EFFICIENT RESOURCE ALLOCATION IN COOPERATIVE COGNITIVE RADIO NETWORKS: A COALITIONAL GAME APPROACH

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Abstract

In this paper, we consider the resource allocation problem of cooperative spectrum sharing among multiple primary users (PUs) and multiple secondary users (SUs), where the PUs lease their spectrum to the SUs in a time-division manner while the SUs in return help the data transmission of the PUs by relaying. The PU’s strategy is to decide a proper fraction of spectrum leasing time to SUs for maximizing its payoff, while the SU’s strategy is to decide the proper power for relaying PU’s traffic, and for transmitting its own data so as to satisfy its payoff. We formulate the problem as a coalitional game with transferable utility (TU), and propose an algorithm to show the game converges to the solution concept (the core). We also show that the grand coalition formed by grouping all PUs and SUs is stable. Finally, we demonstrate in simulations the benefits of our proposed approach comparing to other approaches of power control.

1. Introduction

In the past few years, it has been grown significantly in the demand for wireless services. The FCC report [1] stated the problem of unbalanced spectrum utilization in the traditional spectrum allocation mechanism which indicates that some frequency bands are unoccupied most of the time; whereas others are heavily used. Cognitive radio has emerged as a promising approach for improving the utilization of spectrum resource. Recently, with the emergence and success of distributed cooperative communications in wireless networks, a new communication paradigm, termed cooperative cognitive radio networks (CCRN), in cognitive radio networks is proposed [2–5]. In CCRN, licensed primary users (PUs) actively select those secondary users (SUs) with good channel conditions as their cooperative relays and lease available spectrum to them. By relaying data for the PUs, the SUs gain opportunities to access the spectrum otherwise unavailable to them. Therefore, a “win-win” situation is created where PUs can potentially increase their transmission rates and SUs can access the available frequency bands to transmit their own traffic.

In [2], a distributed power control mechanism was designed to obtain the optimal power allocation for SUs in the CCRN. In [3], the authors also studied the resource allocation problem of CCRN, but no power control was considered. In [2] [3], the spectrum leasing problem was formulated for one PU and multiple SUs as a Stackelberg game and the Nash equilibrium was derived. In such systems, a single channel was considered and different transmissions were divided in time. However, the consideration of one channel and one PU in [2] [3] presents a simplification for practical scenarios where there are typically multiple channels and multiple PUs coexisting in the coverage area of a base station in the cellular network. Therefore, in this paper, we consider the problems of power control and time fraction allocation for multiple PUs and multiple SUs in CCRN. To tackle these problems, we formulate the problem as a canonical coalition game with transferable utility (TU) in the proposed framework. In addition, we propose an iterative algorithm to solve the non-convex problem which considers the power control and time fraction allocation, and enables the derived equilibrium
to converge to the core solution [6]. Furthermore, we show that the core is nonempty (i.e., the grand coalition formed by all PUs and SUs is stable) in the system.

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Figure 1: An illustration of the CCRN with two PUs and two SUs. (a) Primary transmission. (b) Cooperative transmission. (c) Secondary transmission.

2. System model

2.1 System setup

We consider the uplink transmission of a cooperative cognitive radio network, which entirely consists of \( N_b \) available channels, \( N_p \) PUs, a primary receiver called the primary base station (PBS), \( N_s \) SUs, and a cognitive receiver called the secondary access point (SAP). Let the set of PUs and SUs be \( N_p = \{1, \ldots, N_p\} \) and \( N_s = \{1, \ldots, N_s\} \), respectively. In this system, we assume that the ith PU communicates with PBS using the ith channel only, and the set of all PUs’ channels in the primary network is \( N_b = \{1, \ldots, N_p\} \). The unlicensed cognitive SUs can only transmit their own traffic using the channels leased from a group of PUs who are willing to cooperate, in which together the cooperating SUs and PUs form a coalition.

A channel licensed for PU \( m \), \( m \in N_p \) can be used by SU \( k \), \( k \in N_s \) only if both of them are in the same coalition. On the other hand, an SU can assist the transmission of a PU only when they are in the same coalition. Figure 1 depicts a simple example of our proposed model with two PUs and two SUs. The formal definition of a coalition is as follows.

Definition 1 A coalition \( S \) is a set of players (e.g., PUs and SUs) that cooperate with each other. In CCRN, a coalition can be partitioned by \( S_p \) and \( S_s \), which respectively represent the set of PUs and SUs in \( S \). The union \( N_p \cup N_s \) of all PUs and SUs is referred to as the grand coalition.

