Rate-per-link adaptation in cooperative wireless networks with multi-rate combining

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Abstract—Rate adaptation based on Signal-to-Noise Ratio (SNR) measurements is a common channel adaptation scheme to increase throughput in wireless communication systems. To use rate adaptation efficiently in cooperative wireless networks, an adaptation algorithm must consider multiple channels (source-destination, source-relays, and relays-destination) to select modulation and code rates that maximize throughput. In this paper we analyze the potential gains that combining cooperation with rate adaptation brings in three steps: (1) We derive the theoretical capacity bounds for ideal rate adaptation schemes for typical topologies. (2) We propose an offline heuristic for computing SNR thresholds aimed at reaching the derived bounds. (3) Using this heuristic, we compare rate adaptation for Maximal Ratio Combining (MRC), where links are equally adapted, with Soft-Bit MRC (SBMRC), where links are individually adapted. We find that adapting the rate per link is superior in terms of throughput.

I. INTRODUCTION

High-quality voice and video streaming in wireless networks requires high throughput at a specific error bound, even if the wireless channel varies over time. Two different techniques have been established to improve throughput: Rate adaptation trades off the transmitter’s data rate and robustness to improve performance under varying channel conditions [1], e.g. by using the observed SNR for adapting the rate. It chooses higher-rate yet less robust modulations and Forward Error Correction (FEC) codes when channel conditions should still allow for meeting the error bound. Cooperative relaying, on the other hand, is a powerful technique for combating the detrimental effects of fading for a fixed rate. It achieves a lower error rate by letting relays forward overheard messages due to the broadcast nature of the channel (see Fig. 1) and by combining their independently faded signals at the receiver. If only correct messages are relayed, error propagation is avoided and full diversity can be reached [2]. Integrating this so-called Selection Decode-and-Forward (SDF) relaying into IEEE 802.11a Wireless Local Area Networks (WLANs) shows a significant increase in throughput for low transmission power [3]. In summary, rate adaptation benefits from lower error rates while SDF achieves lower error rates. Since both techniques contribute to throughput, albeit approaching it from opposite sides, the question arises whether a combination of both techniques can yield further gains than what each technique on its own has to offer. The major contribution of this paper is to answer the question with “Yes, if rate-per-link adaptation is used.”

This question has first been raised in [4], but due to a wrong path loss model, unrealistic power gains distorted the performance results. Better results were reported in [5] for cooperative networks using adaptive M-QAM transmissions using Amplify-and-Forward (AF) relaying. Source and relay always use the same modulations and the destination applies MRC on symbol level. It was observed that the channel’s capacity could be approached within 5 dB when rates were continuously adapted, with a 1.5 dB penalty using discrete rates. Unfortunately, requiring the same modulation on both source and relay uplinks, which in the presence of fading may experience quite different channel conditions, is a drawback as it reduces the spectral efficiency. For this reason, a combining scheme for different modulations was proposed in [6] that, after separately demodulating the signals from source and relay, weights and adds the resulting soft-bit symbols similar to MRC. The so-called SBMRC has been shown to be close to the optimal detector and outperforms selection combining (using the same, possibly different modulations) by almost 2 dB. SBMRC makes it possible to adapt the rates of source and relay per link, but to the best of our knowledge no such adaptation has been proposed for cooperative networks yet.

Therefore, after introducing the system model in Section II, we derive the outage capacity for SDF cooperative relaying with N relays in Section III to provide a theoretical benchmark for our practical rate adaptation schemes. Such a scheme is then developed in Section IV. Here we answer the question on how to adapt in practical systems where multiple links (as shown in Fig. 1) must be considered. We propose a heuristic for deriving a suitable set of SNR thresholds for adaptive SDF. After having determined thresholds for link-equal and per-link adaptation, we study the performance of adaptive SDF, lever-
aging SBMRC as the actual combining scheme, in different topologies in Section V. We conclude that using rate-per-link adaptation yields substantial gains for low transmission powers in slow-fading scenarios (Section VI).

