Jointly Optimal Power Assignment for Multi-Source Multi-Destination Relay Networks

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Abstract—In this paper, the total transmission power of a multi-source multi-destination relay network is minimized under the constraint that the signal to interference plus noise ratio (SINR) requirement of each source-destination pair is satisfied. The optimization problem involves $K$ power variables, where $K$ is the number of source-destination pairs in the network, and an exhaustive search is prohibitive for large $K$. In this work, we develop an asymptotically tight approximation of the SINR that allows us to reformulate the original optimization problem to a single-variable optimization problem, which can be easily solved by numerical search of the single variable. Then, the corresponding optimal transmission power at each source and relay can be calculated directly. The proposed optimization scheme is scalable and leads to a power assignment algorithm that exhibits the same optimization complexity for any number ($K$) of source-destination pairs in the network. Moreover, for the special case of transmission over orthogonal channels, we derive analytically the solution to the optimization problem. Extensive numerical studies illustrate our theoretical developments.

Index Terms: Multi-source multi-destination relaying, interference relay channel, optimum power allocation, total power consumption, cooperative networks.

I. INTRODUCTION

Relaying has been known as an effective technique to improve signal quality and detection reliability in wireless networks ([1]–[3] and the references therein). Recently, there is increasing interest in investigating the advantages of relaying in multi-source multi-destination networks [4]–[10]. The simplest multi-source multi-destination relay network is modeled as an interference relay channel (IRC) [4] where a relay helps two independent source-destination pairs by using various relaying strategies such as decode-and-forward, amplify-and-forward, or compress-and-forward.

Most previous work on multi-source multi-destination relay networks focused on information-theoretic studies including achievable rate regions or bounds of capacity region [4]–[8]. For example, [4] used a rate splitting technique to study the problem of achievable rate region for a Gaussian IRC channel. The achievable rate region of [4] was further improved in [5] by considering both intended message and interference forwarding at the relay. The capacity region of the interference channel with a single-relay was addressed in [6], [7] where it was shown that by only forwarding the intended message of one source, the achievable rates for both source-destination pairs are improved. By assuming that the relay knows the source message a priori, a relaying strategy was proposed in [8] where generalized beamforming with dirty paper coding was considered for a two-source two-destination system.

Power control is important to improve overall performance of multi-source multi-destination relay networks. In [9], power allocation was optimized by exhaustive search for a two-source two-destination relay network where a half-duplex decode-and-forward relay was considered. Unfortunately the exhaustive search is not scalable and leads to prohibitive searching complexity for networks with larger number of source-destination pairs. Another power allocation scheme was proposed in [10] for a multi-source multi-destination relay network based on a geometric programming approach, where it was assumed that different sources’ signals were sent through orthogonal channels.

In this paper, we analyze and optimize a general multi-source multi-destination relay network with $K$ sources ($K$ can be large). The network allows simultaneous multi-source transmissions through non-orthogonal, in general, channels. Our objective is to minimize the total power consumption of all sources and the relays under the constraint that the signal to interference plus noise ratio (SINR) requirement of each source-destination pair is satisfied. Thanks to an asymptotically tight approximation of the SINR that we develop, we are able to reformulate the original optimization problem, which involves $K$ power variables, to a single-variable optimization problem. The resulting optimization problem can be easily solved by a simple numerical search of the single variable. The proposed optimization approach is scalable and leads to a power assignment algorithm that has the same optimization complexity for any number ($K$) of source-destination pairs in the network. Moreover, for the special case of transmission over orthogonal channels, we are able to further simplify the single-variable optimization and obtain analytical solutions for a symmetric system. Extensive numerical studies included in this paper illustrate and validate our theoretical developments.

