On the Optimality of Frequency-Domain Equalization in DFT-Spread MIMO-OFDM Systems

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Abstract—In this paper, we compare linear receivers for DFTspread MIMO-OFDM Systems. It is proven mathematically and illustrated by numerical simulations that the advanced DFT RAKE receiver with linear multiuser detection is equivalent to the popular frequency-domain equalization (FDE) with IDFT despreading. This result holds also for multi-antenna systems. As a consequence, broadband wireless transmission chains with high diversity can be implemented with low complexity at the receiving end. The benefit of using the minimum mean-square error (MMSE) criterion in the FDE is also illustrated.

I. INTRODUCTION

Orthogonal frequency-division multiplexing (OFDM) is used in the next generation cellular systems. In some transmission chains, OFDM is combined with precoding based on the digital Fourier transform (DFT). The resulting signal is mapped on localized or distributed bunches of OFDM subcarriers in order to realize multiuser and multipath diversity. This is called single-carrier frequency-division multiple access (SCFDMA) [1]. In this paper, we consider the case where the DFT precoding covers a block of N subcarriers in a given spectral range. The energy of a data symbol is dispersed in frequency domain by multiplication with a sequence taken out of the N-DFT matrix. This process can be considered as DFT spreading. After transmission over a multipath channel, the data signal is optimally recovered in the RAKE receiver, where the multipath components are isolated and coherently combined in order to exploit the diversity of the channel [2]

DFT spreading is a special form of multycarrier code-division multiple access (MC-CDMA) which has been widely studied throughout the recent years [3], [4]. The RAKE receiver can be derived from the maximum-likelihood sequence estimator (MLSE) for MC-CDMA and extensions to multiple antennas are possible [5]. Typically, there is crosstalk between the resources after the RAKE when multiple sequences are transmitted in parallel over a multipath channel. This crosstalk can be removed by linear multuser detection (see e.g. [6]), but the diversity advantage comes at the price of substantially increased computational complexity.

In the literature, a second approach is based on FDE and subsequent inverse DFT. It captures multipath diversity as well, in particular when the equalizer is based on the well-known MMSE criterion. Although this is evident in numerous simulation results, e.g. [7], the relation between both, the optimal RAKE and the FDE-based receivers has not yet been clear.

We demonstrate here that there is an exact equivalence between the advanced DFT RAKE Receiver and FDE with subsequent IDFT which holds in the particular case when DFT spreading is applied. Hence, the optimal properties of the advanced RAKE can already be realized with much simpler receiver structures. Based on DFT spreading, one can establish broadband wireless transmission chains with high diversity using reduced computational effort at the receiving end. This result holds also for multiple antennas. It may be of interest for multimedia broad- and multicast services (MBMS) and also for broadband vehicular communication systems at highest velocities. For both applications, we assume that instantaneous feedback on the fast fading channel cannot be provided.

The paper is organized as follows: In section II the signal model is presented. The two receiver structures are studied in sections III and IV. The equivalence of both receivers is stated in section V and proven in the Appendix. Numerical results are reported in Section VI.

II. SIGNAL MODEL

A two-antenna transmitter based on DFT spreading is given in figure 1. The data stream is demultiplexed, modulated onto mQAM or mPSK constellations and spread using DFT sequences. The sum of spread signals is mapped as a block onto a set of OFDM subcarriers and converted to the time domain using IFFT. In the signal model, the DFT is expressed as a Vandermonde matrix

\[ W = \begin{bmatrix} \omega_N^{1} & \omega_N^{2} & \ldots & \omega_N^{N} \\ \omega_N^{2} & \omega_N^{2} & \ldots & \omega_N^{2N} \\ \vdots & \vdots & \ddots & \vdots \\ \omega_N^{N} & \omega_N^{2N} & \ldots & \omega_N^{N} \end{bmatrix} \quad (1) \]

where

\[ \omega_N = e^{-2\pi i/N} \quad (2) \]

