MULTIUSER MULTICHANNEL ESTIMATION AND DETECTION WITH SIDE USER INFORMATION IN SPACE-TIME CODED CDMA SYSTEMS

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ABSTRACT

In this work, we develop a new second-order statistics based multiuser multipath channel estimation algorithm for uplink wireless space-time coded CDMA systems. The estimation procedure is based on the parameterization, with respect to the multiuser channel response vectors, of the received data covariance matrix. As a side result, we also obtain an improved covariance matrix estimator. Then we utilize both the channel and the covariance matrix estimates to obtain an estimate of the linear MMSE receiver. Simulation studies illustrate the performance improvements of the proposed estimators relative to existing methods in terms of channel estimation mean-square error as well as receiver filter output SINR and receiver BER.

1. INTRODUCTION

Multiple-transmit multiple-receive (MTMR) antenna elements promise significant capacity improvements in rich scattering fading wireless communication links relative to single-transmit single-receive systems. The MTMR architecture is further supported by recent results on space-time coding (STC) [1–3].

In this work, we are interested in the problem of blind multiuser multipath channel estimation and subsequent equalization for uplink space-time coded CDMA networks. In the rest of the paper we use the following notation: Upper-case bold letters denote matrices while lower-case bold letters denote column vectors, the superscripts $\ast$, $T$ and $H$ denote conjugate, transpose and Hermitian, respectively. $E \{ \cdot \}$ refers to statistical expectation, $I$ represents the identity matrix, while $\text{det} \mathbf{A}$ and $\text{tr} \mathbf{A}$ denote the determinant and trace of a square matrix $\mathbf{A}$, respectively.

2. SIGNAL MODEL

For simplicity in presentation, we assume a two-transmit, two-receive antenna synchronous CDMA system where the Alamouti coding scheme is used at the transmitter side. There are $K$ known users in the network and $M$ unknown interferers (possibly from neighboring cells). User $k$, $k = 1, \ldots, K$, is assigned two signatures, $\mathbf{s}_{1k}$ and $\mathbf{s}_{2k}$, one for each transmit antenna TX1 and TX2, respectively. The developments that follow can be extended to other, more general, space-time block coded schemes.

The discrete time received signal at the receive antenna $\text{RX}_n$, $n = 1, 2$, at time $(2i - 1)T$ and $2iT$, after chip matched filtering and chip-rate sampling, is given, respectively:

$$
\mathbf{r}_n(2i - 1) = \sum_{k=1}^{K} \left[ \mathbf{S}_{1k} \mathbf{h}_{1k,n} b_k(2i - 1) + \mathbf{S}_{2k} \mathbf{h}_{2k,n} b_k(2i) \right] + \text{ISI} + \mathbf{n}_n(2i - 1)
$$

$$
\mathbf{r}_n(2i) = \sum_{k=1}^{K} \left[ \mathbf{S}_{2k} \mathbf{h}_{2k,n} b_k^*(2i - 1) - \mathbf{S}_{1k} \mathbf{h}_{1k,n} b_k^*(2i) \right] + \text{ISI} + \mathbf{n}_n(2i)
$$

where $\mathbf{S}_{jk}$ and $\mathbf{h}_{jk,n}$, $j, n = 1, 2$, denote the zero-padded signature matrix for user-$k$ at $\text{TX}_j$, and the corresponding channel response vector to $\text{RX}_n$, respectively, $b_k$ refers to the binary information bits of user-$k$ while $\mathbf{n} \sim \mathcal{CN}(0, \sigma^2 \mathbf{I})$ denotes circularly symmetric complex additive white Gaussian noise (AWGN). Since, in general, the number of multipaths $P$ is quite small compared to the spreading gain $L$, the intersymbol interference (ISI) is negligible and it will be ignored in our developments.