Notation: Bold letters in uppercase and lowercase denote matrices and vectors, respectively. $(\cdot)^*$, $(\cdot)^T$ and $(\cdot)^H$ represent the conjugate, transpose and Hermitian transpose operation, respectively. $\mathbf{I}_L$ is an $L \times L$ identity matrix. $A_{k}$ denotes a sub-matrix of $\mathbf{A}$ obtained by deleting the $k^{th}$ column and $k^{th}$ row of $\mathbf{A}$. If $\mathbf{a}_k$ is the $k^{th}$ column of the matrix $\mathbf{A}$, then $a_{k}$ denotes the vector obtained after removing the $k^{th}$ entry from $\mathbf{a}_k$.

II. SYSTEM MODEL

For illustration purpose and simplicity in presentation, we consider a single relay code division multiplexing system with

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Let $S_k$ denotes the $k$-th source and $D_k$ as its corresponding destination, $k = 1, 2, \cdots, K$, and let $R$ denotes the relay. The relay forwards the signals of all sources simultaneously. Let $b_k$ denote the transmitted information symbol of the source $S_k$. The signal sent by the source $S_k$ can be expressed as

$$s_k = c_k b_k,$$

where $c_k$ is a unit-energy column vector with length $L$, which is the code/signature of the source $S_k$. The codes/channles of different sources are, in general, correlated. Let $c_k^T c_j = \rho_{kj}$ for $k \neq j$ and $\rho_{kk} = 1$, where the code correlation $\rho_{kj} \in [0, 1]$ for $k \neq j$. Let $R \triangleq (\rho_{kj})$ denote the $K \times K$ cross-correlation matrix, i.e. $R = (c_1, c_2, \cdots, c_K)^T (c_1, c_2, \cdots, c_K)$.

We consider the following two-phase amplify-and-forward relay strategy. In Phase 1, each source $S_k$ transmits the signal $s_k$ with transmitted power $P_k$. Then, the received signals at the destination $D_k$ and at the relay $R$ can be modeled as

$$y_{s,d_k} = \sum_{i=1}^{K} \sqrt{P_i} h_{s,j,d_k} c_i b_i + n_{s,d_k},$$

$$y_{s,r} = \sum_{i=1}^{K} \sqrt{P_i} h_{s,j,r} c_i b_i + n_{s,r},$$

(1) (2)

In Phase 2, the relay amplifies the received signal and forwards it to the destination with an amplification factor $\alpha$ and transmission power $P_r$. The received signal at the destination $D_k$ can be written as

$$y_{r,d_k} = \sqrt{P_r} h_{r,d_k} \alpha y_{s,r} + n_{r,d_k}.$$

(3)

Substituting (2) and (4) into (3) yields

$$y_{r,d_k} = \sqrt{P_r} \alpha h_{r,d_k} \sum_{i=1}^{K} \sqrt{P_i} h_{s,j,r} c_i b_i + \sqrt{P_r} \alpha h_{r,d_k} n_{s,r} + n_{r,d_k}.$$

(4)

The combined received signal from Phase 1 and Phase 2 at destination $D_k$ can be expressed in vector form as follows:

$$y_k = \left( \begin{array}{c} y_{s,d_k} \\ y_{r,d_k} \end{array} \right) = H_{k,k} c_k b_k + \sum_{l=1 \neq k}^{K} H_{l,k} c_l b_l + n_k,$$

(5)

where $H_{k,k} = \left( \begin{array}{c} \sqrt{P_k} h_{s,j,d_k} I_L \\ \sqrt{P_r} \alpha h_{r,d_k} \end{array} \right)$ is a $2L \times L$ virtual channel matrix from the source $S_k$ to the destination $D_k$, and $n_k = \left( \begin{array}{c} \sqrt{P_k} \alpha h_{r,d_k} n_{s,r} + n_{r,d_k} \end{array} \right)$ is an equivalent noise vector of length $2L$. We note that $H_{k,k}$ is the channel matrix associated with the desired source $S_k$, while $H_{k,l}$ is the channel matrix of interfering sources.