With n and m as row- and column-index, entries in W read

\[ W_{n,m} = e^{-2\pi i (\frac{n-1}{N}(m-1))} \quad (3) \]

The matrix W is orthogonal but not unitary. The inverse is

\[ W^{-1} = \frac{1}{N} \cdot W^H \quad (4) \]

Spreading is done by DFT of the data symbol vector

\[ \vec{x}_n = \frac{1}{\sqrt{N}} \sum_{m=1}^{N} W_{n,m} \cdot \vec{d}_m \quad (4) \]
where the arrow indicates the signal and data vectors $\bar{x}_n$ and $d_{mn}$ containing all signal components in the spatial domain, respectively. We have introduced a normalization constant $1/\sqrt{N}$ to ensure that the in- and output signals have the same power. The received signal is given in frequency domain as

$$\hat{y}_n = H_n \cdot \bar{x}_n + \nu_n \tag{5}$$

where $H_n$ is the channel matrix with the channel coefficients between all transmit-receive antenna pairs.

III. THE ADVANCED DFT RAKE RECEIVER

RAKE receivers can be derived for any waveform using the well known maximum likelihood sequence estimation (MLSE) approach. This has been shown for block-wise WCDMA [8], continuous multicode WCDMA [9] and multicode MC-CDMA [5]. Note that all these results can be applied to multiple transmit and receive antennas which is important to achieve high spectral efficiency. The MC-CDMA multiantenna receiver from [5] can in fact be reused for DFT-spread OFDM by replacing the binary sequence with a DFT-based one. Following equation (A.7) in [5], the first stage of the DFT RAKE produces the sufficient statistics vector

$$\bar{e}_m = \frac{1}{\sqrt{N}} \sum_{n=1}^{N} \bar{W}_{n,m}^* \cdot H_n^H \cdot \hat{y}_n \tag{6}$$

having one signal component per antenna at the transmitter, $\bar{W}_{n,m}$ denotes the conjugate transpose of $W_{n,m}$. Using (5) and (4) in (6), the output of the first RAKE receiver stage can be decomposed into three parts: The sufficient statistics vector $\bar{e}_m$, the effective channel $G_{m,o}$ and an effective noise term $\nu_m$

$$\bar{e}_m = \frac{1}{N} \sum_{o=1}^{N} \sum_{n=1}^{N} \bar{G}_{m,o} \cdot \bar{d}_o + \nu_m \tag{7}$$

$$G_{m,o} = \sum_{n=1}^{N} \bar{W}_{n,m}^* \cdot H_n^H \cdot H_n \cdot W_{n,o} \tag{8}$$

$$\nu_m = \frac{1}{\sqrt{N}} \sum_{n=1}^{N} \bar{W}_{n,m}^* \cdot H_n^H \cdot \nu_n \tag{9}$$

The matrix $G$ has a size of $ntx \cdot N \times ntx \cdot N$ and it consists of sub-matrices of size $ntx \times ntx$. Both vectors $\bar{e}$ and $\nu$ have $ntx \cdot N$ elements. According to (7) there is a linear relation between all transmitted data vectors on all DFT inputs and the sufficient statistics vectors measured at a given output of the first RAKE receiver stage, which is called inter-code or multiple-access interference (MAI) in the literature. Such interference can be reduced by a multiuser detector (MUD) placed after the DFT RAKE as a second stage. We have denoted this dual-stage receiver as the advanced RAKE.

A. The Effective Channel

Consider at first some unique properties of the effective channel matrix in case of DFT spreading. According to (8), the effective channel is constructed using scalar coefficients taken from the DFT matrix $W$. If we insert these elements in (8) according to (3), the result is

$$G_{m,o} = \sum_{n=1}^{N} W_{n,o} \cdot H_n^H H_n \tag{10}$$

Obviously, there is a cyclic permutation in the elements of $G$. The expression $(o - m)$ is interpreted modulo $N$. Therefore, it is only necessary to calculate the first row of $G$ and rotate each of the following rows in a cyclic manner. The first row of the circulant effective channel matrix describes the entire effective channel:

$$G_{o} = G_{1,o} = \sum_{n=1}^{N} W_{n,o} \cdot H_n^H H_n \tag{11}$$

$G_{o-m}$ indicates this cyclic shift in $G_o$ by $m$ columns clockwise.

