Pilot Design for MIMO OFDM Systems with Virtual Carriers

Qinfei Huang, Mounir Ghogho and Steven Freear

Abstract—We consider the problem of pilot design for channel estimation in multiple-input multiple-output (MIMO) orthogonal frequency division multiplexing (OFDM) systems in the presence of virtual carriers. The design criterion is the average mean-square error (MSE) of the least square (LS) estimates of the channel frequency response over the data carriers. To maximize bandwidth efficiency, the number of pilots is set to its minimum which is equal to the number of unknown channel coefficients. Both the phase shift orthogonal and disjoint pilot schemes are investigated. For a given pilot placement, a closed-form expression for the pilot power distribution for the disjoint scheme is provided. It is found that the optimal power distribution allocates less power to the pilots that are close to the virtual carriers. Pilot placement is obtained using an exhaustive search. The latter is too complex in systems with large number of subcarriers. A suboptimum pilot placement design for the disjoint pilot scheme is presented. It is shown that, unlike fully loaded systems where the two pilot design schemes have similar performance, in the presence of virtual carriers the optimal disjoint scheme not only has lower complexity but also yields more accurate channel estimation and bit error rate performance.

Index Terms—Channel estimation, MIMO, OFDM, pilot design, virtual carriers, disjoint pilot scheme

I. INTRODUCTION

Channel estimation is a critical task in coherent communication systems. Judicious pilot design for channel estimation not only provides better estimation performance but also reduces the resources allocated to training. This is particularly true in the case of fast fading channels where pilots have to be inserted in each OFDM block in order to track channel variations. Pilot design for OFDM systems has therefore attracted a lot of attention (see e.g. [1]–[4]). More recently, pilot design for MIMO-OFDM was addressed in [5]–[8], among others. In [6], equipowered and equispaced phase shift orthogonal pilot design is proposed to achieve the minimum MSE on channel estimation. More general design schemes, such as the code-division multiplexed and equispaced equipowered disjoint schemes, were proposed in [8]. However, most of the pilot design assumes a fully loaded OFDM system, i.e. all carriers are activated. Such an assumption often does not hold true in practice. In order to avoid data being distorted by non-ideal filters, some carriers at the edges of the spectrum are deactivated in practical OFDM systems. The number of these virtual carriers is dictated by system design and is around 10% of the total number of carriers. The optimal pilot sequences designed for fully loaded systems are not suitable for such systems.

Considering the effect of placement of equipowered pilot carriers on channel estimation performance, some results were drawn in [4] and [9] for SISO OFDM with virtual carriers. The work in [4] showed that the minimum MSE can be achieved with non-uniformly spaced pilot sequences and pointed out that the optimal placement can only be determined through exhaustive search. By using the asymptotic equivalence between Toeplitz and associated circulant matrices, [9] showed that, given a number of pilot carriers, pilot sequences with the largest distance between adjacent pilot carriers provide the best channel estimate within the class of partially equispaced and equipowered pilot design. In [10], the placement of equipowered pilot carriers was optimized for MIMO OFDM systems in the presence of virtual carriers. Since the design proposed in [10] minimized the Frobenius norm of a matrix relevant to the MSE instead of minimizing the MSE itself, the obtained placement is sub-optimal. Power allocation is another important issue for pilot design. Unlike [10] which only optimized pilot placement for MIMO OFDM systems, both placement and power allocation were discussed in [11], where the optimization problem was formulated as a linear programming (LP) problem. In [11], the optimal pilot design was obtained by an iterative gradient method. However, the bandwidth efficiency was not maximum because the minimum number of pilot carriers was supposed to be nearly twice as large as the number of unknown channel coefficients. Moreover, the iterative method had the strict constraint that a proper initial matrix was needed for each pilot placement.

In this paper, we revisit the problem of pilot design for channel estimation in MIMO OFDM systems with virtual carriers. Since equalization in OFDM is carried out in the frequency domain, we propose to minimize the MSE of the frequency domain channel estimation at the data carriers, unlike [9]–[11] where the MSE of the time domain channel estimation was minimized. Both placement and power distribution are investigated. Based on the disjoint pilot scheme, we show that the dimension of the optimization problem can be reduced by applying the lagrange multiplier method to derive the optimal power distribution in closed-form.