The destination $D_k$ combines the signal received from the source in Phase 1 and the signal received from the relay in Phase 2 to jointly detect the information symbol transmitted by the source $S_k$. Based on the maximum ratio combining (MRC) detection [12], the transmitted signal from the source $S_k$ is detected as

$$\hat{b}_k = \text{sign}(R e\{w_k^H y_k\}),$$

where the weight vector $w_{k,o}$ is chosen to maximize the SINR at the destination $D_k$, which is given by

$$\text{SINR}(w_k) = \frac{E\{|w_k^H H_{k,k} c_k b_k|^2\}}{E\{|w_k^H (\sum_{l=1 \neq k}^{K} H_{l,k} c_l b_l + n_k)|^2\}},$$

(6)

i.e.

$$w_{k,o} = \text{arg max}_{w_k} \text{SINR}(w_k).$$

**III. SYSTEM PERFORMANCE ANALYSIS**

To determine the maximum SINR weight vector $w_{k,o}$ and the corresponding SINR at the destination $D_k$, we first define

$$U_k \triangleq \sum_{l=1 \neq k}^{K} H_{l,k} c_l c_l^H H_{l,k} + \Gamma_k,$$

$$\Gamma_k \triangleq E\{n_k n_k^H\} = \left( \begin{array}{cc} I_L & 0 \\ 0 & (P_r \alpha^2 |h_{r,d_k}|^2 + 1) I_L \end{array} \right),$$

Then (5) becomes

$$\text{SINR}(w_k) = \frac{|w_k^H H_{k,k} c_k|^2}{w_k^H U_k w_k}.$$

(7)

Using Schwartz inequality

$$|w_k^H U_k^2 w_k|^2 \leq |w_k^H U_k|^2 |w_k^H U_k^2 w_k|^2,$$

the maximum SINR weight vector $w_{k,o}$ is given by

$$w_{k,o} = (U_k)^{-1} H_{k,k} c_k,$$

while the corresponding maximum SINR at the destination $D_k$ with the optimum weight vector $w_{k,o}$ is equal to

$$\text{SINR}_k = c_k^H H_{k,k} c_k.$$

(8)
In order to design a scheme that optimally allocates powers to all sources, we further exploit SINR in (7). If we define \( \hat{C}_k \triangleq diag(c_1, \ldots, c_{k-1}, c_{k+1}, \ldots, c_K) \), then \( C_k \) is a \( L(K-1) \times (K-1) \) block diagonal matrix formed by placing all code vectors except \( c_k \) in the diagonal positions, and \( H_k \triangleq \begin{bmatrix} H_{1,k}, & \cdots, & H_{k-1,k}, & H_{k+1,k}, & \cdots, & H_{K,k} \end{bmatrix} \) which is a \( 2L \times L(K-1) \) interference channel matrix. Using the above notation, SINR can be expressed as

\[
\text{SINR}_k = c_k^H H_k^H (H_k C_k \hat{C}_k^H H_k^H + \Gamma_k)^{-1} H_k c_k
\]  

(8)

According to the Woodbury matrix inversion lemma [11], we have

\[
(H_k C_k \hat{C}_k^H H_k^H + \Gamma_k)^{-1} = \Gamma_k^{-1} - \Gamma_k^{-1} H_k C_k (I_{K-1} + \hat{C}_k^H \Gamma_k^{-1} H_k C_k)^{-1} \hat{C}_k^H \Gamma_k^{-1} \Gamma_k^{-1}.
\]

Let’s denote matrix \( F(k) \triangleq (f_{mn}^{(k)}) \), where

\[
f_{mn}^{(k)} = \rho_{mn} \sqrt{P_m P_n} \left( h_{mn,dk}^* h_{mn,dk} + \alpha^2 P_m |h_{mr,dk}^* h_{mr,dk}^*| + 1 \right),
\]

and denote \( f(k) \) as the \( k \)-th column vector of the matrix \( F(k) \). Then, after some algebraic calculations, we can see that

\[
\begin{align*}
&f_{mn}^{(k)} = C_k^H H_k^H \Gamma_k^{-1} H_k c_k, \\
&f_{k}^{(k)} = H_k^H \Gamma_k^{-1} H_k c_k, \\
&F_{k}(k) = C_k^H \hat{C}_k^H \Gamma_k^{-1} H_k c_k,
\end{align*}
\]