II. SYSTEM MODEL

We consider a network in Fig. 1 consisting of a source $s$, $N$ relays $r_i$ ($1 \leq i \leq N$), and a destination $d$. Source and relay operate in half duplex mode which requires an orthogonal channel per relay for forwarding. For simplicity of exposition and without losing generality, we assume the channels to be separated in time and thus talk about phases. In the first phase, $s$ broadcasts a packet to all relays $r_i$ using the transmission mode $\tau_s$. This packet is overheard by $d$ because of the broadcast nature of the wireless channel. Each relay $r_i$ fully decodes the packet and forwards it to $d$ in the $(i+1)$th phase, using the transmission mode $\tau_r$. We consider the set of IEEE 802.11a transmission modes defined in [7]. The transmission modes $\tau_s$ and $\tau_r$ are assigned at $s$ based on SNR estimates of the $(s,d)$ and $(r_i,d)$ channels. For this, source and relays require feedback from the destination. Since we are interested in general results, we do not focus on specific signaling schemes and, thus, ignore the penalties that feedback naturally incurs except for a constant time delay. This delay is significant as SNR estimates are assumed to be constant only for the channel’s coherence time. Hence, the selected modes are always based on past channel knowledge but are exchanged instantaneously and without errors between the users. Unless noted otherwise, in this paper, all SNR values are linear and not in dB.

The instantaneous SNR of the channels $(s,r_i)$, $(s,d)$, and $(r_i,d)$ are denoted by $\gamma_{s,r_i}$, $\gamma_{s,d}$, and $\gamma_{r_i,d}$, respectively. For the Bit Error Rate (BER) results, the amplitudes of the signals received over the $(s,d)$ and $(r_i,d)$ channels are modeled as independent and identically distributed Rayleigh random variables. Consequently, $\gamma_{s,d}$ and $\gamma_{r_i,d}$ are independent exponential random variables. The links from $s$ to the relays $r_i$ must be error-free, thus a relay only cooperates if it can correctly decode the source’s message. The destination combines the signals received from the source and the relays to achieve spatial diversity and to reduce the undesirable effects of multi-path fading. For the simulations in Section V, time-correlated fading is assumed, but the channel stays constant during the transmission of a packet (i.e. block fading model). We denote the data rate used at the transmitter by $R^{tx}$ (transmitter rate, also known as throughput) and the rate of correctly received bits at the receiver by $R^{rx}$ (receiver rate, also known as goodput).

For the study of the single-relay case, we focus on three characteristic topologies depicted in Fig. 2. In the symmetric topology, the average path losses are identical on all channels, yet the instantaneous path loss of all three channels are independent of each other. Due to symmetry, all channels experience equal mean SNRs and, thus, on average the same error rate. In the asymmetric topology, the source to relay distance is halved, the power received from $(s,r)$ will be $2^\alpha$ higher ($\alpha$ is the path loss exponent), increasing the chances for the relay to cooperate. Finally, the chain topology also benefits the relay’s uplink to the destination by $2^\alpha$.

III. CAPACITY BOUNDS

We now derive the maximum throughput that an ideal communication system can achieve without exceeding a given error rate bound. This so-called outage capacity is a common theoretical measure for slow fading channels [8] and serves as a benchmark for our practical rate adaption schemes.

A. Outage probability of direct transmission and SDF

The outage capacity relies on an accurate characterization of the error rate in terms of outage probability – a metric which we discuss, first, for direct transmission and, second, for SDF cooperation with $N$ relays.

The error rate of a block fading channel can be characterized by outage probability. A single channel from node $s$ to node $d$ is in outage if its Shannon capacity $C(\gamma_{s,d}) = \log_2(1 + \gamma_{s,d})$ falls below a spectral efficiency $R$. The probability of this event is called outage probability $P_{out}^{DIR} = \Pr\{C(\gamma_{s,d}) < R\} = \Pr\{\gamma_{s,d} < 2^R - 1\}$.

If fading is the only error event, as assumed in this paper, $C$ solely depends on the instantaneous SNR, e.g. $\gamma_{s,d}$ for channel $(s,d)$. This random variable is defined by the Probability Density Function (PDF) of the fading process and is exponentially distributed for Rayleigh fading. For a single Rayleigh fading channel, we obtain

$$P_{out}^{DIR} = \frac{1}{\Gamma_{s,d}} \int_0^{2^R - 1} \exp\left(-\frac{\gamma_{s,d}}{\Gamma_{s,d}}\right) d\gamma_{s,d} = 1 - \exp\left(-\frac{2^R - 1}{\Gamma_{s,d}}\right)$$

by solving the exponential PDF at $2^R - 1$. Here, we denote the channel’s mean SNR as $\Gamma_{s,d}$ since factoring out the reference mean SNR $\Gamma$ allows us to study the channel-dependent terms (e.g. $\Gamma_{s,d}$) separately. Beside the mean SNR, (1) depends on the spectral efficiency $R = R^{tx}/W$ in bit/s/Hz chosen at the transmitter. In this paper, we assume that each transmitter (source and each relay) uses a constant bandwidth $W$ but employs rate adaptation to select the transmitter rate $R^{tx}$ in bit/s.