B. Linear Multiuser Detection

Such a matrix is called circulant. Circulant matrices have unique properties (see [10] pp. 31) that can be useful to calculate their inverse

1) The eigenvalues of $G$ are $\psi_k = \frac{1}{N} G_{o} \cdot \rho^{o-1}$ where $\rho^{o-1}$ is a complex root of unity $\rho_k = e^{-j2\pi(k-1)/N}$ $(k = 1...N)$. The $k$ eigenvalues can be comput as

$$\psi_k = \sum_{o=1}^{N} G_{o} \cdot e^{-j2\pi(k-1)(o-1)/N} \tag{12}$$

2) The corresponding eigenvectors $\bar{\psi}_k$

$$\bar{\psi}_k = \frac{1}{\sqrt{N}} \begin{pmatrix} e^{-j2\pi(k-1)/N} \\ e^{-j2\pi(2(k-1))/N} \\ ... \end{pmatrix}^T$$

compose the unitary DFT matrix $1/N \cdot W$. All circulant matrices share the same eigenvectors.

3) Inverses, products and sums of circulant matrices are also circulant. The inverse of $G$ is

$$G^{-1} = \frac{1}{N} \cdot W \cdot \text{diag}[\psi]^{-1} \cdot W^H \tag{12}$$

Due to these properties, the inverse channel $G^{-1}$ is fully described by its first row

$$G^{-1}_{m} = \frac{1}{N} \sum_{k=1}^{N} \left( \frac{1}{N} \sum_{o=1}^{N} W_{k,o} \cdot G_o \right)^{-1} \cdot W_{k,m} \tag{13}$$
The ZF MUD output signal is finally given as
\[ \hat{d}_q = N \cdot \sum_{m=1}^{N} G_{m-q}^{-1} \cdot \tilde{e}_m \] (14)

The MMSE MUD can be obtained by using the well known MMSE equalizer [11] together with the effective channel \( G \):
\[ M = \left( n_{tx} \cdot N \cdot \sigma_v^2 \cdot I_{n_{tx} \cdot N} + G^H G \right)^{-1} \cdot G^H \] (15)
Due to the effective noise term (9) follows:
\[ \sigma_v^2 = \frac{1}{\text{SNR} \cdot N} \cdot G_m \] (16)

As for the ZF approach, one can use the special properties of circulant matrices to simplify \( G^H G \) in (15) to:
\[ (G^H G)_m = \sum_{o=1}^{N} G_o \cdot G_{o-m}^H \] (17)

In this way, the MMSE MUD can be obtained using (16) and (17) in (15):
\[ M_m = \sum_{r=1}^{N} \left( \frac{n_{tx}}{\text{SNR}} \cdot G_m + \sum_{o=1}^{N} G_o \cdot G_{o-m}^H \right)^{-1} \cdot G_{r-m}^H \] (18)
as
\[ \hat{d}_q = N \cdot \sum_{m=1}^{N} M_{m-q} \cdot \tilde{e}_m \] (19)

The RAKE-MUD receiver is shown in fig. 2. After downconversion from the RF and the individual fast Fourier transform (FFT) on each antenna, the RAKE calculates the sufficient statistics vector and the effective channel matrix which are used by the multiuser detector (based on ZF or MMSE) to generate a weight matrix \( M \) that enables the reconstruction of the data symbols.