Next, we describe the cooperation mechanism and system assumptions in the following:

1. Decode-and-forward (DF) protocol [7] is adopted at SUs to relay PUs’ traffic, and SU’s power control is considered. Besides, we assume flat Rayleigh faded channel for each link that is invariant within each slot. At the PBS, the receiver employs maximal ratio combining (MRC) to combine signals from the direct link and the relay links. The channel state information (CSI) is assumed available at the PBS.

2. Cooperative transmissions for PUs and SUs are scheduled in a time-division fashion. As shown in Fig. 1, each time slot is divided into three parts: the first \( 1 - \alpha \) fraction of the slot is used for PUs’ transmission \((0 < \alpha < 1)\), and the second \( \beta \) fraction is used for SUs to relay the received data to PBS \((0 < \beta < 1)\). The value of \( \beta \) is designed to be less than and equal to \( \alpha \) so as to ensure the fraction of time is feasible. In the last fraction of slot, SUs can access channels for transmitting their own traffic to SAP.

3. Each PU has only one available channel for lease in the formed coalition. Since each SU helps to relay the traffic from multiple PUs in the coalition, each SU gains the opportunity to access multiple channels simultaneously. To differentiate SUs in the same channel, each SU is assigned a unique spreading code. We assume that through the spreading code mechanism, the interference among SUs can be avoided.

2.2 Utility functions definition

First, the achievable data rate for PU \( m \) with the help of SU \( k \) is determined by two stages, they are PUs-SUs,
and SUs-PBS, respectively. In the first stage, the transmission rate of PU \( m \) achieved from PU \( m \) to SUs is dominated by the worst channel which is given by

\[
    r_{m,s} = (1 - \alpha) \log_2 \left( 1 + \frac{P_{pm} \min_{k \in S_s} |h_{pm,s,k}|^2}{\sigma^2} \right),
\]

where \( P_{pm} \) is the transmission power of PU \( m \), which is set to be fixed. \( h_{pm,s,k} \) denotes the channel gain between PU \( m \) and SU \( k \); \( (1 - \alpha) \) is the fraction of time for PU’s transmission. In addition, for notational simplicity, we have assumed that all receivers have the same noise level, denoted as \( \sigma^2 \).

Next, in the second stage, with SUs’ received data from PUs, the PU \( m \)’s transmission rate can be presented as:

\[
    r_{s,p_m} = \beta \log_2 \left( 1 + \frac{P_{pm} |h_{pm}|^2}{\sigma^2} + \sum_{k \in S_s} \frac{P_{sk,p_m} \max_{j \in S_s} |h_{sk,j,p}^j|^2}{\sigma^2} \right),
\]

where \( P_{sk,p_m} \) is the power of SU \( k \) for relaying PU \( m \)’s traffic. \( h_{pm} \) denotes the channel gain between PU \( m \) and PBS; \( h_{sk,j,p}^j \) denotes the channel gain of channel \( j \) from SU \( k \) to PBS. For simplicity of the system design, we have assumed that SUs’ power consumed for relaying PU \( m \)’s traffic are identical. Therefore, the overall achievable transmission rate of PU \( m \) equals to the minimum transmission rate of the above two stages,

\[
    r_{pm} = \min \{ r_{m,s}, r_{s,p_m} \}, \quad m \in S_p.
\]

The transmission rate of SU \( k \) can be calculated by the sum rate of available channels,

\[
    r_{sk} = (\alpha - \beta) \sum_{j \in S_s} \log_2 \left( 1 + \frac{P_{sk} |h_{sk,j}^j|^2}{\sigma^2} \right),
\]

where \( P_{sk} \) is the power of SU \( k \) for transmitting its own traffic. \( h_{sk,j}^j \) is the channel gain of channel \( j \) from SU \( k \) to SAP.

The cost of SU \( k \) is represented by,

\[
    \zeta_k = c_s \cdot \beta \sum_{m \in S_p} P_{sk,p_m} + w_s \cdot (\alpha - \beta),
\]

where \( \beta \) is the fraction of time for SU’s relaying, \( c_s \) is SU’s cost per relaying energy, \( (\alpha - \beta) \) is the fraction of time for SU’s accessing, and \( w_s \) is SU’s cost per access time.

