Formulation of Optimal Tree Construction for Maximum Lifetime Multicasting in Wireless Ad-hoc Networks with Adaptive Antennas

Song Guo and Oliver Yang
School of Information Technology and Engineering
University of Ottawa
Ottawa, Canada
{sguo, yang}@site.uottawa.ca

Abstract—We consider the problem of maximizing the lifetime of a given multicast connection in wireless networks that use directional antennas and have limited energy resources. We present a constraint formulation for the MLM (Multicast Lifetime Maximization) problem in terms of MILP (Mixed Integer Linear Programming) for wireless ad hoc networks, which we can use to evaluate and compare the realistic performance of different heuristic algorithms.

Keywords—wireless ad-hoc network; multicast lifetime maximization; adaptive antenna; mixed integer linear programming

I. INTRODUCTION

Nodes in wireless ad-hoc networks are usually powered by a limited capacity battery, and operate in an unattended manner. As the batteries of nodes are drained and stop functioning, a wireless ad hoc network will eventually cease to be usable. Consequently, energy efficiency is an important design consideration for these networks. In this paper, we explore the energy conservation offered by the use of directional antennas for providing long-lived broadcast/multicasting in wireless ad hoc networks.

In order to distribute packet-relaying loads for each node in a manner that prevents nodes from being overused or abused and therefore prolongs lifetime, a main type of optimization problem, the BLM/MLM (Broadcast / Multicast Lifetime Maximization) problem, has received more and more attention. A few algorithms, e.g. [1, 2, 3], use the broadcast / multicast lifetime as an explicit optimization metric to build a maximum lifetime broadcast / multicast tree in the case of using omni-directional antennas.

Wieselthier et al [4] first studied the BLM/MLM problem considering use of directional antennas. The incremental power philosophy in BIP/MIP [5], originally developed for use with omni-directional antennas, can be applied to broadcast /multicast tree construction in networks with directional antennas as well. Two heuristic algorithms called RB-BIP/RB-MIP and D-BIP/D-MIP were proposed for the situation of using adaptive antennas. Furthermore, they have incorporated residual energy into the cost metric, so that their heuristic algorithms would discourage the inclusion of energy-poor nodes as transmitting nodes in the broadcast/multicast tree (by increasing the cost associated with their use), and therefore provide long-lived broadcast / multicast trees.

While the performances of these algorithms can certainly be compared among themselves, it has not been possible to judge the quality of the solutions with respect to the optimal solution, which has been absent so far. This paper attempts to fill that void by proposing a general analytical MILP (Mixed Integer Linear Programming) model for the MLM problem in an ad-hoc network equipped with adaptive antennas. An optimal solution of the MLM problem using our MILP model can always be obtained in a timely manner for small and moderately sized network, and it also provides a way to evaluate the realistic performance of different heuristic algorithms.

II. NETWORK MODEL

We consider a wireless ad hoc network modeled by a simple directed graph $G(N, A, p)$ in which the node locations are fixed, and the channel conditions unchanging. The directed graph $G$ has a finite node set $N$ with $|N| = n$ nodes and an arc set $A$ corresponding to the unidirectional wireless communication links. The arc weight function $p: A \rightarrow R^+$ assigns RF power to each arc. That is, for each arc $(v, u) \in A$, $p_{vu}$ is the RF power needed for the link from node $v$ to node $u$. We assume that any node $v \in N$ can choose its power level, not to exceed some maximum value $p_{\text{max}}$. Any directed arc $(v, u) \in A$ if and only if $p_{vu} \leq p_{\text{max}}$.

In an ad-hoc wireless network each node is equipped with adaptive array antennas, which permit energy savings by concentrating RF transmission energy to where it is needed. We use an idealized adaptive antenna propagation model, where the antenna orientation $\varphi_v\ (0 \leq \varphi_v < 2\pi)$ of node $v$ is defined as the angle measured counter-clockwise from the horizontal axis to the antenna boresight, and the antenna directionality is specified as the angle of beamwidth $\theta_v\ (\theta_{\text{min}} \leq \theta_v \leq \theta_{\text{max}})$. By ignoring the possibility of sidelobe interference, the transmitted energy is assumed to be uniformly distributed across the beamwidth. Based on this model, the minimal transmitted power required by node $v$ to support a link between two nodes $v$ and $u$ separated by a distance $r_{vu}$ ($r_{vu} > 1$) is proportional to $r_{vu}^{-\alpha}$ and beamwidth $\theta_v$, where the propagation loss exponent $\alpha$ typically takes on a value between 2 and 4. Without loss of generality, all receivers are assumed to have the same signal detection threshold, which is typically
normalized to one. Then the transmission power \( p_{vu} \) needed by node \( v \) to reach node \( u \) can be expressed as

\[
p_{vu} = r_{vu}^\theta / 2\pi \tag{1}
\]