IV. FREQUENCY-DOMAIN EQUALIZATION

Starting from (5), there is a simple way of reconstructing the transmitted signal using the (pseudo-)inverse
\[ V_n^{(ZF)} = (H_n^H H_n)^{-1} \cdot H_n^H \] (20)
of \( H \) on each subcarrier needed in the ZF FDE
\[ \hat{\tilde{x}}_n = V_n^{(ZF)} \cdot \tilde{y}_n \] (21)
However, ZF equalization leads to noise enhancement, in particular when \( H \) is ill-conditioned. The FDE is considerably improved using the MMSE equalizer [11]
\[ V_n^{(MMSE)} = \left( \frac{n_{tx}}{\text{SNR} + H_n^H H_n} \cdot I_{n_{tx}} \right)^{-1} \cdot H_n^H \] (22)
instead of \( V_n^{(ZF)} \) in (21). Finally \( \hat{\tilde{x}}_n \) is despread as
\[ \hat{\tilde{x}}_n = \frac{1}{\sqrt{N}} \sum_{n=1}^{N} W_{n,q}^* \cdot \hat{\tilde{x}}_n \] (23)
to obtain an estimate of the data symbol.

V. COMPARISON OF BOTH RECEIVERS

A. Computational effort

The complexity of the advanced RAKE is significant, in particular since the calculation of the linear MUD weight matrix in (15) involves the inversion of a matrix with dimension \( n_{tx} \cdot N \times n_{tx} \cdot N \) in real time, i.e. once per coherence interval. Also, multiplication of the weight matrix with the \( n_{tx} \times N \) dimensional sufficient statistics vector is needed once per OFDM symbol. Complexities for brute-force matrix inversion and matrix-vector multiplication scale with \( (n_{tx} \cdot N)^3 \) per coherence interval and \( (n_{tx} \cdot N)^2 \) per OFDM symbol, respectively.

In contrast, the FDE approach with subsequent IDFT requires \( N \) inversions of much smaller matrices with dimension \( n_{tx} \times n_{tx} \) once per subcarrier and per coherence interval as well as \( N \) multiplications of these smaller matrices with the \( n_{tx} \times 1 \) dimensional received signal vector on each subcarrier.
random numbers with zero mean and variance 1 independently and identically distributed complex (i.i.d.) Gaussian simulated over a multipath channel with \( L \) of an OFDM system with \( N=64 \) subcarriers. Transmission is over a DFT spread OFDM system with one antenna at transmitter and receiver side. It uses DFT spreading with 64 points on top of an OFDM system with \( N=64 \) subcarriers. Transmission is simulated over a multipath channel with \( L = 4 \) components where the fading channel coefficients are modelled as independently and identically distributed complex (i.i.d.) Gaussian random numbers with zero mean and variance \( 1/\sqrt{L} \).

Firstly, our simulation results confirm the analytical result. For the ZF and MMSE receivers, within statistical errors, the same bit error rate is achieved by the RAKE. It is also observed how important it is to use the MMSE instead of the ZF FDE. This has been previously observed also for the space-frequency RAKE used for MC-CDMA when the system is fully loaded, i.e. all spreading codes are used. It is typical for the linear ZF receiver, that in the fully loaded system almost no more multipath diversity can be realized, see [5]. Note that the maximal diversity order is 4 in this simulation. With the linear MMSE receiver, a significant portion of the diversity can be realized, provided that the noise power is known.

Note that an equivalence between the advanced RAKE and the MMSE equalizer with subsequent despreading has been previously observed in the case of multiantenna receivers for WCDMA with multiple codes [9]. But in case of WCDMA, which uses binary spreading sequences, there was not such a dramatic complexity reduction as in the present case of using DFT spreading codes. Our main result allows the realization of optimal linear receivers for broadband wireless communication channels with much lower complexity than in previous approaches.

The complexity is significantly lower. It scales with \( n^3_{tx} \cdot N \) per coherence interval and \( n^2_{tx} \cdot N + N \cdot \log(N) \) per OFDM symbol. Obviously, the larger \( N \), the more efficient is the FDE approach.

### B. Main result

In the case of DFT spreading, the favorite properties of the circulant effective channel matrix can be exploited to reduce the complexity of the optimal RAKE approach. After a straightforward but complex calculation, detailed in the appendix, it can be proven that the advanced RAKE is indeed exactly equivalent with the FDE-based receiver when DFT spreading is used.