The rest of this paper is organized as follows. In Section II, we briefly introduce the signal model of MIMO OFDM systems. The optimal pilot design is discussed in Section IV, and the optimal power distribution for the disjoint scheme is
II. SIGNAL MODEL

Consider a MIMO OFDM system with $N_t$ transmit and $N_r$ receive antennas. Since the same channel estimation procedure is performed at each receive antenna, we only need to consider one receive antenna when designing optimum pilot sequences.

We model the frequency selective channels as FIR filters with channel impulse response (CIR) $h_i = [h_i(0), \ldots, h_i(L-1)]^T$, where $i$ is the transmit antenna index, $L$ is the length of the longest CIR of the transmit-receive antenna pairs. Let $N$ denote the total number of carriers, $N_p$ (resp. $N_d$) denote the number of pilot (resp. data) carriers. Since we focus on block-by-block processing, we omit the block index in what follows. To avoid inter-block interference, we assume that each OFDM block is preceded by a cyclic prefix (CP) whose minimum length is $L - 1$. After removing the CP and performing FFT, the received signal at the data and pilot carriers can be modeled as

$$
x(d_n) = \sum_{i=1}^{N_t} H_i(d_n) S_i(n) + v(d_n), \quad n = 0, \ldots, N_d - 1; \quad d_n \in \mathcal{D}$$

$$
x(p_m) = \sum_{i=1}^{N_t} H_i(p_m) C_i(m) + v(p_m), \quad m = 0, \ldots, N_p - 1; \quad p_m \in \mathcal{P}$$

where $\mathcal{D}$ denotes the subset of data carriers, $\mathcal{P}$ denotes the subset of pilot carriers, $S_i(n)$ is the $n$th data symbol transmitted from the $i$th transmit antenna, $C_i(m)$ is the $m$th pilot symbol transmitted from the $i$th transmit antenna, $v(n)$ is an AWGN with variance $\sigma_v^2$ and $H_i(n) = \sum_{l=0}^{L-1} h_i(l) e^{-j \frac{2\pi}{N} n l}$.

The received pilot symbols can be re-expressed in vector form as

$$
x_P = C(I_{N_t} \otimes \tilde{W}_P) h + v_P$$

where $x_P = [x(p_0), \ldots, x(p_{N_p-1})]^T$, $C = [C_1, C_2, \ldots, C_{N_t}]$, $C_i = \text{diag}(C_i(0), C_i(1), \ldots, C_i(N_p-1))$, $h = [h_1^T \ldots h_{N_t}^T]^T$, $v_P = [v(p_0), \ldots, v(p_{N_p-1})]^T$ and $\tilde{W}_P$ denotes the sub-matrix of $W$, obtained from the $P$ rows and first $L$ columns of $W$.

III. CHANNEL ESTIMATION AND MSE

From eq. (3), the LS estimator of $h$ is given by

$$
\hat{h} = \left( ( I_{N_t} \otimes \tilde{W}_P^H ) C^H C ( I_{N_t} \otimes \tilde{W}_P ) \right)^{-1} \left( I_{N_t} \otimes \tilde{W}_P^H \right) C^H x_P
$$

The LS estimate of $H_i(n)$ is

$$
\hat{H}_i(n) = w_n \hat{h}_i
$$

where $w_n = W(n,:)$. The channel identifiability condition is

$$\text{rank} \{ C(I_{N_t} \otimes \tilde{W}_P) \} = N_t L$$

which requires (necessary condition)

$$N_p \geq N_t L$$

For the purpose of maximizing the bandwidth efficiency, we set $N_p = N_t L$ in this paper.

The MSE of the LS channel estimate is given by

$$\Sigma_h := E \left\{ \| \hat{h} - h \|^2 \right\} = \sigma_v^2 \text{Tr} \{ \Pi \}$$

where $\Pi := \left( ( I_{N_t} \otimes \tilde{W}_P^H ) C^H C ( I_{N_t} \otimes \tilde{W}_P ) \right)^{-1}$.