Next, we define:

$$
\mathbf{r} \triangleq \begin{bmatrix} \mathbf{r}_1(2i - 1) \\ \mathbf{r}_1^T(2i) \\ \mathbf{r}_2(2i - 1) \\ \mathbf{r}_2^T(2i) \end{bmatrix}, \quad \mathbf{n} \triangleq \begin{bmatrix} \mathbf{n}_1(2i - 1) \\ \mathbf{n}_1^T(2i) \\ \mathbf{n}_2(2i - 1) \\ \mathbf{n}_2^T(2i) \end{bmatrix}, \quad \mathbf{h}_k \triangleq \begin{bmatrix} \mathbf{h}_{1k,1} \\ \mathbf{h}_{1k,1}^* \\ \mathbf{h}_{1k,2} \\ \mathbf{h}_{1k,2}^* \end{bmatrix},
$$

$$
\mathbf{C}_{1k} \triangleq \mathbf{I}_2 \otimes \begin{bmatrix} \mathbf{S}_{1k} & 0 \\ 0 & \mathbf{S}_{2k} \end{bmatrix}, \quad \mathbf{C}_{2k} \triangleq \mathbf{I}_2 \otimes \begin{bmatrix} 0 & -\mathbf{S}_{1k}^* \\ -\mathbf{S}_{1k} & \mathbf{S}_{2k} \end{bmatrix},
$$

$$
\mathbf{b}_k \triangleq \begin{bmatrix} b_k(2i - 1) \\ b_k(2i) \end{bmatrix}, \quad \mathbf{J} \triangleq \mathbf{I}_2 \otimes \begin{bmatrix} 0 & \mathbf{I} \\ -\mathbf{I} & 0 \end{bmatrix},
$$

$$
\mathbf{c}_{1k} \triangleq \mathbf{C}_{1k} \mathbf{h}_k, \quad \mathbf{c}_{2k} \triangleq \mathbf{C}_{2k} \mathbf{h}_k^*,
$$

$$
\mathbf{C} \triangleq \begin{bmatrix} \mathbf{c}_{11} & \mathbf{c}_{12} & \cdots & \mathbf{c}_{1K} & \mathbf{c}_{21} & \cdots & \mathbf{c}_{2K} \end{bmatrix}, \quad \mathbf{b} \triangleq \begin{bmatrix} \mathbf{b}_1^T, \ldots, \mathbf{b}_K^T \end{bmatrix}^T
$$

Taking into account possible unknown interference (that may be due to neighboring cells), the received data vector $\mathbf{r}$ can be compactly written as

$$
\mathbf{r} = \mathbf{Cb} + \mathbf{C_Jb_J} + \text{ISI} + \mathbf{n}
$$

(3)
where \( C_I \) is the \( L_p \times 2M \) unknown interference composite signature matrix. We assume that the composite interfering signatures are paired, i.e. for each \( c_{I,m} \in C_I \), \( Jc_{I,m} \in C_I \). The composite processing gain is the product of the multipath extended signature length, the frame-size of the space-time code and the number of receive antennas, i.e. if the assigned signature length is \( L \) and the number of multipaths is \( P \), then \( L_p = 4(L + P - 1) \).

To facilitate our analysis, we utilize some important properties that are induced by STC [4]. These properties are summarized below (the proof is straightforward and thus omitted).

**Alamouti encoding induced properties:**

1. \( J^T J = I \), \( J^T = -J \);
2. \( C_{2k} = J C_{1,k} \), \( R^* = J^T R J \);
3. \( c_{2k} = Jc_{1,k}^* \), \( c_{2k}^H R_{1,k} = c_{2k}^H R_{2,k} \),
   \( c_{2k}^H C_{1,k} = 0 \), \( c_{2k}^H R_{1,k} = 0 \).

We note that if we substitute \( R \) by any matrix \( A \) such that \( A = \sum_i [a_i a_i^H + (Ja_i^*)^T J a_i^H] + \sigma^2 I \), for any arbitrary set of vectors \( a_i \), then properties (2) and (3) above will still hold.

3. **ALGORITHMIC DEVELOPMENTS**

3.1. **SOS based Channel Estimators**

All SOS based algorithms exploit the statistical properties of the received data covariance matrix \( R \triangleq E \{ r r^H \} \). The most popular SOS channel estimators are the Capon estimator and the MUSIC subspace estimator.

The Capon channel estimator for MTMR CDMA systems was developed in [4]:

\[
\hat{h}_k = \mathcal{U}_{\text{min}} \left( C_{1,k}^H R^{-1} C_{1,k} + C_{2,k}^H \left( R^{-1} \right)^* C_{2,k}^* \right)
\]  

(4)

where \( \mathcal{U}_{\text{min}}(A) \) denotes the eigenvector corresponding to the minimum eigenvalue of the matrix \( A \).