In Fig. 4, the performance is investigated with a variable number of multipath components (MPC). Note that the ZF receiver is sensitive to an increased number of MPC, while the MMSE receiver is not. In Fig. 7 with two transmit and receive antennas, the power is shared among the transmit antennas which shifts all curves based on ZF by 3 dB to higher SNR. However, the MMSE receiver does not suffer as much as the ZF receiver and it still provides a high diversity order when the number of MPCs is large.

In addition, the N-point IDFT must be performed. Altogether, the complexity is significantly lower. It scales with \( n^3_{tx} \cdot N \) per coherence interval and \( n^2_{tx} \cdot N + N \cdot \log(N) \) per OFDM symbol. Obviously, the larger \( N \), the more efficient is the FDE approach.

### C. Performance

This analytical result has been verified by numerical simulations. In fig. 4, we have plotted the link performance for a DFT-spread OFDM system with one antenna at transmitter and receiver side. It uses DFT spreading with 64 points on top of an OFDM system with \( N=64 \) subcarriers. Transmission is simulated over a multipath channel with \( L = 4 \) components where the fading channel coefficients are modelled as independently and identically distributed complex (i.i.d.) Gaussian random numbers with zero mean and variance \( 1/\sqrt{L} \).

In Fig. 6, the performance is investigated with a variable number of multipath components (MPC). Note that the ZF receiver is sensitive to an increased number of MPC, while the MMSE receiver is not. In Fig. 7 with two transmit and receive antennas, the power is shared among the transmit antennas which shifts all curves based on ZF by 3 dB to higher SNR. However, the MMSE receiver does not suffer as much as the ZF receiver and it still provides a high diversity order when the number of MPCs is large.

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**Figure 4.** Comparison of the performance for the advanced DFT RAKE Receiver (dashed lines) and the FDE with consecutive despreading (solid lines) over a multipath fading channel: Both receivers types, ZF and MMSE are evaluated.

**Figure 5.** Performance of the MMSE Receiver with SNR Estimation Errors

**Figure 6.** Comparison of the performance for ZF and MMSE equalization in single antenna system

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Conclusions

We have shown analytically that the signal processing in the optimal advanced RAKE receiver can be dramatically simplified when DFT spreading is used, which creates a circulant structure of the effective channel matrix. The receiver is mathematically equivalent to the well known frequency-domain equalizer (FDE) with consecutive despreading based on the inverse DFT. This major result holds also for multi-antenna systems. We have shown by numerical simulations that it is beneficial to use the minimum mean-square error approach in the FDE, and the sensitivity against estimation errors for the noise power has been investigated. Nonetheless, significant multipath diversity can be realized with simple signal processing at the receiving end. For this reason, we believe that DFT spreading might be an interesting concept for the downlink of future mobile communication systems, in particular when no or little feedback can be provided as in broadcast or vehicular applications at high velocity.

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Appendix

Theorem 1 (Equality of MUD and FDE with ZF): When using ZF equalization, the DFT-Rake receiver [equation (14)] and DFT precoded frequency domain equalization [equations (20, 21, 23)] are equal:

\[
\tilde{d}_q = \frac{1}{\sqrt{N}} \sum_{n=1}^{N} W_{n,q}^* \cdot H_n^{-1} \cdot \hat{y}_n = \sum_{m=1}^{N} G_{m}^{-1} \cdot \hat{e}_m
\]

Proof: Substituting \( G_o \) in equation (13) with the corresponding equation (11), one gets for the inverse effective channel \( G_{m}^{-1} \):

\[
G_m^{-1} = \frac{1}{N} \sum_{k=1}^{N} \left( \sum_{\alpha=1}^{N} W_{k,\alpha}^* \cdot \sum_{n=1}^{N} W_{n,o} \cdot H_n^H H_k \right)^{-1} \cdot W_{k,m}
\]