Since equalization for OFDM-based systems is performed in the frequency domain, we propose to minimize the MSE of the frequency domain channel estimates at the data carriers instead of minimizing $\Sigma_h$. The average of the MSEs of $\hat{H}_i(n)$, $n \in \mathcal{D}$, can be expressed as

$$\gamma := \frac{\sigma_v^2}{N_t N_d} \text{Tr} \left\{ \left( I_{N_t} \otimes \tilde{W}_P \right) \Pi \left( I_{N_t} \otimes \tilde{W}_P^H \right) \right\}$$

IV. PILOT DESIGN

We assume that the total transmit powers from the different antennas are equal. The same assumption is made on the total powers allocated to the pilot carriers. In this section, we first address the case of fully-loaded systems, then we address the more general case of systems with virtual carriers. Although fully-loaded have been well studied in the literature, here we will show that the existing designs are still optimum when using the MSE on the estimation of the frequency response of the channel at the data carriers as a criterion.

A. Pilot design for fully loaded systems

The optimization problem for fully loaded systems can be expressed as

$$\rho^* = \arg \min_{\rho^P} \gamma$$

under the constraints

$$\mathcal{P} \subset \{0, \ldots, N-1\} \text{ and } \sum_{m=0}^{N_p-1} \rho_i(m) = \sigma_p^2$$

for $i = 1, \ldots, N_t$, where $\rho = [\rho_1, \ldots, \rho_{N_t}]$, $\rho_i = [\rho_i(0), \ldots, \rho_i(N_p-1)]$ and $\rho_i(m) = |C_i(m)|^2$ is the power of the $m$th pilot transmitted from the $i$th antenna, and $\sigma_p^2$ is
the total power of the pilot carriers of each OFDM block and from each transmit antenna.

For any $M \times M$ positive-definite matrix $B = \{bk,l\}_{k,l=0}^{M-1}$, the following result is derived in [12].

$$\text{Tr}\{B^{-1}\} \geq \sum_{m=0}^{M-1} \frac{1}{b_{m,m}}$$

with equality if and only if $B$ is diagonal. Applying this result to eq. (6), we obtain

$$\gamma \geq \frac{\sigma^2}{\bar{P}} L$$

with equality if both $\Pi$ and $\hat{W}_D^H W_D$ are diagonal matrices. Assuming that $N$ is an integer multiple of $N_p$, we next give two pilot schemes that achieve the minimum MSE.

#a: Phase shift orthogonal equispaced equipowered pilot design. Using the time-domain channel MSE, this scheme was proposed and shown to be optimal in [6]. In this scheme, we have that

$$C_i(m) = \sqrt{\frac{\sigma^2}{N_p}} e^{-j\frac{2\pi m}{N_p}} \quad m = 0, \ldots, N_p - 1$$

where $i = 1, \ldots, N_t$ and $n_i = (i-1)L$, $P = \{t + nQ \mid n = 0, \ldots, N_p - 1, Q = N/N_p, t \in [0, Q-1]\}$. In this scheme, each antenna transmit non-zero pilot symbols over the $N_t L$-element set of pilot carriers.

#b: Disjoint equispaced equipowered pilot sequences. This pilot scheme can be described as follow.

$$|C_i(m)|^2 = r_i(m) \frac{\sigma^2}{\bar{P}} L \quad m = 0, \ldots, N_p - 1$$

where $i = 1, \ldots, N_t$ and

$$r_i(m) = \begin{cases} 1, & P(m) \in P_i \\ 0, & \text{others} \end{cases}$$

$P_i$ denotes the active pilot (not nulled) at the $i$th transmit antenna, $P_i = \{t_i + nQ \mid n = 0, \ldots, L-1, Q = N/L, t_i = t_{i-1} + Q/N_t\}$, and $t_0 \in [0, Q/N_t - 1)$, $P_i \cap P_j = \emptyset$, $P = (P_1 \cup P_2 \cup \cdots \cup P_{N_t})$. In this scheme, the antennas transmit non-zero pilot symbols over disjoint $L$-element carrier sets.

Fig. 1 illustrates pilot schemes #a and #b with two transmit antennas. Hence, in the case of fully-loaded systems, the two design schemes provide the same estimation accuracy and achieve the maximum bandwidth efficiency. It is worth pointing out that the above optimal design schemes also minimize the time-domain channel MSE $\Sigma_k^*$, which is not the case for systems with virtual carriers.