The subspace channel estimator was developed in [5]. It is a two step procedure that proceeds as follows:

Step 1: Compute the eigen-decomposition of the data covariance matrix \( R \):
\[
R = U_s \Sigma_s U_s^H + \sigma^2 U_n U_n^H
\]

Step 2: Estimate the channel of user-\( k \), \( k = 1, \ldots, K \)
\[
\hat{h}_k = \mathcal{U}_{\text{min}} \left( C_{1,k}^H U_n U_n^H C_{1,k} + C_{2,k}^H U_n^* U_n^T C_{2,k} \right)
\]  

(5)

To avoid any confusion, we emphasize that throughout this work the returned eigenvalues of any eigen-decomposition procedure are sorted in a non-increasing order. Furthermore, the eigenvalue/eigenvector matrices (denoted by capital bold letters) are composed of eigenvalues/eigenvectors that are denoted by the *same* though lower-case letters.

After all channels are estimated, the composite signatures are computed by \( \hat{c}_{1,k} = S_{1,k} \hat{h}_k \), \( \hat{c}_{2,k} = S_{2,k} \hat{h}_k \), and the multiuser composite signature matrix is reconstructed by \( \hat{C} = \hat{c}_{11}, \ldots, \hat{c}_{1K}, \hat{c}_{2K} \). Then, various multiuser receivers can be formed, e.g. the Rake receiver, the linear MMSE receiver, or the decorrelator. The MMSE multiuser filter is simply \( W_{\text{MMSE}} = R^{-1} \hat{C} \), and the multiuser decorrelator is given by \( W_{\text{DEC}} = \hat{C} \left( \hat{C}^H \hat{C} \right)^{-1} \). To resolve the phase ambiguity that is inherent to SOS based methods, differential encoding/decoding is performed, e.g. if \( y(i) = \hat{w}^H r_i \) denotes the receiver filter output, then the information bits are estimated by \( \hat{\theta}(i) = \text{sign} \{ \text{Re} \{ y(i)(i-1)^* \} \} \).

In [6], a family of group multiuser receivers was proposed, and it was shown that the hybrid decorrelator-MMSE receiver performs the best in the family. In particular, one implementation of the hybrid receiver (named form-2 in [6]) suffers very little bit error rate (BER) performance loss compared to the other (form-1) implementation while it maintains the least computational complexity. It is given by

\[
W_{\text{hyb}} = U_s \Sigma_s^{-1} U_s^H \hat{C} \left( \hat{C}^H U_s \Sigma_s^{-1} U_s^H \hat{C} \right)^{-1} \hat{C}
\]  

(6)

We recall that the standard subspace multiuser detector \( W_{aa} = U_s \Sigma_s^{-1} U_s^H \hat{C} \) is a subspace implementation of the optimum linear MMSE filter.

In practice, \( R \) is unknown. Given \( N \) received data vectors, the most commonly used covariance matrix estimate is the sample average estimate

\[
R_{\text{sa}} = \frac{1}{N} \sum_{i=1}^{N} r_i r_i^H
\]  

(7)

Substitution of \( R \) by \( R_{\text{sa}} \) in the corresponding receiver expression provides an estimate of the receiver. For example, the sample average inversion (SAI) MMSE receiver utilizes the filter \( W_{\text{SAI}} = R_{\text{sa}}^{-1} \hat{C} \).

The sample average estimate \( R_{\text{sa}} \) is very simple to evaluate. Indeed, it is the maximum likelihood estimate when the data are independent identically Gaussian distributed. However, if the data are not Gaussian distributed, and when the number of observations is small and/or when the user signal-to-noise ratio (SNR) is low, then \( R_{\text{sa}} \) could be an unreliable estimate that is far away from the true covariance matrix \( R \).