Consequently, the use of narrow beams allows energy saving for a given communication range or extends the antenna range for a given RF transmission power level, when compared to the use of omni-directional antennas.

In addition to RF propagation, energy may be also expended for transmission processing (on modulation, encoding, etc) and reception processing (on demodulation, decoding, etc). For simplicity, these quantities are the same for any node, denoted as \( p_{\text{trans}} \) and \( p_{\text{recv}} \) respectively. Thus, the actual reception power \( q_v \) expended at node \( v \) is

\[
q_v = \begin{cases} p_{\text{recv}} & \text{if } v \text{ is a receiver;} \\ 0 & \text{otherwise.} \end{cases}
\tag{2}
\]

We neglect any energy consumption occurring when the node is simply “on” without transmitting or receiving, although it would be straightforward to incorporate it into our model. Let the energy supply set \( \mathcal{E} = \{ e_u \mid u \in N \} \) be the initial energy level associated with each node in \( G \). The maximal lifetime \( \tau_v \) of an arc \( (v, u) \in \mathcal{A} \) is therefore

\[
\tau_v = \frac{e_v}{p_{vu} + p_{\text{trans}} + q_v} \tag{3}
\]

III. PROBLEM STATEMENT

A. Multicast Tree

We consider a source-initiated multicast in wireless ad-hoc networks. Any node is permitted to initiate multicast sessions. The set of nodes \( M \) that support a multicast session, including the source node and all destination nodes, is referred to as a multicast tree. Any multicast tree is rooted at a directed acyclic graph with a source node called root with no incoming arcs, and all its other nodes with exactly one incoming arc. A node with no out-going arcs is called a leaf node, and all other nodes are internal nodes (relay nodes). Formally, a multicast tree is modeled by a directed tree \( T(N', A') \) rooted at a source node \( s \in N \), with a multicast node set \( N' \subseteq N \), and an arc set \( A' \subseteq A \).

Definition 1. A directed tree \( T(N', A') \) is a multicast tree in a wireless ad hoc network with adaptive antennas if and only if the following properties are satisfied: (1) the RTP (Rooted Tree Property) which requires \( T \) can span all the multicast members from node \( s \), i.e. \( M \subseteq N' \), and (2) the ACP (Antenna Coverage Property) which requires nodes to be located within the antenna beam of node \( v \), for any \( (v, u) \in A' \).

Given a multicast tree \( T_s \), let \( p(T_s) \) be the minimal RF transmission power required at node \( v \) to reach its farthest child in the tree \( T_s \) and \( \theta(T_s) \in [\theta_{\text{min}}, \theta_{\text{max}}] \) be the minimal antenna beamwidth that can cover all its children in \( T_s \). Considering the wireless multicast advantage property, we can obtain the actual RF power \( p_{vu}(T_s) \) assigned to node \( v \) in \( T_s \) as follows:

\[
p_{vu}(T_s) = \max \{ p_{vu} \mid (v, u) \in \mathcal{A}(T_s) \} \tag{4}
\]

B. Multicast Lifetime Maximization Problem

The multicast lifetime \( \tau(T_s) \), with respect to the multicast tree \( T_s(N', A') \), is defined as the duration of the network operation time until the battery depletion of the first node in \( N' \). We assume that once a multicast tree is established at the beginning of a multicast session, the same tree is used for the whole session duration. Therefore, for a given initial energy supply \( \mathcal{E} = \{ e_u \mid u \in N \} \) and the RF transmission power assignment \( p(T_s) \) for each multicast node \( v \), the multicast lifetime \( \tau(T_s) \) related to the lifetime of a tree arc can be obtained as follows:

\[
(ttt) \quad \tau(T_s) = \min \left\{ \frac{e_v}{p_{vu}(T_s) + p_{\text{trans}} + q_v} \mid v \in N' \right\} = \min \left\{ \frac{e_v}{p_{vu}(T_s) + p_{\text{trans}} + q_v} \mid (v, u) \in A' \right\} = \min \left\{ \tau_{uv} \mid (v, u) \in A' \right\} \tag{5}
\]