B. Optimal design for systems with virtual carriers

In the presence of virtual carriers, matrix $\Pi$ may be diagonalized if $L$ is much smaller than the number of virtual carriers. However, matrix $\hat{W}_D^H W_D$ cannot be diagonal. This implies that in the presence of virtual carriers, a design that minimizes the average time-domain channel MSE does not necessarily minimize the average frequency-domain channel MSE.

Solving the optimization problem (7) analytically does not seem tractable. Numerical optimization is possible but computationally inefficient because $\rho$ is a continuous-valued vector. However, the optimization problem can be simplified in the case of the disjoint pilot scheme as follows.

By using the disjoint scheme and reorganizing the receiving pilot symbols $x_P$, the LS channel estimate can be re-expressed as

$$\hat{h}_i = \left(\hat{W}_D^H A_i \hat{W}_P\right)^{-1} \hat{W}_P C_i \hat{W}_P x_P, \quad i = 0, \ldots, M-1$$

and eq. (6) can be written as

$$\gamma := \frac{\sigma^2}{N_t N_d} \sum_{i=1}^{N_t} \text{Tr}\left\{ \hat{W}_D^H \left(\hat{W}_P A_i \hat{W}_P\right)^{-1} \hat{W}_D^H \right\}$$

where $A_i = \text{diag}\{\hat{\rho}_i(0), \hat{\rho}_i(1), \ldots, \hat{\rho}_i(L-1)\}$; $C_i = \text{diag}\{C_i,0, C_i,1, \ldots, C_i,L-1\}$, $C_i,n$ is the $n$th active pilot transmitted from the $i$th antenna, and $\hat{\rho}_i(n)$ is the power allocated to $C_i,n$; $x_P = [x_P(0), \ldots, x_P(L-1)]^T$, $p_{i,j} \in P_i$.

Since $\hat{W}_P$ and $A_i$ are non-singular square matrices, the optimization problem can be re-expressed as

$$\left(\hat{\rho}^o, \bar{\rho}^o\right) = \arg\min_{\hat{\rho}, \bar{\rho}} \sum_{i=1}^{N_t} \sum_{n=0}^{L-1} \frac{\Psi_i(n,n)}{\hat{\rho}_i(n)}$$

under the constraints

$$\bar{\rho} = (\bar{P}_1 \cup \bar{P}_2 \cup \cdots \cup \bar{P}_N_t) \subset A \quad \text{and} \quad \sum_{i=0}^{L-1} \hat{\rho}_i(n) = \sigma^2$$

for $i = 1, \ldots, N_t$, where $\Psi_i = \hat{W}_P^H \hat{W}_D^H W_D \hat{W}_P^{-1}$, and $A = \{0,1, \ldots, \frac{N_p}{2}-1, N - \frac{N_p}{2}, \ldots, N - 2, N - 1\}$ denotes the subset of active carriers, $N_a = N_p + N_d$ is the number of active carriers.

The Lagrange multiplier method is employed to solve the above optimization problem with respect to $\rho$ under the power constraint. The Lagrangian $L$ can be constructed as

$$L_i(\lambda_i, \hat{\rho}_i) = \sum_{n=0}^{L-1} \frac{\Psi_i(n,n)}{\hat{\rho}_i(n)} + \lambda_i \left(\sum_{n=0}^{L-1} \hat{\rho}_i(n) - \sigma^2\right)$$

for $i = 1, \ldots, N_t$, where $\lambda_i = [\lambda_i(0), \lambda_i(1), \lambda_i(L-1)]^T$, and the $\lambda_i$’s are unknown scalars. Taking the gradients of $L_i$ with respect to $\rho_i$, we obtain the following equations.

$$\nabla \hat{\rho}_i(0) L_i = 0$$

$$\vdots$$

$$\nabla \hat{\rho}_i(L-1) L_i = 0$$

Since $\Psi_i = \left(\hat{W}_D^H \hat{W}_P\right)^H \hat{W}_D \hat{W}_P^{-1}$, we know the diagonal elements of matrix $\Psi_i$ are positive. The optimal power
distribution can be readily derived as
\[
\hat{\rho}_i(n) = \sigma_p^2 \frac{\sqrt{\Psi_i(n,n)}}{\sum_{m=0}^{L-1} \sqrt{\Psi_i(m,m)}} \quad n = 0, \ldots, L-1
\]  \hspace{1cm} (14)