3.2. **A New Channel Estimator based on Structured Data Covariance Matrix**

If the received vectors are identically Gaussian distributed, then the maximum likelihood estimate of the covariance matrix is the solution of the following optimization problem:

\[
\hat{R} = \arg \min_R \{ \log \det R + \text{tr}(R^{-1} R_{\text{sa}}) \}
\]  

(8)
Motivated by (8), we propose a new estimator, denoted by \( \hat{R} \), that exploits the structure of the data covariance matrix and also utilizes \( R_{na} \). More specifically, we propose to estimate the data covariance matrix by solving the following constrained optimization problem:

\[
\hat{R} = \arg \min_{R} \left[ \log \det R + \text{tr}(R^{-1}R_{na}) \right] \quad \text{s. t. } R = \sum_{k=1}^{K+M} \left( c_{1k}c_{1k}^{H} + c_{2k}c_{2k}^{H} \right) + \sigma^{2}I, \quad c_{2k} = Jc_{1k}^{*}
\]

We note that, unlike single-antenna transceiver systems where each user (or interferer) has only one composite signature, in space-time coded multi-antenna systems a pair of composite signatures for each user (or interferer) arises. The estimation of both the covariance matrix and the multiuser channel is a coupled procedure and can be decomposed into several modules. Theorems 1–3 below justify our final proposal for channel and covariance matrix estimation that is given at the end of this section (the proofs are omitted due to lack of space).

Theorem 1 (Interference covariance matrix estimation)

Let \( R_{na} \) be given by (7), and \( \hat{R} = \hat{R}_{\text{L}} + \hat{C}_{I}\hat{C}_{I}^{H} \) i.e. \( \hat{R}_{\text{L}} = \hat{C}_{C}^{H} + \sigma^{2}I \) is the component of \( \hat{R} \) that is due to the known users and AWGN. \( \hat{R}_{\text{L}} \) is fixed and satisfies \( \hat{R} = \hat{R}_{\text{L}} \).

Let \( R_{\Delta} = \left( R_{na} + J^{\dagger}R_{na}^{*}J \right) / 2 \). Then, the component \( \hat{C}_{I}\hat{C}_{I}^{H} \) that is due to interference can be estimated as follows:

\[
\hat{C}_{I}\hat{C}_{I}^{H} = Q\Phi^{\frac{1}{2}}T(A - \mathbf{I})^{+}T^{H}\Phi^{\frac{1}{2}}Q^{H}
\]

where \( (A - \mathbf{I})^{+} = \text{diag}(\lambda_{1}^{+}, \ldots, \lambda_{M}^{+}) \), \( \lambda_{m}^{+} = \max(\lambda_{m} - 1, 0) \), and the matrices in the right hand side of (10) satisfy the following eigen-decomposition expressions

\[
\hat{R}_{\text{L}} = Q\Phi Q^{H}
\]

\[
\Phi^{-\frac{1}{2}}Q^{H}R_{\Delta}Q\Phi^{-\frac{1}{2}} = [T T_{2}] \text{diag}(\Lambda A_{2}) [T T_{2}]^{H}
\]

Theorem 2 (Noise variance estimation) Let \( R_{na} \) be given by (7), and \( \hat{R} = \hat{R}_{\text{L}} + \sigma^{2}I \) i.e. \( \hat{R}_{\text{L}} \) is the component of \( \hat{R} \) that is due to the known users plus interference. If the eigen-decomposition of \( \hat{R}_{\text{L}} \) is \( \hat{R}_{\text{L}} = Q\Phi Q^{H} \), then, the noise variance \( \sigma^{2} \) that optimizes (9) is

\[
\sigma^{2} = \max(r, 0)
\]

where \( r \) is the minimum positive root of the equation

\[
\sum_{i=1}^{L} \frac{r + q_{i}^{H}R_{na}q_{i}}{(r + \phi_{i})^{2}} = 0.
\]

Theorem 3 (Channel estimation) Let \( R_{na} \) be given by (7), and \( \hat{R} = \hat{R}_{\text{L}} + c_{1k}c_{1k}^{H} + c_{2k}c_{2k}^{H} \) i.e. \( \hat{R}_{\text{L}} \) is the remainder of the estimated covariance matrix after we subtract the component that is due to user-k. \( \hat{R}_{\text{L}} \) is fixed and satisfies \( \hat{R}_{\text{L}}^{*} = J^{\dagger}\hat{R}_{\text{L}}J \).