For direct transmission, the outage probability can be expressed in closed form as in (1). Nevertheless, deriving the outage probability for cooperative relaying requires more effort. Unlike direct transmission, with SDF an end-to-end transmission involves many channels between the cooperating

![Fig. 2. Three topologies for the single-relay case are studied.](image-url)
nodes and the destination. So far, for SDF cooperation with \( N \) relays, no exact closed-form outage probability is known. However, for these networks, we can approximate \( P^{\text{out}} \) for high SNR by cut set analysis [9]. With this method, SDF’s outage probability has the general form

\[
P^{\text{out}}_N \approx \frac{1}{L!} \Theta \left( \frac{2^{KR} - 1}{\Gamma} \right)^L \tag{2}
\]

if we assume i.i.d. Rayleigh fading channels [9], [10]. Here, \( K \) accounts for the number of used channels, \( \Theta \) includes all gains of these channels, \( \Gamma \) denotes the reference mean SNR, and exponent \( L \) the diversity order. If SDF employs \( N \) relays, \( K = N + 1 \) channels are employed and the full diversity order \( L = N + 1 \) is reached [2]. The channel-dependent term \( \Theta \) can be derived by cutting the network graph into sets as described in detail in [10].

With this general form, we can now derive the outage probability for the scenario used in this paper. As only a single relay \( r_1 \) is employed we have \( N = 1 \) and, thus, \( K = L = 2 \). With the instantaneous SNR \( \gamma_{d}, \gamma_{r_1}, \) and \( \gamma_{d} \) of the three used channels we obtain the channel-dependent term \( \Theta_T = \frac{\Gamma_{r_1} + \Gamma_{r_1,d}}{\Gamma_{r_1}} \) by cut set analysis. Finally, the outage probability of SDF using a single relay is

\[
P^{\text{out}}_{N=1} \approx \frac{1}{2 \Gamma_{s,d}} \Theta_T \left( \frac{2^{2R} - 1}{\Gamma} \right)^2 . \tag{3}
\]

Note that this specific solution of (2) is well-known [2] which provides a sanity check for (2). We will now employ these \( P^{\text{out}} \) results to derive SDF’s outage capacity for \( N \) relays.

B. Outage capacity of direct transmission and SDF

Outage capacity is defined as the largest transmitter rate (in terms of spectral efficiency \( R \)) supported at a given outage probability level \( \varepsilon \) [8]. Unlike Shannon or ergodic capacity, the outage capacity \( C^e \) reflects the error bound as an important design criteria of many wireless communication systems; e.g. IEEE 802.11a/g WLANs systems must not exceed a Packet Error Rate (PER) of 10% [7]. In this paper, such PER bound is expressed by \( \varepsilon \) as we assume block fading per packet as the only error event. Consequently, for an \( \varepsilon \)-constrained system, \( C^{e} \) provides the theoretical maximum for the received rate, i.e. the maximum goodput it can achieve under ideal conditions. No system with practical a rate adaptation scheme can reach a higher performance than \( C^{e} \).

According to its definition as largest \( R \) supported at a given \( \varepsilon \) we can derive \( C^{e} \) by solving \( P^{\text{out}}(R) = \varepsilon \) in \( R \). With our above \( P^{\text{out}} \) results we obtain

\[
C^e_1 := R \approx \frac{1}{K} \log_2 \left( \frac{\Gamma_{r_1}}{\Theta} \cdot \Gamma + 1 \right) \tag{4}
\]

as outage capacity in bit/s/Hz for SDF in cooperative networks with \( N \) relays. For a single relay, solving (3) in \( R \) gives

\[
C^e_{N=1} = \frac{1}{2} \log_2 \left( \frac{2 e \Gamma_{s,d}}{\Theta} \cdot \Gamma + 1 \right) \tag{5}
\]

and, finally, with (1) we obtain \( C^e_{\text{DIR}} = \log_2(\varepsilon \Gamma_{s,d} \Gamma + 1) \) for direct transmission.

