Frequency dependence of magnetic flux profile in the presence of metamaterials for wireless power transfer

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Abstract—We discuss the change in the magnetic flux profile by introducing a negative refractive index material (metamaterial) in between the source and receiver. The environment parameters, \( \varepsilon \) and \( \mu \), has a significant effect on the propagation of electromagnetic wave. The behavior of Transverse Magnetic (TM) wave when the medium in the path of propagation is changed to negative permittivity and permeability is simulated and discussed. The effect of size, shape and anisotrophy of the metamaterials, for near-field regions, on the magnetic flux density has been studied using finite element analysis. An enhancement in the magnetic flux density when a metamaterial is introduced in between the source and receiver was observed. The results show that the static and quasi-static behavior of the system is same.

Keywords—metamaterials, quasi static, magnetic flux transverse magnetic

I. INTRODUCTION

The idea of charging on the go is an exciting option for various high power applications like Electric Vehicle. Wireless power charging can be done by radiative or non-radiative processes. Use of microwave and optical frequencies falls into the radiative category while non-radiative process refers to the near-field domain. This concept was put forward by Nikola Tesla when he invented an apparatus for transmitting electrical energy wirelessly [1]. Later, with the advent of microwave transmission technology in 1960’s researchers dreamed power transfer from satellite space station to earth [2]. For short distances inductive coupling is very convenient [3-4]. The enhancement in coupling efficiency is obtained by replacing coils with resonators [5-7]. The efficiency can further be improved by introducing a negative refractive index material between the source and the receiver [8-12]. The negative refractive index material or metamaterial has the unique property of enhancing the evanescent as well as non-evanescent waves [10].

In this paper we present the magnetic flux density variations for quasi-static scenarios when a metamaterial is introduced in between the source and the receiver. The model used for simulation is a 2-dimensional one as we are interested only in the profile in that direction which is in the direction of propagation.

II. THEORY

Our system consists of a source, receiver and a metamaterial as shown in fig. 1. The source is a circular loop of radius ‘a’ located in free space. The receiver is a point of interest ‘P’ where the magnetic flux density enhancement is observed. The metamaterial in between the source and the receiving point is a rectangular block which enhances the magnetic flux density at the point ‘P’. The transmitter is a single turn coil carrying current ‘I’ which in turn generates the magnetic field \( \mathbf{H} \) in the surrounding medium. The magnetic field \( \mathbf{H} \) at a distance ‘z’ from the center of the coil is given by

\[
H = \frac{2Ia^2}{2(a^2+z^2)^2} \tag{1}
\]

The coil is fed with a current of ‘I’ amperes as given by the equation below

\[
I = \int \int \mathbf{J} \cdot d\mathbf{s} \tag{2}
\]

![Fig. 1. Schematic of Wireless Power transfer](image-url)
where, \( \mathbf{J} \) is the current density and \( \mathbf{ds} \) is the vector component perpendicular to the cross section of the circular coil. Then the applied potential \( V \) related to the current density \( \mathbf{J} \) is given by the equation

\[
\mathbf{J} = \frac{\sigma V}{2\pi a}
\]

(3)

where \( \sigma \) is the conductivity of the coil and \( a \) is the radius of the coil.

The direction of propagation is \('z'\), towards the point P in fig. 1. To enhance the flux density the metamaterials of optimized dimension is placed in between the source and the receiver at a distance \('z/2'\) from the center of the transmitter coil as shown in figure 1. The metamaterial is one whose permittivity \( \varepsilon \) and permeability \( \mu \) is negative. The materials can be classified on the values of \( \varepsilon \) and \( \mu \) as dielectric, plasmas, ferri-magnetic or metamaterial [11]. These parameters also constitute the propagation of electromagnetic waves in matter. Since the permittivity and permeability are negative for the metamaterial the refractive index also becomes negative as explained below. The refractive index is related to relative permittivity and relative permeability as mentioned below

\[
n = \sqrt{\varepsilon_r \mu_r}
\]

(4)

If \( \varepsilon_r = -1 \) and \( \mu_r = -1 \) then substituting \( e^{ix} \) for \(-1\) in the above equation gives refractive index as \(-1\).

We can solve the problem using FDTD (Finite Difference Time Domain) or the FEM (Finite Element Method). In both cases, the Maxwell equations are solved by dividing the solution region into nodes or elements considering suitable boundary conditions. We have used COMSOL Multiphysics, an FEM based tool for our simulation. The governing equation for quasi static situation is given by

\[
(j \omega \sigma - \omega^2 \varepsilon) \mathbf{A} + (\nabla \times \mathbf{H}) = \mathbf{J}
\]

(5)

The magnetic flux density related to the magnetic potential and magnetic field intensity are given by

\[
\mathbf{B} = \nabla \times \mathbf{A} = \mu \mathbf{H}
\]

(6)

### III. SIMULATION

#### A. Model

The transmitter is a circular loop of radius 400mm excited with a current of 0.1 A. The coil is made up of copper of radius 1mm. From fig. 1 we can clearly see the structure of the system is symmetry and we are interested in the direction of propagation (\(z\)-direction). So it is sufficient to choose a 2D axisymmetry model for simulation. The metamaterial considered is anisotropic and characterised by uniaxial permittivity and permeability tensors as shown in equation below

\[
[\varepsilon_r] = \begin{bmatrix}
\varepsilon_{xx} & 0 & 0 \\
0 & \varepsilon_{yy} & 0 \\
0 & 0 & \varepsilon_{zz}
\end{bmatrix}
\]

(7a)

\[
[\mu_r] = \begin{bmatrix}
\mu_{xx} & 0 & 0 \\
0 & \mu_{yy} & 0 \\
0 & 0 & \mu_{zz}
\end{bmatrix}
\]

(7b)

The distance between the source and the receiver is at a distance of 500mm. The magnetic flux density between the source and the receiving point of interest without the metamaterial is computed analytically and found to be be \(3.87 \times 10^{-8}\) Tesla. This scenario is simulated using COMSOL and found to be \(3.74 \times 10^{-8}\) Tesla.