Therefore, the utility of PU \( m \) is defined as

\[
    U_{pm} = F(r_{pm})
\]

where \( F(r_{pm}) \) is a concave increasing function mapping each PU’s rate to a utility gain. For example, it can be a linear function, i.e. \( F(r_{pm}) = r_{pm} \). On the other hand, the utility of SU \( k \) is defined as

\[
    U_{sk} = G(r_{sk}) - \zeta_k
\]

where \( G(\cdot) \) is a concave increasing function representing the gain of SU per unit of rate. \( \zeta_k \) is the consuming cost of SU \( k \).

In the following, we will adopt a coalitional game theory to give an analysis of the optimal cooperative strategies in the proposed network.

### 3. Coalitional game formulation and solution concept

A TU coalitional game is one in which the yield of the coalition value can be apportioned arbitrarily between the coalition members [6]. In our system, since the coalition value can be properly defined as the sum of utility generated by PUs and SUs in the coalition, a TU game applies to our considered network. Therefore, we formulate our considered cooperative cognitive radio network as a canonical coalitional game with TU.

In the following, we will present the defined coalition value \( v(S) \) and show that this coalitional game has desired properties that ensure a stable grand coalition and a feasible payoff allocation.

By Definition 1 and using the above defined utilities of PUs and SUs, the coalition value \( v(S) \) is calculated as follows:

\[
    v(S) \triangleq \text{Maximize: } \sum_{m \in S_p} F(r_{pm}) + \sum_{k \in S_s} \left( G(r_{sk}) - \zeta_k \right)
\]

Subject to:

1. \( 0 \leq \beta \leq \alpha \leq 1 \),
2. \( P_{sk,p_m} + P_{sk} \leq P_{max}, \quad m \in S_p, \quad k \in S_s \),
3. \( P_{sk}, \quad P_{sk,p_m} \geq 0, \quad m \in S_p, \quad k \in S_s \).

(6)

To ensure the time fraction allocation to be feasible, \( \alpha \) is no less than \( \beta \) and confined in one unit of time by constraint (1). Constraint (2) guarantees the SU’s total consumption power which is used for relaying primary transmission and for its own traffic is less than and equal to the maximum power it has. Constraint (3) guarantees that the SU’s power for relay or for its own traffic is feasible.

To examine the problem’s convexity, the second derivative test of \( v(S) \) with respect to \( \alpha, \beta, P_{sk,p_m}, \) and \( P_{sk} \)
are performed. The derivation result equals zero, that mean the optimal point is not guaranteed to be existed. Therefore, we propose the following Algorithm 1 to solve the problem formulated in (6), which is specified in the following section.

**Algorithm 1** Iterative algorithm for power control and time fraction allocation

0: Define: \( v(S)^{(n)} \) denotes the coalition value at the \( n \)th iteration. And \( \Omega^{(n)} \) denotes the maximum coalition value at the \( n \)th iteration.
1: Initialize: Initialize \( \alpha^{(1)} \), \( \beta^{(1)} \), and \( \Omega^{(1)} \).
2: Repeat: Initialize \( P_{sk}, P_{pm} \), and \( P_{sk}^* \).
3: Condition 1: Check whether \( \alpha^{(n)} \) and \( \beta^{(n)} \) are feasible or not.
4: Repeat: Optimize \( P_{sk}, P_{pm} \), and \( P_{sk}^* \) to achieve \( v(S)^{(n)} \).
5: Condition 2: Check whether \( P_{sk}, P_{pm} \), and \( P_{sk}^* \) are feasible or not.
6: End repeat
7: If \( v(S)^{(n)} \geq \Omega^{(n-1)} \),
   \( \Omega^{(n)} = v(S)^{(n)} \).
Else
   \( \Omega^{(n)} = \Omega^{(n - 1)} \).
8: End if
9: Update \( \alpha^{(n+1)} = \alpha^{(n)} + q^{(n)}(v(S)^{(n)} - \Omega^{(n-1)}) \),
   \( \beta^{(n+1)} = \beta^{(n)} + t^{(n)}(v(S)^{(n)} - \Omega^{(n-1)}) \).
10: End repeat until the optimal \( \Omega^* \) is achieved.
11: We obtain the optimal system utility \( \Omega^* \), and the parameters \( \alpha^*, \beta^*, P_{sk}^*, P_{pm}^* \), and \( P_{sk}^* \).