Let \( \Omega_M \) be the family of the trees \( T_s \) of \( G \) spanning all the nodes in \( M \). The objective of our MLM problem is to find a multicast tree with the maximum multicast lifetime \( \tau^* \) defined as

\[
\tau^* = \max \left\{ \tau(T_s) \mid T_s \in \Omega_M \right\} \tag{6}
\]

C. C. Min-Max Steiner Tree Problem

The statement of the MLM problem now allows us to formulate it as a min-max problem defined as follows.

Definition 2. Given a source node \( s \) and a subset \( M \) of nodes \( s \in M \subseteq N \), the min-max Steiner tree (MMST) problem is to determine a multicast tree rooted at node \( s \) that spans all nodes in \( M \) such that the maximum weight of the tree arc is minimized.

We observe from Equation (6) that if we define

\[
w_{uv} = 1 / \tau_{uv} = (p_{vu} + p_{\text{trans}} + q_v) / e_v \tag{7}
\]

as the weight for each arc \( (v, u) \), then the MLM problem is equivalent to the MMST problem. The optimal min-max Steiner tree \( T_s^* \) corresponding to the optimal solution of the MMST problem is therefore a multicast tree with the maximum multicast lifetime. Let \( w(T_s) \) denote the maximum arc weight of a tree \( T(N', A') \) and \((s, x, y)\) the bottleneck arc of the tree \( T_s^* \). Then

\[
w(T_s) = w_T = \max \{ w_{uv} \mid (v, u) \in A' \} \tag{8}
\]

Then the minimum of \( w(T_s) \) over \( \Omega_M \) is the optimal solution of Problem MMST.

IV. PROBLEM FORMULATION

The definition of multicast tree given in the context of directional antenna applications allows us to formulate the
MLM Problem as a MILP (Mixed Integer Linear Programming) model. The main idea is to extract a sub-graph \( T_s^{*} \) from the original graph \( G \), such that \( T_s^{*} \) is a multicast tree rooted at node \( s \) with maximum lifetime. In order to formulate the problem, we define the following variables:

(i) \( w \) is a nonnegative continuous variable that represents the weight of bottleneck arc in \( T_s^{*} \);
(ii) \( Z_{vu} \) is a binary decision variable which is equal to one if the arc \((v, u)\) is in the sub-graph \( T_s^{*} \) of \( G \), and zero otherwise;
(iii) \( F_{vu} \) is a nonnegative continuous variable that only represents a flow produced by the multicast initiator \( s \) going through arc \((v, u)\). It is only a fictitious variable to prevent loops.

We shall prove that if \((x)\) is the optimal solution of variable \( x \) obtained from this MILP model, then the graph \( T_s^{*}(N', A') \) is the optimal tree associated with this solution, where \( A' = \{(v, u) \mid Z_{vu} = 1\} \) is its arc set, and \( N' = \{u \mid \exists (v, u) \in A' \text{ or } (u, v) \in A'\} \) is its node set. In other words, \( T_s^{*}(N', A') \) is a multicast tree of \( G \) with maximum lifetime.

### A. Objective Function

Due to the equivalence to Problem MMST, the objective function of Problem MLM is

\[
\text{Minimize: } w
\]

We shall provide a set of constraints to guarantee that the variable \( w \) represents the maximum arc weight of \( T_s^{*} \). By substituting Equations (1) and (7) into Equation (8), \( w \) can be rewritten in terms of the optimization variables as follows:

\[
w = \max \left\{ \frac{r_{vu}^{*}}{2\pi \cdot e_{v}} \cdot \theta_{v} + \frac{p_{run} + p_{rec}}{e_{v}} \mid (v, u) \in A' \right\},
\]

where \( q_{e} = \begin{cases} p_{rec} & v \neq s \\ 0 & v = s \end{cases} \).

The “max” form in Constraint (10) is obviously nonlinear. The following theorem illustrates how this constraint can be linearized.