Using \(\hat{\rho}_i\), and after removing irrelevant terms, the optimal pilot placement is found to be
\[
\hat{\rho}^o = \arg \min_{\rho \in A} \sum_{i=1}^{N_t} \left( \sum_{n=0}^{L-1} \sqrt{\Psi_i(n,n)} \right)^2
\]  \hspace{1cm} (15)

Hence, the dimension of the optimization problem is reduced from \((\hat{\rho}, \hat{\rho})\) to \(\hat{\rho}\). It is worth pointing out that this reduction of the dimension of optimization is not possible for the non-disjoint pilot schemes such as the phase shift orthogonal design. Moreover, \(\tilde{W}_{P_i}\) is a non-square matrix if \(N_p > N_t L\), and the optimization is intractable in this case. To overcome this drawback, boosted pilots (allocating more power to pilots) can be adopted instead of using more pilot carriers. The advantages of boosted pilots have been discussed in [13].

According to eq. (15), an exhaustive search over all \(N_p\)-pilot subsets of \(A\) or other more efficient discrete search methods (such as grid search) can be used to find \(\hat{\rho}^o\). Since the complexity of the discrete exhaustive search is \(\varphi = \tau (N_a - N_p)^{N_t (L/d)} N_r^4\), where \(\tau\) is a scalar coefficient that represents the computational complexity per placement candidate, this computer exhaustive search is efficient for small \(N_a\) or \(N_p\), but it becomes unacceptable for large values even if the search process is carried out during the system design phase. It is therefore valuable to develop a suboptimal pilot placement scheme, which is what we address next.

C. Suboptimal placement for disjoint pilot schemes

Recalling eq. (15), we have
\[
\sum_{i=1}^{N_t} \left( \sum_{n=0}^{L-1} \sqrt{\Psi_i(n,n)} \right)^2 \leq L \sum_{i=1}^{N_t} \{\Psi_i\}
\]  \hspace{1cm} (16)

Since both matrices \(\tilde{W}_{D}^H \tilde{W}_{D}\) and \((\tilde{W}_{P_i}^H \tilde{W}_{P_i})^{-1}\) are positive semi-definite, we get
\[
\{\Psi_i\} \leq \{\tilde{W}_{D}^H \tilde{W}_{D}\} \{\tilde{W}_{P_i}^H \tilde{W}_{P_i}\}^{-1}
\]  \hspace{1cm} (17)

Using (17) and \(\{\tilde{W}_{D}^H \tilde{W}_{D}\} = L N_d\), we replace (16) by the following inequality
\[
\sum_{i=1}^{N_t} \left( \sum_{n=0}^{L-1} \sqrt{\Psi_i(n,n)} \right)^2 \leq L^2 N_d \sum_{i=1}^{N_t} \{\tilde{W}_{P_i}^H \tilde{W}_{P_i}\}^{-1}
\]

After neglecting irrelevant terms, the suboptimal placement is given by
\[
\hat{\rho}^o = \arg \min_{\rho \in A} \sum_{i=1}^{N_t} \{\tilde{W}_{P_i}^H \tilde{W}_{P_i}\}^{-1}
\]  \hspace{1cm} (18)

It can be seen that, when \(N_t = 1\), eq. (18) reduces to the optimization problem discussed in [9] for SISO OFDM systems. However, the multiple-antenna case has not been addressed in [9]. Considering the class of partially equispaced schemes as illustrated in Fig. 2, it was shown in [9] that the pilot sequences with the largest distance between adjacent carriers achieve the minimum \(\text{Tr} \{\tilde{W}_{P_i}^H \tilde{W}_{P_i}\}^{-1}\). For our MIMO-OFDM problem, the suboptimal placement of eq. (18) within the class of partially equispaced schemes can be found using the following steps.

Step 1) Let \(\overline{P}_i\) denote the set of \(L\) partially equispaced pilots allocated to \(i\)th antenna. For initialization, we let \(d = \lfloor N_a/(L-1) \rfloor\), \(i = 1\) and \(A_1 = A\). Choose a subset \(\overline{P}_i \subset A_1\) within partially equispaced schemes with distance \(d\).