**Definition 2** A payoff allocation \( \mathbf{x} = [x_p, x_{pm}, x_a, x_{sk}, x_{tm}, x_{tk}, x_{sk}] \) for the grand coalition is said to be an imputation if \( \sum_{m \in \mathcal{N}_p} x_p + \sum_{k \in \mathcal{N}_s} x_{sk} = v(\mathcal{N}_p \cup \mathcal{N}_s) \) and \( x_i \geq v(\{i\}) \), \( \forall i \in (\mathcal{N}_p \cup \mathcal{N}_s) \), where \( x_p \) and \( x_{sk} \) represent the payoff value of PU \( m \) and SU \( k \), respectively. The core \( \mathcal{C} \) is the set of imputations for which \( \sum_{m \in \mathcal{N}_p} x_p + \sum_{k \in \mathcal{S}_k} x_{sk} \geq v(S) \) for all \( S \subseteq (\mathcal{N}_p \cup \mathcal{N}_s) \).

Definition 2 states that the core, if exists, guarantees that the grand coalition is stabilized. Therefore, to show that our proposed game has a nonempty core and obtain the payoff allocation for PUs and SUs, we employ the duality technique [8]. We first define the Lagrangian functions of PUs and SUs as

\[
J_m(\pi) = \max_{r_{pm} \geq 0, m \in \mathcal{S}_p} \left( F(r_{pm}) + \pi_m r_{pm} \right)
\]

\[
g_k(\gamma, \tau) = \max_{r_{sk} \geq 0, k \in \mathcal{S}_s} \left( G(r_{sk}) - \gamma_k r_{sk} + \tau_k \right)
\]

where \( \pi_m, \gamma_k, \) and \( \tau_k \) are the Lagrangian multipliers. Then, the dual problem of the problem in (6) is given by

**Minimize** \( \sum_{m \in \mathcal{S}_p} (J_m(\pi) + \lambda_m) + \sum_{k \in \mathcal{S}_s} (g_k(\gamma, \tau)) \)

**Subject to**

\[
\gamma_k, \tau_k \geq 0, k \in \mathcal{S}_s, \quad \eta_j \geq 0, j \in \mathcal{S}_b,
\]

\[
\pi_m, \lambda_m \geq 0, m \in \mathcal{S}_p.
\]

We formulate the dual problem by appropriately defining vectors \( \pi, \lambda, \gamma, \tau, \) and \( \eta \). Denote \( \mathcal{D} \) as the set of optimal solutions of the dual problem for \( \mathcal{S}_p = \mathcal{N}_p \) and \( \mathcal{S}_s = \mathcal{N}_s \). Then, the core is represented as

\[
\mathcal{O} = \left\{ \mathbf{x}^* \in \mathbb{R}^{\mathcal{N}_p + \mathcal{N}_s} : \right.
\]

\[
x^*_{pm} = J_m(\pi^* + \lambda_m^*), m \in \mathcal{N}_p
\]

\[
x^*_{sk} = g_k(\gamma^*, \tau^*) + \sum_{j \in \mathcal{S}_b} \eta^*_j, k \in \mathcal{N}_s
\]

for some \( \pi^*, \lambda^*, \gamma^*, \tau^*, \eta^* \in \mathcal{D} \).
where $x_{pm}^*$ and $x_{sk}^*$ represent the set of PU’s and SU’s optimal payoff allocation. Then, we show the core of the proposed game is nonempty in the following theorem.

**Theorem 1:** The core of the considered canonical coalitional game with TU is nonempty and $O \subseteq C$.

**Proof 1** Since the set $O$ is the subset of the set $D$, and the set $D$ is nonempty, the set $O$ is nonempty. We consider an arbitrary $x^* \in O$ corresponding to some $(\pi^*, \lambda^*, \gamma^*, \tau^*, \eta^*) \in D$. Now, $\sum_{m \in \mathcal{S}_p} x_{pm}^* + \sum_{k \in \mathcal{S}_s} x_{sk}^*$ is the optimal value of the objective function of the dual problem. Since $F(\cdot)$ and $G(\cdot)$ are concave functions, the objective function of (6) is a concave function. Additionally, the constraints of (6) are linear. Then, according to the definition of strong duality [10], we can conclude that the duality gap is zero. Thus, $\sum_{m \in \mathcal{S}_p} x_{pm}^* + \sum_{k \in \mathcal{S}_s} x_{sk}^* = v(N_p \cup N_s)$. The first necessary condition of the group rationality of the core has been proven to be satisfied. Then, we only need to show that $v(S) \leq \sum_{m \in \mathcal{S}_p} x_{pm}^* + \sum_{k \in \mathcal{S}_s} x_{sk}^*$ for all $S \subseteq N_p \cup N_s$.