(10)

where \( f_{mn}^{(k)} \) includes the channels and the cross-correlation between the intended source \( S_k \) and the interfering sources, while \( F_{k}(k) \) includes the channels and the cross-correlation among interfering sources. Based on (8)-(10), we can represent SINR as

\[
\text{SINR}_k = f_{kk}^{(k)} - f_{k}^{(k)} (I_{K-1} + F_{k}(k))^{-1} f_{k}^{(k)}.
\]

(11)

### IV. OPTIMUM POWER ASSIGNMENT

#### A. General Optimization

We try to minimize the total transmission power of all sources and the relay under the condition that the SINR requirement of each source-destination pair is satisfied. Assume that the SINR requirement for the pair between source \( S_k \) and destination \( D_k \) is \( \gamma_k, k = 1, 2, \ldots, K \). Then, the optimization problem can be formulated as:

\[
\begin{align*}
&\min_{P_1, \ldots, P_k, P_r} \sum_{k=1}^{K} P_k + P_r, \\
&\text{s.t.} \quad \text{SINR}_k \geq \gamma_k, \quad k = 1, 2, \ldots, K.
\end{align*}
\]

(12)

Let us define a parameter

\[
x = \alpha^2 P_r,
\]

where \( \alpha \) is the amplification factor specified in (4). The parameter \( x \) will play a key role in the optimization. Furthermore, let us denote matrix \( G(k) = (g_{mn}^{(k)}) \) with elements

\[
g_{mn}^{(k)} \triangleq \rho_{mn} |h_{mn,dk}^* h_{mn,dk} + x |h_{mr,dk}^* h_{mr,dk}^*| x |h_{mr,dk}^*|^2 | + 1 |, \quad (14)
\]

for any \( m, n = 1, 2, \ldots, K \). Then, (9) implies that we can represent each entry in \( F(k) \) by \( f_{mn}^{(k)} = \sqrt{P_m P_n} g_{mn}^{(k)} \). It is straightforward to verify that

\[
det(F(k)) = \left( \prod_{l=1}^{K} P_l \right) \det(G(k)) \quad \text{and} \quad det(F_k^{(k)}) = \left( \prod_{l=1, l \neq k}^{K} P_l \right) \det(G_k^{(k)}).
\]

Based on the Schur complement formula, we have

\[
det(I_k + F(k)) = det \left( \begin{pmatrix} I_{K-1} + f_{k}^{(k)} & f_{k}^{(k)} \end{pmatrix} \begin{pmatrix} \Gamma_k^{-1} & 0 \\ 0 & \Gamma_k^{-1} \end{pmatrix} \begin{pmatrix} I_{K-1} + f_{k}^{(k)} \end{pmatrix} \right)
\]

\[
= \frac{det(I_{K-1} + F(k))}{det(I_{K-1} + f_{k}^{(k)})}
\]

\[
\times \left[ 1 + f_{kk}^{(k)} \right]^{-1} \left( (I_{K-1} + f_{k}^{(k)})^{-1} f_{k}^{(k)} \right).
\]

(15)

where the last equality follows from the expression of SINR in (11). Thus, we have

\[
1 + \text{SINR}_k = \frac{det(I_k + F(k))}{det(I_{K-1} + f_{k}^{(k)})}.
\]

(16)

We note that for moderate or high SINR region, we can approximate \( 1 + \text{SINR}_k \approx \text{SINR}_k \) and \( 1 + g_{kk}^{(k)} \approx g_{kk}^{(k)} \), \( \forall k, l = 1, \ldots, K \). So, based on (16), we can approximate SINR as

\[
\text{SINR}_k \approx \frac{det(F(k))}{det(F_k^{(k)})} = \frac{P_k \det(G(k))}{\det(G_k^{(k)})}.
\]