Step 2) Let \(A_{i+1} = A_i - \overline{P}_i\), find a subset \(\overline{P}_{i+1} \subset A_{i+1}\) within partially equispaced pilot schemes with distance \(d\).

Step 3) If no \(L\) partially equispaced pilots with distance \(d\) exist in \(A_{i+1}\), replace \(d\) by \(d - 1\) and goto step 2); else let \(i = i + 1\) and goto step 4).

Step 4) Repeat Step 2) and 3) until \(i = N_t\).

It is worth pointing out that the suboptimal placement is not unique. Fig. 3 shows a suboptimum placement obtained by the above-mentioned method and optimum power distribution in eq. (14). It can be seen that the pilot symbol power \(\hat{\rho}_i(n)\) decreases when the pilot is close to the virtual carriers zone. This can be explained by the fact that there are less data carriers in this zone and therefore when the criteria is the averaged MSE over the data carriers, one should assign more power to the pilot carriers that are surrounded by more data carriers.

V. SIMULATION RESULTS AND COMPLEXITY ANALYSIS

We assume that the channels of different transmit-receive antenna pairs are independent, and the \(h_i(l)\)'s are uncorrelated zero-mean Gaussian random variables with exponential power delay profile \(E\{|h_i(l)|^2\} = C \exp(-0.2l), \ l = 0, \ldots, L - 1\), where \(C\) is a scaling factor that ensures that \(\sum_{l=0}^{L-1} E\{|h_i(l)|^2\} = 1\). In our simulation setup, we consider a V-BLAST OFDM uncoded system with two transmit antennas and four receive antennas.

A. Simulation Results

Since the optimal placement for small number of carriers can be easily obtained by using exhaustive search, we focus on

\[\text{The subset of active carriers is } \{-N_a/2, -N_a/2 + 1, \ldots, N_a/2 - 1\}\]  \hspace{1cm} (1)}

for illustration purpose.
the systems with large number of carriers, where the optimal placement is untractable. We consider an OFDM system with $N = 1024$ carriers, $N_a = 640$ of which are active while the other carriers are unused. The data carriers are modulated by QPSK symbols.

In our simulations, we evaluate the performance for the following pilot designs.

1) Phase shift orthogonal sequences with equipower distribution and conventional equispaced placement (PS). The conventional equispaced placement is given by

$$\mathcal{P} = \left\{ \frac{d}{2}, \frac{3d}{2}, \ldots, \frac{(N_p - 1)d}{2}, N - \frac{(N_p - 1)d}{2}, \ldots, N - \frac{3d}{2}, N - \frac{d}{2} \right\}$$

where $d = \lfloor N_a/(N_p - 1) \rfloor$. Such conventional placement has been adopted by many protocols, e.g. IEEE 802.16d. The pilot placement subset $\mathcal{P} = \{21, 63, 105, 147, 189, 231, 273, 315, 709, 751, 793, 835, 877, 919, 961, 1003\}$ when $L = 8$ and $\mathcal{P} = \{10, 30, 50, 70, 90, 110, 130, 150, 170, 190, 210, 230, 250, 270, 290, 310, 714, 734, 754, 774, 794, 814, 834, 854, 874, 894, 914, 934, 954, 974, 994, 1014\}$ when $L = 16$.

2) Disjoint pilot sequences with equipower distribution and proposed suboptimal placement (DE). The suboptimal placement $\tilde{\mathcal{P}}_1 = \{44, 135, 226, 317, 704, 795, 886, 977\}$ and $\tilde{\mathcal{P}}_2 = \{45, 136, 227, 318, 705, 796, 887, 978\}$ for $L = 8$; $\tilde{\mathcal{P}}_1 = \{16, 58, 100, 142, 184, 226, 268, 310, 704, 746, 788, 830, 872, 914, 956, 998\}$ and $\tilde{\mathcal{P}}_2 = \{17, 59, 101, 143, 185, 227, 269, 311, 705, 747, 789, 831, 873, 915, 957, 999\}$ for $L = 16$.

3) Disjoint pilot sequences with optimal power distribution and proposed suboptimal placement (DO). The suboptimal placement is the same as DE.