(25)

where \( W_{k,\alpha} \cdot W_{n,o} \) equals \( e^{-j2\pi \frac{(n-k)}{N}} \). The sum \( \sum_{\alpha=1}^{N} \sum_{n=1}^{N} e^{-j2\pi \frac{(n-k)}{N}} \) is one for \( n = k \) and zero for \( n \neq k \) which leads to the inverse channel:

\[
G_m^{-1} = \frac{1}{N} \sum_{k=1}^{N} \left( H_k^H H_k \right)^{-1} \cdot W_{k,m}
\]

(26)

Plugging (26) into (14) results in:

\[
\tilde{d}_q = \frac{1}{\sqrt{N}} \sum_{n=1}^{N} W_{n,q}^* \cdot \left( H_n^H H_n \right)^{-1} \cdot W_{k,m-q} \cdot \hat{e}_m
\]

= \sum_{m=1}^{N} \left[ \sum_{k=1}^{N} \left( H_k^H H_k \right)^{-1} \cdot W_{k,m-q} \right] \cdot \hat{e}_m

\[
\frac{1}{\sqrt{N}} \sum_{n=1}^{N} W_{n,m}^* \cdot H_n^H \cdot \hat{y}_n
\]

Due to equation (3), \( W_{k,m-q} \cdot W_{n,m}^* \) equals \( e^{-j2\pi \frac{(m-q)(n-m)}{N}} \cdot e^{j2\pi \frac{(k-m)(n-m)}{N}} \). The sum is again only one for \( n = k \) and zero for \( n \neq k \). It follows for the Rake-ZF receiver:

\[
\tilde{d}_q = \frac{1}{\sqrt{N}} \sum_{n=1}^{N} W_{n,q}^* \cdot \left( H_n^H H_n \right)^{-1} \cdot \hat{y}_n
\]

(27)

Since \( \left( H_n^H H_n \right)^{-1} \) it follows, that this is equivalent to the MIMO-ZF equalizer, which concludes the proof.

Theorem 2 (Equality of MUD and FDE with MMSE): When using MMSE equalization, the DFT-Rake receiver [equation (19)] and DFT precoded frequency domain equalization [equations (22, 21, 23)] are equal:

\[
\tilde{d}_q = \frac{1}{\sqrt{N}} \sum_{n=1}^{N} W_{n,q}^* \cdot \left( H_n^H H_n + \frac{n_{iz}}{\text{SNR}} \cdot I_{n_{iz}} \right)^{-1} \cdot H_n^H \cdot \hat{y}_n
\]

(28)

MMIMO-MMSE

Rake-MMSE

Proof: One can use the effective channel representation (11) to replace the term \( \sum_{m=1}^{N} G_m \cdot G_{m-o}^H \) in eq. (18) with

\[
\sum_{o=1}^{N} W_{o-o} \cdot W_{p,o-m}^* \cdot H_{n}^H H_{n} \cdot \sum_{j=1}^{N} W_{p,j} \cdot H_{j}^H H_{j}
\]

For the two DFT spreading coefficients follows due to eq. (3):

\[
W_{n,o} \cdot W_{p,o-m} = e^{-j2\pi \frac{(n-o)(p-o)}{N}} \cdot e^{-j2\pi \frac{(n-o)(m-o)}{N}}
\]
The sum $\sum_{n=1}^{N} \sum_{p=1}^{N} e^{-j2\pi \frac{(n-1)(n-p)}{N}}$ is always one for $n = p$ and zero for $n \neq p$. It follows:

$$\sum_{n=1}^{N} G_{\alpha}G_{\beta}^H = \sum_{p=1}^{N} W_{p,m} \left( H_p^H H_p \right)^2$$

(28)

The next step is to rewrite (18) with (11) and (28) to:

$$G_m = \frac{\tilde{r}_{tx}}{\text{SNR}} \sum_{n=1}^{N} W_{n,m} H_n^H H_n + \sum_{p=1}^{N} W_{p,m} \left( H_p^H H_p \right)^2$$

$$- (G_m^H G_m)$$

$$M_m = \sum_{r=1}^{N} \left[ G_m \right]_{r}^{-1} \cdot G_{r-m}^H$$

(29)