(17)

Therefore, the optimization problem in (12) can be written as

\[
\begin{align*}
&\min_{P_1, \ldots, P_K, P_r} \sum_{k=1}^{K} P_k + P_r, \\
&\text{s.t.} \quad \frac{P_k \det(G(k))}{\det(G_k^{(k)})} \geq \gamma_k, \quad k = 1, 2, \ldots, K.
\end{align*}
\]

(18)

Let \( V \) denote the feasible set for the optimization problem in (18), i.e., \( V = \{ P_1, \ldots, P_K, P_r \mid \text{SINR} \geq \gamma_k, \forall k = 1, 2, \ldots, K \} \). We may partition \( V \) into disjoint subsets such as \( V = \bigcup_{x \geq 0} V_x \), where

\[
V_x = \{ P_1, \ldots, P_K, P_r \mid \text{SINR} \geq \gamma_k, \forall k = 1, \ldots, K, \alpha^2 P_r = x \}
\]

for any \( x \geq 0 \).

From (13), we can see that for any given \( x = \alpha^2 P_r \), the transmission power at the relay \( P_r \) can be determined as

\[
P_r = \frac{x}{\alpha^2} = x \sum_{k=1}^{K} \rho_{sk} g_{sk,r}^2 + x L.
\]

(19)

Thus, the optimization problem in (18) over the feasible set \( V_x \) for any given \( x \geq 0 \) becomes

\[
\begin{align*}
&\min_{P_1, \ldots, P_K} \sum_{k=1}^{K} (x \rho_{sk}^2 x + 1) c_k + x L, \\
&\text{s.t.} \quad P_k \geq \gamma_k \frac{\det(G_k^{(k)})}{\det(G(k))}, \quad k = 1, 2, \ldots, K.
\end{align*}
\]

(20)

We observe that in (20), for any fixed \( x \), \( \gamma_k \frac{\det(G_k^{(k)})}{\det(G(k))} \) is a constant that is independent of \( P_k (k = 1, 2, \ldots, K) \). Hence the minimal value in (20) is obtained when all constraints hold with equality, i.e.,

\[
P_k = \gamma_k \frac{\det(G_k^{(k)})}{\det(G(k))}, \quad k = 1, 2, \ldots, K.
\]

(21)

2Assume that the matrix \( D \) is invertible, then [11]

\[
det \begin{pmatrix} A & B \\ C & D \end{pmatrix} = det(D) \cdot det(A - BD^{-1} C).
\]
Then, the corresponding minimal total power of (20) is

\[ v(x) \triangleq \sum_{k=1}^{K} \gamma_k(x \sigma_{k,r}^2 + 1) \frac{\det(G_k^{(k)})}{\det(G^{(k)})} + xL, \quad (22) \]

which is a function of \( x \geq 0 \). Let us denote \( v^* \) as the minimal value of the object function in (18) in the feasible set \( V \). Then, we can see that

\[ v^* = \min_{x \geq 0} v(x). \quad (23) \]

The above discussion implies that we can reformulate the optimization problem in (12) over a multi-dimensional searching space to the minimization problem in (23), which depends only on one variable. The minimization of \( v(x) \) in (22) can be easily solved by numerically searching for the optimal value of the parameter \( x \geq 0 \). With the optimal value \( x^* \) that minimizes the function \( v(x) \) in (22), we can obtain the corresponding optimal power \( P_k^* \) and \( P_r^* \) based on (21) and (19), respectively.

