), and that of the internal cylindrical feature is \( 5.7 - 0.05 = 5.3 \). So we know that the two parts are guaranteed to assemble. Now let’s see what constraints on deviation parameters we can get by our tolerance mapping, so we can do the tolerance analysis or synthesis by computer.

Fig. 11 Planar feature (rectangular) with size tolerance

Fig. 12 Deviation space for size tolerance on planar surface

Fig. 13 Assembly of two parts specified with positional tolerance
Let \( a = r + dr , \quad b = \Delta \theta_x , \quad c = \Delta x , \)

\[ d = -\Delta \theta_y , \quad e = \Delta y \]

\[ f_{\max} = \max \{- \left( \frac{\pm 1}{\sqrt{d^2 + b^2}} \right) \left( (ad^2 + ab^2) - be + cd \right)^2 \}, 0 \}

\[ f_{\min} = \min \{- \left( \frac{\pm 1}{\sqrt{d^2 + b^2}} \right) \left( (ad^2 + ab^2) - be + cd \right)^2 \}, 0 \}

The final set of constraints for the external feature are:

\[ f_{\max} \leq \left( \frac{5 + 0.25 + 0.05}{2} \right)^2 , \quad -0.25 \leq dr \leq 0.25 \]

When the toleranced feature is internal, as for example, for a hole, the constraints are:

\[ f_{\min} \geq \left( \frac{5.7 - 0.35 - 0.05}{2} \right)^2 , \quad -0.35 \leq dr \leq 0.35 \]

Above equations couldn’t be reduced further in compact form and as such no further derivation and/or plotting is possible. However, the inequalities as such could be used in computations.

CONCLUSION

We have presented a method for transforming tolerance specifications as per ASME Y14.5 into a generalized coordinate system (deviation space) using the SDT representation and we have indicated possible usage of these methods in tolerance synthesis and related computational works. We have elaborated the method with two examples. In our future work we intend to publish results of analysis using these transformations.

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