Fig. 4 and 5 show the average MSE performance of channel estimation and the BER performance of the system. The pilot power $\sigma_p^2$ per antenna is set to 2.5% of the total transmit power $\sigma_T^2$ when $L = 8$ and $\sigma_p^2 = 5% \sigma_T^2$ when $L = 16$. Compared to the conventional placement, the proposed suboptimal placement can provide approximately 5dB and 20dB gains when $L = 8$ and $L = 16$, respectively. The performance of the disjoint optimally powered design (DO) has a slight gain compared to the equipowered disjoint design (DE) when $L = 8$, and it has nearly 2dB gain over the DE when $L = 16$. Such slight gain when $L = 8$ can be ascribed to the fact that optimal power distribution is close to the equipower distribution. Using the BER as a metric, the pilot scheme with the conventional placement (PS) was again outperformed by the other pilot schemes; we can see a slight gain in $E_s/N_o$ for the disjoint scheme with optimal power distribution over DE at a BER of $10^{-4}$ when the CIR length is 8 and 2dB gain when $L = 16$, and the performance gains increase with the $E_s/N_o$. 

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**Fig. 3.** Optimal power distribution for suboptimal placement (disjoint pilot scheme); $N_t = 2$, $\sigma_p^2 = 32$

**Fig. 4.** The averaged MSE of evaluated schemes versus $E_s/N_o$

**Fig. 5.** The BER performance of evaluated schemes versus $E_s/N_o$
such as peak-to-average-power ratio. In the disjoint scheme, it is possible to add further constraints, phase constraint between different pilot symbols and antennas coefficients for the disjoint scheme. Further, since there is no need to store at the receiver as opposed to only $N_t L^2$ coefficients for the disjoint scheme. Further, since there is no phase constraint between different pilot symbols and antennas in the disjoint scheme, it is possible to add further constraints, such as peak-to-average-power ratio.

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B. Complexity of Channel Estimation

We briefly compare the complexities of the LS channel estimate for the disjoint and non-disjoint schemes. Considering the number of complex multiplications as a complexity metric, the inversion of an $n \times n$ matrix requires $O(n^3)$ operations, and the product of an $m \times r$ matrix with a $r \times n$ matrix requires $O(mnr)$ operations.

In the case of fully-loaded systems, for the optimal design schemes #a, the LS channel estimate in (4) is greatly simplified since matrix $\Pi = 1/\sigma_p^2 I_{N_t L}$. For the disjoint scheme #b, a further simplification is obtained since the MIMO estimation problem is reduced to $N_t$ SISO estimation problems; see eq. (12). For systems with virtual carriers, eq. (4) and (12) cannot be simplified. However, if the pilot design is fixed across the OFDM blocks, the matrices that multiply $x_{\mathbf{r}}$ and $x_{\mathbf{r}'}$, in eq. (4) and eq. (12), respectively, can be computed offline and stored in memory, and thus the complexity of channel estimation for systems with virtual carriers is the same as that of fully-loaded systems. However, if the pilot design scheme varies across the OFDM blocks, these matrices have to be computed online and hence complexity of channel estimation is higher than that of fully-loaded systems provided optimality is maintained in the latter. A varying design scheme may be relevant in cognitive radio for example where the available carriers depend on the activities of the primary users. Table I shows the computational complexities of LS channel estimation in different scenarios. The complexity advantage of the disjoint scheme becomes more pronounced when $N_t$ is large. Moreover, for the fixed pilot scheme, the non-disjoint design has a higher memory requirement since $N_t^2 L^2$ coefficients need to be stored at the receiver as opposed to only $N_t L^2$ coefficients for the disjoint scheme. Further, since there is no phase constraint between different pilot symbols and antennas in the disjoint scheme, it is possible to add further constraints, such as peak-to-average-power ratio.

VI. CONCLUSIONS

In this paper, we addressed the problem of pilot design for channel estimation in MIMO OFDM systems with virtual carriers. Both placement and power distribution were investigated. A closed-form expression for the power distribution and a suboptimal placement solution for the disjoint pilot scheme were provided. The simulation results show that the proposed disjoint pilot design not only achieves smaller MSE on channel estimation and bit error rate than the conventional phase shift orthogonal design, but also has a lower channel estimation complexity. The proposed pilot design is particularly useful in large-array MIMO OFDM systems.

REFERENCES