B. Simplified Optimization with Orthogonal Signatures

When the signature vectors \( c_k \) for different sources are orthogonal to each other, we are able to further simplify the minimization of \( v(x) \) in (22). More specifically, since for his case, the cross-correlation matrix \( R \) is an identity matrix, both \( G_k^{(k)} \) and \( G^{(k)} \) are diagonal matrices, and

\[ \frac{\det(G_k^{(k)})}{\det(G^{(k)})} = \prod_{k=1}^{K} \frac{\det(9_k^{(k)})}{\det(9^{(k)})} = \frac{1}{g(k)}, \quad (24) \]

where \( g_k^{(k)} \) is specified in (14). Thus, by substituting (24) into (22), the minimization function \( v(x) \) is given by

\[ v(x) = \sum_{k=1}^{K} \gamma_k(x \sigma_{k,r}^2 + 1) \left( \frac{|h_{sk,dk}|^2 + x|h_{sr,dk}|^2|h_{sk,dr}|^2}{x|h_{sr,dk}|^2 + 1} \right)^{-1} + xL \]

\[ = \sum_{k=1}^{K} \gamma_k (c_k + b_k x + \frac{a_k}{x + d_k}), \quad (25) \]

where

\[ a_k = \frac{|h_{sk,dr}|^2}{A|h_{sr,dk}|^2}, \quad b_k = \frac{L}{K \gamma_k} + \frac{\sigma_{sr,r}^2}{A}, \quad c_k = 1 + \frac{\sigma_{sr,r}^2}{A|h_{sr,dk}|^2}, \quad d_k = \frac{|h_{sk,dr}|^2}{A|h_{sr,dk}|^2}, \]

and

\[ A = \frac{|h_{sk,dr}|^2 + |h_{sk,dr}|^2}{|h_{sr,dr}|^2}. \]

We note that for any \( k = 1, 2, \ldots, K \), each term \( c_k + b_k x + \frac{a_k}{x + d_k} \) in (25) is convex with respect to \( x \), and it can be minimized by \( x_k = \max(0, -d_k + \sqrt{\frac{2a_k}{b_k}}) \).

Let us denote \( x_{\text{min}} = \min(x_1, x_2, \ldots, x_K) \) and \( x_{\text{max}} = \max(x_1, x_2, \ldots, x_K) \). Then, the optimal solution \( x^* \) that minimizes the function \( v(x) \) in (25) is bounded as \( x_{\text{min}} \leq x^* \leq x_{\text{max}} \). Thus, to find the optimal solution \( x^* \), we only need to search within the range \( [x_{\text{min}}, x_{\text{max}}] \).

Note that the necessary condition for \( x_k > 0 \) is \( a_k > b_k d_k \), which implies that

\[ \frac{|h_{sk,dr}|^2}{|h_{sr,dr}|^2} > \frac{|h_{sk,dr}|^2}{|h_{sr,dr}|^2}. \quad (26) \]

In a non-fading or slow-fading scenario, we can safely assume that \( h_{s_i,j}^2 \approx \sigma_{s_i,j}^2 \). Then (26) is equivalent to \( \sigma_{r,sk}^2 > \sigma_{sk,dk}^2 \).

If none of the inequalities is true, i.e., \( \sigma_{r,sk}^2 \leq \sigma_{sk,dk}^2 \) for all \( k = 1, 2, \ldots, K \), then \( x_{\text{max}} = 0 \), which implies that the optimal solution \( x^* = 0 \). So, the corresponding optimum power at the relay \( P_r^* = 0 \), which means that the relay is not needed. In other words, if any relay-destination channel link is weaker than the intended source-destination channel link, then we should not use relay.

V. Numerical Results

In this section, we perform numerical studies to illustrate the proposed optimum power assignment strategy. In our studies, we consider a slow fading scenario and then approximate \( h_{s_i,j}^2 \) by \( \sigma_{s_i,j}^2 \). We assume that the channel gain for each channel link follows a path loss model, where the variance of channel coefficients is given by \( \sigma_{s_i,j}^2 = \delta_{s_i,j} \) (i.e., \( \delta_{s_i,j} \geq 0 \)) with \( \delta_{s_i,j} \) being the distance of the channel link and \( \beta \) the path-loss exponent (\( \beta \) is equal to 2 in our numerical studies).