Solving \( C^e_{\text{DIR}} \) and (5) for several values of \( \Gamma \), a path loss exponent \( \alpha = 3 \), and for \( \varepsilon = 0.1 \) (i.e. a WLAN’s 10% error rate bound) yields the numerical results in Fig. 3. Here, the results for direct transmission are equal in all topologies as the mean SNR is not changed among them (Section II). However, with SDF, a strong variation is shown between the topologies. In the symmetric case, SDF can reach only a negligible gain compared to direct transmission. Here, even ideal rate adaptation can only slightly improve throughput.

This is a result of the linear multiplexing loss \( 1/K \) in (4) which dominates \( C^{e} \) compared to the merely logarithmic effect of the diversity order \( L \).

With a weaker direct channel, however, still significant capacity gains can be found. The largest potential for rate adaptation is shown in the chain topology where the direct channel is weakest compared to both relay links. Nonetheless, even in the asymmetric topology significant throughput gains are achievable.

IV. A PRACTICAL HEURISTIC FOR RATE-PER-LINK ADAPTATION

SNR is a practical criterion for rate adaptation that can be used to predict the quality of the channel [1]. For an indoor scenario, it has been found that with 20 ms between two channel measurements, the prediction error is only around 1 dB. For this reason, we also use SNR as an adaptation criterion.

In a cooperative network such as the one in Fig. 1, \( 2N+1 \) channels need to be considered, i.e. one direct channel, \( N \) inter-user channels, and \( N \) uplink channels. It is not clear which set of transmission modes a rate adaptation algorithm should choose for SDF’s cooperative relaying to maximize the receiver rate. To answer this question, we propose a heuristic that iterates over the entire range of discretized SNR values
Coding. We denote the PER for reference transmissions that we use to look up the BER after and its results are to be used at run-time in the network to look channels to find the appropriate transmission modes. This SDF cooperative relaying with rate adaptation.

Transmit mode probability of successful reception at the destination. The rate that a transmitter achieves with mode \( \tau \) is given, assuming that mode \( \tau \) is used. with SNR tuple (Line 1), the heuristic determines the transmission modes used by source \( s \) and relays \( r_i \) that maximize the receiver rate \( R_{rx} \). To do so, the heuristic must be able to compute the PER of a transmission when a particular transmission mode \( \tau \) is used, given the SNR \( \gamma \). For rate adaptation, we use the transmission mode \( \tau \) found with our heuristic for the link-equal (Line 3), i.e. \( \tau_s = \tau_1 = \cdots = \tau_N \), because the block fading assumption, \( \gamma \) holds for the entire packet. Correspondingly, the PER achieved using a cooperative transmission from all users to the destination can be stated as \( P_{coop}(\tau_s, \tau_1, \ldots, \tau_N, \gamma_d, \gamma_1, \ldots, \gamma_N,d) \) where each user may use an individual transmission mode with all uplink SNRs given. Due to the SDF protocol, we require the transmissions to all relays to be successful (Line 4) and \( S_r \) denotes the probability for that. Similarly, \( S_d \) denotes the probability of successful reception at the destination. The rate \( R_{rx} \) accounts for the correctly received bits (Line 6) during a cooperative transmission cycle, where \( R_{rx}(\tau) \) denotes the rate that a transmitter achieves with mode \( \tau \). For all possible combinations of transmission modes (Line 3), the receiver rate is computed and the rate-maximizing modes (Line 8) are finally stored in a \((2N + 1)\)-dimensional matrix \( A \) (Line 11). At run-time, the measured SNR values will be used to look up the transmission modes to use for SDF cooperative relaying.

Conventional MRC [12] requires all transmitters to use the same modulations and code rates. In this case, the heuristic can only consider equal modes in Line 3, i.e. \( \tau_s = \tau_1 = \cdots = \tau_N \), severely reducing the search space. In the case of IEEE 802.11a, only 8 out of \( 8^{N+1} \) possibilities remain. The limitation of equal rates is no longer needed for SBMRC. We identify the additional gain that this more flexible combining brings in Fig. 5 for the case of \( N = 1 \) relay. It shows the difference in receiver rate for the single-relay case between an unrestricted run of the heuristic, allowing for rate-per-link adaptation, and a restricted run with \( \tau_s = \tau_N \), only offering link-equal adaptation. We find that a notable difference in rate of up to 5 Mbit/s exists for an IEEE 802.11a physical layer when the SNRs vary by at least 5 dB and at most 15 dB. No difference is found below or beyond, which indicates that a minimum exists above which unequal rates better utilize the channel as well as a maximum in what a flexible combining algorithm such as SBMRC can achieve.