#### B. Results and Discussions

The propagation region can be subdivided into near field and far-field. In the far field region the efficiency is related to absorption and scattering by the environment. Also at high power electromagnetic interference becomes a critical issue. In reality propagation can occur by a simple radiative process. The radiative efficiency is reduced by losses due to absorption and scattering by the environment. At high intensities we will also have to deal with electromagnetic interference. In the near field, E/H ratio can be made negligible, thereby not causing any harm to biological and other objects in the vicinity. This in turn becomes a trade-off between the environmental benefits versus distance and efficiency of propagation. The propagation efficiency can be increased by introducing metamaterial in the medium between the source and receiver [8, 12].

Fig. 2 shown below gives the magnetic flux density with when the transmitter coil is excited at a frequency of...
10 MHz without and with the metamaterial. It is clearly seen from the figure that there is an increase in the magnetic flux density when the metamaterial is introduced. Y. Urzhumov et al., [8] have also observed enhancement of transmission of TM polarized wave i.e. magnetic field normal to propagation direction. Fig. 3 shows the relation between the size of the metamaterial and magnetic flux density. As the size of the material is increased radially in steps of 10mm we get a peak at a specific dimension. The same is observed when the material size is varied axially as seen in fig. 3. Similarly the distance between the material and receiver, size of the material, anisotropy characteristics places a major role in enhancing the flux density [12]. Fig. 4 shows the magnetic flux density measured when the degree of anisotropy in the direction of propagation $\varepsilon_{zz}/\mu_{zz}$ are varied from -1 to 1 in steps of 0.1. From the graph we can conclude that the magnetic flux density is high when $\varepsilon_{zz}$ and $\mu_{zz}$ are -1.

Y. Urzhumov et al., [8] suggests that the transverse wave propagation properties of a metamaterial slab of thickness ‘d’ and anisotropy ‘$\alpha_{TE}$’ are indistinguishable due to the invariance of the Maxwell’s equation, with respect to the coordinate transformation. This simply states that the metamaterial slab thickness can be reduced without effective change in performance. This is possible if the magnetic loss tangent is kept constant while increasing the negative real part of ‘$\mu$’.

Fig. 5 shows the magnetic flux density is almost same from dc to 20 MHz. This gives an impression that the static and the quasi static behaviour of the system is same. This system can be used for High frequency RFID system (13.56 MHz) for enhancing the range and act as a power source for charging the wireless devices in the Near-field regime.

G. Shvet et al. [13], state that, for a single split ring resonator, SSR, of a metamaterial, the effective permeability approximately is

$$\begin{align*}
\mu_{eff} &= 1 - F \omega^2 / \omega^2 - \omega_M^2 - i \omega \Gamma \\
\epsilon_{eff} &= 1 - (\omega_p^2 - \omega_E^2) / \omega^2 - \omega_M^2 + i \omega \Gamma
\end{align*}$$

where $F$ is the fractional area of a unit cell occupied by the SRR, $\omega_M$ is the magnetic resonance frequency, and $\Gamma$, is the resistive loss coefficient, $\omega_p$ is the characteristic ‘plasma’ frequency and $\omega_E$ is the cut-wire resonance frequency. The filling factor $F$ is typically kept small to avoid strong interaction between adjacent unit cells. Developing a metamaterial may require finding a resonant structure that has adjacent electric and magnetic resonances.

For a periodic structure the effective permeability will reduce to [14]

$$\mu_{eff} = \mu_0 \left[ 1 - (0.47)^2 / \{ 1 - (1.05)^2 / f^2_GHz \} \right]$$

where $\epsilon_{eff} = 1.8 \epsilon_0$. Here it is possible to show that a complex NIM structure can be equivalent to a uniform medium. The effective ($\epsilon_{eff} - \mu_{eff}$) of the medium will provide the width of the band around resonance. Here the losses are ignored, but from practical point it has a major impact. At the resonance, a very strong current is circulated in the loops and generates a considerable conduction loss.
These are the issues that have to be resolved for practical purposes.

IV. CONCLUSION

There is an enhancement in the magnetic flux density when a metamaterial is introduced in between the source and receiver. This enhancement depends on the size, shape and anisotropy characteristics of the material. The above result implies that the static and quasi-static behavior of the system are same. Thus, the metamaterial can be used in between the source and receiver to enhance the range, which in turn increase the efficiency. It is also possible to further enhance the range if the source can be replaced with a resonator instead of the circular coil [6, 7]. This has immense possibility of application in high power energy transfer like electric vehicle wireless charging. An experimental setup is being fabricated and the results will be discussed in future. We are also looking at micro and nano-array architecture for the negative refractive index material. Preliminary results have been discussed elsewhere [15].

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