Calculating the inverse of the inner matrix $G_\alpha$ can be done according to equation (13) where the special properties of circulant matrices are used to find an inverse circulant matrix. It follows for eq. (29):

$$\left[ G_m \right]_{r}^{-1} = \frac{1}{N} \sum_{k=1}^{N} \left( \frac{\tilde{r}_{tx}}{\text{SNR}} \sum_{n=1}^{N} W_{k,m} \cdot G_m \right)_{r}^{-1} \cdot W_{k,r}$$

$$= \frac{1}{N} \sum_{k=1}^{N} \left( \frac{\tilde{r}_{tx}}{\text{SNR}} \sum_{n=1}^{N} W_{k,m} \cdot H_n^H H_n + \sum_{p=1}^{N} W_{p,m} \left( H_p^H H_p \right)^2 \right)_{r}^{-1} \cdot W_{k,r}$$

(30)

Again, one can replace $W_{k,m} \cdot W_{n,m}$ and $W_{k,m} \cdot W_{p,m}$ with (3), which leads to:

$$W_{k,m} \cdot W_{n,m} = e^{-j2\pi \frac{(k-n)}{N}}$$

$$W_{k,m} \cdot W_{p,m} = e^{-j2\pi \frac{(k-p)}{N}}$$

In both cases, there is only a nonzero solution for $n = k$ and $p = k$. One can replace $p$ and $n$ with $k$ and (30) becomes:

$$\left[ G_m \right]_{r}^{-1} = \frac{1}{N} \sum_{k=1}^{N} \left( \frac{\tilde{r}_{tx}}{\text{SNR}} \cdot H_k^H H_k + \left( H_k^H H_k \right)^2 \right)_{r}^{-1} \cdot W_{k,r}$$

(31)

(29) can therefore be rewritten to:

$$M_m = \frac{1}{N} \sum_{r=1}^{N} \sum_{k=1}^{N} \left( \frac{\tilde{r}_{tx}}{\text{SNR}} \cdot H_k^H H_k + \left( H_k^H H_k \right)^2 \right)_{r}^{-1} \cdot W_{k,r}$$

$$= \sum_{p=1}^{N} W_{p,r-m} \cdot H_p^H H_p$$

(32)

The product $W_{k,r} \cdot W_{p,r-m}$ results in $e^{-j2\pi \frac{(p-r)}{N}(k-p)}$. Due to (3), So again, the sum reveals only nonzero solutions for $p = k$. Hence, eq. (32) equals:

$$M_m = \frac{1}{N} \sum_{k=1}^{N} W_{k,m} \left( \frac{\tilde{r}_{tx}}{\text{SNR}} \cdot I_{n_{tx}} + H_k^H H_k \right)_{r}^{-1}$$

(33)

With (19) and (6), the decoded data symbol is:

$$\tilde{d}_q = \frac{1}{N} \sum_{m=1}^{N} \sum_{k=1}^{N} W_{k,m-q} \left( \frac{\tilde{r}_{tx}}{\text{SNR}} \cdot I_{n_{tx}} + H_k^H H_k \right)_{r}^{-1} \cdot M_m$$

(34)

One can use (3) to substitute $W_{k,m-q} \cdot W_{n,m}$ which reveals that $k = n$ and $e^{-j2\pi j \frac{(k-n)}{N}} = W_{k,q} = W_{n,q}$. The final result is:

$$\tilde{d}_q = \frac{1}{\sqrt{N}} \sum_{n=1}^{N} W_{n,q} \left( \frac{\tilde{r}_{tx}}{\text{SNR}} \cdot I_{n_{tx}} + H_n^H H_n \right)_{r}^{-1} \cdot H_n^H \cdot \hat{y}_n$$

(35)

which is exactly the MIMO-MMSE equalizer described in the equations (22) and (23).

REFERENCES