V. PERFORMANCE EVALUATION

To characterize the performance that a combination of SDF cooperative relaying and rate adaptation brings, we compare it in terms of receiver rate with the following schemes:

Direct transmission – The transmission on the \((s,d)\) channel does not have multiplexing losses and achieves the best receiver rate for high SNR, but will be more susceptible to fading effects for low SNR.

Selection Decode-and-Forward (SDF) – This is cooperative relaying using the SDF protocol with diversity combining at the destination. For rate adaptation, we use the transmission modes found with our heuristic for the link-equal \( (\tau_s = \tau_N) \) case with MRC and the per-link case with the more general SBMRC. We also provide results for a static choice of the most robust transmission mode (BPSK 1/2), as it is the most pessimistic assumption and no rate adaptation scheme can adapt below it.

Non-Cooperative Relaying (NCR) – Here, the destination processes only packets received from the relay which, in turn, forwards the source’s correctly decoded packets. Direct
rate adaptation is used independently for the $(s, r)$ and $(r, d)$ channels for maximum spectral efficiency. This scheme does not exploit cooperation diversity but also benefits from the additional power injected by the relay.

We study the topologies shown in Fig. 2 with path loss exponent $\alpha = 3$ and distance $d = 50$ m. Fig. 6 shows the receiver rate for the asymmetric topology and $v = 1$ m/s, corresponding to e.g. an indoor scenario. SDF with BPSK 1/2 as the most robust static transmission mode achieves an increase of up to 1 Mbit/s for low transmission powers due to diversity combining that direct transmission with rate adaptation cannot compensate for as it cannot switch down lower than BPSK 1/2, i.e. it cannot become more robust. NCR can adapt individually to the $(s, r)$ and $(r, d)$ channels with the same multiplexing loss like SDF, but it does not enjoy cooperative diversity. Its performance is even worse than direct transmission because two channels are predicted separately and bad estimates on both hurt twice. Cooperation with rate adaptation is not feasible at high speeds (whereas static cooperation is), so we focus on $v = 1$ m/s for the remaining two topologies.

At this low speed, for the symmetric topology, direct transmission with rate adaptation outperforms any other scheme as the multiplexing losses dominate the diversity gains (Section III), which is especially severe if the source optimizes its transmitter rate (see Fig. 8). If, however, the relay is placed half way in-between source and destination, the additional power injected by the relay offers tremendous diversity gains (see Fig. 9). Adaptive SDF achieves a maximum gain of 4 Mbit/s in almost twice the power range; although adaptive NCR outperforms direct transmission as well, the difference to adaptive SDF is with a maximum of roughly 1.8 Mbit/s still significant.

Since rate adaptation is susceptible to the channel’s coherence time due to prediction errors, we also study the effect of increasing the relative speed $v$ depicted in Fig. 7. For $v = 50$ m/s, corresponding to e.g. a moving train, the channel state decorrelates, making it impossible for rate adaptation to predict the channel state. Consequently, the static choice of BPSK 1/2 achieves the best receiver rate for low power. Interestingly, adaptive SDF performs worse than direct transmission because two channels are predicted separately and bad estimates on both hurt twice. Cooperation with rate adaptation is not feasible at high speeds (whereas static cooperation is), so we focus on $v = 1$ m/s for the remaining two topologies.
VI. CONCLUSION

We proposed a practical rate adaptation scheme and assessed its performance in WLANs with cooperative relaying. Our main findings are:

1) In symmetric scenarios, cooperation diversity cannot compensate for relaying’s multiplexing loss. Thus, with symmetric links, even with perfect cooperation and rate adaptation, only insignificant throughput gains are possible.

2) Forcing the relay to use the same rate as the source has adverse effects on the received rate for two reasons: On the one hand, if the relay’s channel to the destination is significantly better than that of the source, using the source’s low adapted rate causes an unnecessarily long retransmission. If, on the other hand, the relay experiences a significantly worse channel than the source, then using the source’s adapted rate may be too high causing an unnecessarily error-prone retransmission. Both cases are costly and limit the achievable throughput. Thus, especially for low speeds and slow fading, rate-per-link adaptation should be used with cooperative relaying.

3) For high speeds and fast fading, combining rate adaptation with cooperative relaying is not beneficial anymore as prediction errors are more costly.

Future work should now analyze which performance can still be reached in practical systems where feedback is not for free.

REFERENCES