Let \( R_{\Delta} \) be as in Theorem 1, \( A \triangleq C_{k}^{H}\hat{R}_{\text{L}}^{-1}C_{k} \), and \( B \triangleq C_{k}^{H}\hat{R}_{\text{L}}^{-1}R_{\Delta}\hat{R}_{\text{L}}^{-1}C_{k} \), then the channel response vector \( \hat{h}_{k} \) that optimizes the cost function in (9) is given by

\[
\hat{h}_{k} = \begin{cases} \sqrt{\frac{1/d_{\text{min}} - 1}{u_{\text{min}}}u_{\text{min}}} & \text{if } d_{\text{min}} < 1 \\ 0_{p \times 1} & \text{if } d_{\text{min}} \geq 1 \end{cases}
\]

where \( d_{\text{min}} \) and \( u_{\text{min}} \) are the minimum generalized eigenvalue/eigenvector pair such that \( A_{\text{min}} = d_{\text{min}}B_{\text{min}} \).

Proposed Channel and Covariance Matrix Estimation

1. Initialize \( \hat{C}, \sigma^{2} \).
2. Estimate \( \hat{C}_{I}\hat{C}_{I}^{H} \) according to Theorem 1.
3. Update channel estimates \( \hat{h}_{k}, k = 1, \ldots, K \), according to Theorem 3.
4. Update noise variance estimate \( \sigma^{2} \) according to Theorem 2. Update \( \hat{R}_{\text{L}} = \hat{C}_{C}^{H} + \sigma^{2}I \).
5. Goto 2, until convergence is declared.

Furnished with these theoretic results, both the composite signatures and the data covariance matrix can now be fully determined/estimated. Then, we propose the following linear MMSE filter estimate:

\[
W_{\text{prop}} = \hat{R}^{-1}\hat{C}
\]

4. SIMULATION RESULTS

We consider a (2TX, 2RX) CDMA system with \( K = 7 \) users and \( M = 3 \) interferers from neighboring networks. The users/interferers are each assigned a pair of \( L = 8 \) signatures constructed from the Karystinos-Pados code matrices. We assume that each user experiences \( P = 3 \) multipaths.

The performance measures of interest are channel estimation mean-square error (MSE), receiver filter output signal-to-interference-plus-noise ratio (SINR), and receiver BER. Three channel estimation procedures are implemented and compared, i.e., the Capon method, the subspace method, and the proposed approach. The filter output SINR and BER performance comparisons consider six types of filters in total: The ideal linear MMSE filter, the ideal hybrid filter (that utilizes the ideal covariance matrix), the SMI filter that utilizes the Capon channel estimate, the SMI filter that utilizes the subspace channel estimate, the form-2 hybrid filter and the proposed linear MMSE filter estimate.

Fig. 1 shows the effect of the received data record size on the channel estimation and equalization performance. We plot the performance of user-3 whose input SNR is 9dB, while the other user SNR’s are 12, 12, 9, 6, 6 dB, respectively. The interfering user’s SNR’s are fixed at 6, 6, and 3dB. The

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performance curves of the other users are similar but shifted appropriately according to their input SNR.

Fig. 2 illustrates the effect of the input SNR on the estimation and equalization performance. In the simulations we fix the user/interference energies, and increase the input SNR’s by decreasing the noise variance, such that the relative SNR ratio is fixed. The data record size is fixed to twice the composite processing gain. We, again, plot the performance of user-3, and see similar improvements in both channel estimation and data detection.

We note that in all of our simulation studies we observed that the proposed algorithm converges fast, usually within five iterations, which implies that the complexity of the proposed scheme is slightly higher than the complexity of the Capon and subspace method.

5. CONCLUSIONS
In this work, we proposed a new estimation scheme for both the user channel response vector and the data covariance matrix, for space-time coded multi-antenna uplink wireless CDMA communications networks. The method is based on second-order statistics and in particular, on a constrained estimate of the structured data covariance matrix that is parameterized by the multiuser channel vector. The channel and covariance matrix estimates were subsequently utilized to obtain a linear MMSE receiver estimate. Simulation studies demonstrated that the proposed approach is superior over existing methods in terms of channel estimation MSE as well as receiver output SINR and BER.

6. REFERENCES