By strong duality, $v(S)$ equals the optimal value of the objective function of (9). The sub vectors $(\pi_S, \lambda_S, \gamma_S, \tau_S, \eta_S)$ consisting of the components of $(\pi^*, \lambda^*, \gamma^*, \tau^*, \eta^*)$ in $S$. Thus, $\sum_{m \in \mathcal{S}_p} x_{pm}^* + \sum_{k \in \mathcal{S}_s} x_{sk}^*$ is the value of the objective function of (9) for the above feasible solution. It follows that the optimal value of the objective function of (9) is a lower bound for $\sum_{m \in \mathcal{S}_p} x_{pm}^* + \sum_{k \in \mathcal{S}_s} x_{sk}^*$. Thus, we can conclude that $x^* \in C$.

### 4. Simulation results

In this section, numerical results are presented to illustrate the cooperative strategies among PUs and SUs in our considered network. We simulate a scenario where we set $N_p = \{\text{PU 1, PU 2, PU 3}\}$, $N_s = \{\text{SU 1, SU 2, SU 3, SU 4}\}$, and $N_b = 3$ by using the cvx tool [10]. The channels gains of link pairs among PUs, SUs, the PBS, and the SAP are independently generated from complex Gaussian distribution with zero mean and unit variance. We assume that the noise power is $\sigma^2 = 10^{-2}$ at the receiver. The linear utility functions are adopted, i.e., $F(r_{pm}) = 10 \cdot r_{pm}$ and $G(r_{sk}) = u_s \cdot r_{sk}$, where $u_s$ is the SU’s access rate gain factor, its value is set to be 8. The SU’s cost per relaying energy and per access time are set as $c_e = 8$ and $w_p = 8$, respectively. The total power constraint of SUs is set as $P_{max} = 2$. The fractions of time updating size are set as $\mu = 0.0132$ and $\delta = 0.0045$.

Fig. 2 shows the payoff allocation for PUs and SUs in the core, with the fraction of time $\alpha^* = 0.658$, $\beta^* = 0.224$ obtained from the proposed algorithm. For PUs, the payoff of direct transmission of PU 1, PU 2, and PU 3 are 14.28, 12.87, and 11.44, respectively; after forming the grand coalition, the payoff increases to 16.26, 15.18, and 13.98, respectively. For SUs, after forming the grand coalition, the payoff of SU 1, SU 2, SU3, and SU4 increase from 0 to 32.80, 25.62, 30.36, and 37.68, respectively. Thus, we can clearly see that it is beneficial for all PUs and SUs to join the grand coalition.

Fig. 3 depicts the impact of access gain factor $u_s$ on the cooperation scheme. We can find that, as $u_s$ increases, the total data rate of all PUs decreases. This is because that with a larger $u_s$, increasing $r_{sk}$ will let SUs allocate less power for relaying PUs’ traffic. Thus, PU’s data rate decreases. On the contrary, SUs get such more power for transmitting their own traffic that their total data rate increase.

Fig. 4 shows the performance comparison results between our approach with other approaches which are the
non-cooperative (i.e., the direct transmission) approach, and the equal power approach, respectively. In the direct transmission approach, each PU employs the direct transmission to transmit signals to PBS without SU’s help. In the equal power approach, both PUs and SUs are transmitting at fixed power level.

Fig. 4 illustrates the benefits of forming the grand coalition. We can see that, SU’s payoffs increase significantly. This is because that when PUs and SUs form the grand coalition, SUs have opportunities to access more spare channels for transmitting their own traffic.

5. Conclusion

We have proposed an approach based on coalitional game to address the resource allocation problems in a multi-channel CCRN with multiple PUs and SUs, where the problems of the power control, and the cooperative time fraction-based spectrum sharing interleaved with one another. An iterative algorithm was proposed to achieve the optimal power control and time fraction allocation. Also, we showed that the considered coalitional game with TU had a nonempty core (i.e., a stable grand coalition existed). Finally, numerical results have showed our proposed approach was efficient and beneficial for both primary and secondary networks.

References