First, we consider a system with two source-destination pairs and one relay, i.e., \( K = 2 \). The cross-correlation of the two source signatures is \( \rho = 0.25 \). The values for the distances between nodes are set as follows: \( \delta_{s_1,d_1} = 2, \delta_{s_1,d_2} = 3, \delta_{s_2,r} = 1, \delta_{s_2,d_1} = 1, \delta_{s_2,d_2} = 3 \) and \( \delta_{r,d_2} = 2 \). We study two sets of SINR requirements for the two source-destination pairs: (i) \( \gamma_1, \gamma_2 = [10, 10, 10] \) dB, and (ii) \( \gamma_1, \gamma_2 = [10, 20, 20] \) dB. Fig. 2 plots the total power consumption versus the parameter \( x \). The optimal value \( x^* \) is 0.6253 and 1.523 for \( \gamma_1, \gamma_2 = [10, 10, 10] \) dB and \( [10, 20, 20] \) dB, respectively. Based on the optimal value \( x^* \), we obtain the corresponding optimal power allocation \( P_1, P_2, P_r \) according to (21) and (19), as listed in Table I. In this table, we also compare the optimal power values obtained by our proposed approximation.

<table>
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<th>Table I</th>
<th>Optimal Power Allocation Using the Proposed Method and the Exhaustive Search Method (( K = 2 )).</th>
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Fig. 2. Optimization of multi-source multi-destination relay network (\( K = 2 \)).
method and those obtained by exhaustive search based on the optimization in (12). We observe that the optimal power values obtained by the two methods are almost indistinguishable.

In the second study, we consider a system with three source-destination pairs and one relay, i.e. $K = 3$, in which the signatures of the three sources are orthogonal to each other. We examine both an asymmetric case (Fig. 3) and a symmetric case (Fig. 4). For both figures, we consider two sets of SINR requirements for the three source-destination pairs: (i) $[\gamma_1, \gamma_2, \gamma_3] = [10, 10, 20]$ dB, and (ii) $[10, 20, 20]$ dB. In Fig. 3, we consider an asymmetric system in which the distance values are set as $\delta_{s_k, r} = 1, (k = 1, 2, 3)$, $\delta_{r, d_1} = 1, \delta_{r, d_2} = 2, \delta_{r, d_3} = 3$ and $\delta_{s_1, d_1} = 2, \delta_{s_2, d_2} = 3$ and $\delta_{s_3, d_3} = 4$. In this case, the optimal values are $x^* = 2.9940$ and $x^* = 2.3126$ for the two sets of SINR requirements, respectively. Based on the optimal values, the optimal power allocation $P_1, P_2, P_3$ and $P_r$ can be obtained accordingly based on (21) and (19). In Fig. 4, we study a symmetric system in which the distance between each source and the relay is 1, the distance between the relay and each destination is also 1, and the distance between each source and its intended destination is 2. The optimal values are $x^* = 0.7876$ and $x^* = 0.7816$ for the two sets of SINR requirements, respectively.

VI. CONCLUSION

In this paper, we analyzed and optimized a multi-source multi-destination relay network in which the relay amplifies and forwards signals received from all sources. We minimized the total power consumption of all sources and the relay under the constraint that the SINR requirement of each source-destination pair is satisfied. The original optimization problem involves $K$ power variables, where $K$ is the number of source-destination pairs in the network, which implies that an exhaustive search approach is prohibitive for large $K$. Thanks to an asymptotically tight approximation of the SINR that we developed, we were able to reformulate the optimization problem, and eventually reduce to a single-parameter optimization problem. The resulting optimization problem can be easily solved by numerical search of the single parameter. Then the corresponding optimal transmission power at each source and at the relay can be calculated directly.

The proposed optimization scheme is scalable and the power assignment algorithm has the same optimization complexity for any number of source-destination pairs in the network. For the special case of transmission over orthogonal channels, we were able to further simplify the proposed single-variable optimization scheme, and obtain an analytical solution for a symmetric system. Numerical studies illustrated and validated our theoretical developments.

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