**Theorem 1.** The variable \( w \) represents the weight of a bottleneck arc in \( T_s^{*} \), if the formulation of Problem MLM includes Constraint (11), where \( y_{s} \) is a relatively large number.

\[
w - \left( \frac{r_{vu}^{*}}{2\pi \cdot e_{v}} \cdot \theta_{v} + \frac{p_{run} + p_{rec}}{e_{v}} \right) \geq y(Z_{vu} - 1), \forall (v, u) \in A'
\]

**Proof:** For any arc \((v, u)\) in the tree \( T_s^{*} \), i.e. \( Z_{vu} = 1 \), we obtain

\[
w \geq r_{vu}^{*} \cdot \theta_{v} / (2\pi \cdot e_{v}) + (p_{run} + q_{e}) / e_{v}\text{ directly from Constraint (11). Therefore, } w \text{ must not be less than the maximum among } r_{vu}^{*} \cdot \theta_{v} / (2\pi \cdot e_{v}) + (p_{run} + q_{e}) / e_{v}\text{ for all } (v, u) \in A', \text{ i.e. } w \geq \max \{ r_{vu}^{*} \cdot \theta_{v} / (2\pi \cdot e_{v}) + (p_{run} + q_{e}) / e_{v} \mid (v, u) \in A' \}. \]

The equality is achieved only when the variables \( w \) is minimized. On the other hand, for the arc \((v, u)\) which is not in the tree \( T_s^{*} \), i.e. \( Z_{vu} = 0 \), Constraint (11) becomes redundant. Now we can conclude that Constraint (11) is equivalent to Constraint (10) and is linear.

The constant \( y \) in Constraint (11) should be large enough to make the inequality always tenable when \( Z_{vu} = 0 \). A possible value is given below.

\[
y = \max \left\{ \frac{r_{vu}^{*} + p_{run} + p_{rec}}{e_{v}} \mid (v, u) \in A' \right\}
\]

**B. Linear Constraints for RTP**

We want to provide a set of constraints that would guarantee that \( T_s^{*}(N', A') \) obtained from the formulation satisfies the following two rooted tree properties. Theorem 2 below can achieve these two properties, and its proof also elaborates the construction and interpretation of the linear constraints that \( T_s^{*} \) is a rooted tree spanning all the multicast members, i.e., \( M \subseteq N' \).

RTP (a): Every node \( u \in N' \setminus \{s\} \) has exactly one incoming arc, and node \( s \) has no incoming arcs;

RTP (b): \( T_s^{*}(N', A') \) does not contain cycles.

![Illustration of constraints](image)

**Theorem 2.** \( T_s^{*}(N', A') \) is a rooted tree at node \( s \), provided Problem MLM satisfies the following constraints:

\[
\sum_{v \in N} Z_{uv} = 0;
\]

\[
\sum_{v \in N} Z_{uv} = 1; \text{ for } \forall u \in M \setminus \{s\}
\]

\[
\sum_{v \in N} Z_{uv} \leq 1; \text{ for } \forall u \in N \setminus M
\]

\[
\sum_{v \in N} Z_{uv} \leq (n - 1) \sum_{v \in N} Z_{uv} \leq \sum_{v \in N} Z_{uv}; \text{ for } \forall u \in N \setminus M
\]

\[
\sum_{v \in N} F_{uv} - \sum_{v \in N} F_{vu} = \sum_{v \in N} Z_{uv}; \text{ for } \forall u \in N \setminus \{s\}
\]

**Proof:** We first prove the RTP (a) case. Note that \( \sum_{v \in N} Z_{uv} \) and \( \sum_{v \in N} Z_{uv} \) are the in-degree and out-degree of node \( u \) in \( T_s^{*} \) respectively. Therefore, the root node \( s \) and the other multicast members satisfy RTP (a) directly from the Constraints (13) and (14) respectively. It remains to prove that any non-multicast member in \( T_s^{*} \) supporting the multicast communications must have exactly one incoming arc. Assume \( u \in N' \) is a non-multicast member in \( T_s^{*} \), indicated by a hollow node in Figure 1, its incoming degree must be 1 or 0 from Constraints (15). If \( \sum_{v \in N} Z_{uv} = 0 \), from Constraints (16), it follows that \( \sum_{v \in N} Z_{uv} = 0 \). That means \( u \) must be an isolated node (as shown in Figure 1a), thus \( u \notin N' \). This contradicts the original assumption, and therefore node \( u \) has exactly one incoming arc.
For the RTP (b) case, from the Constraints (13), (14) and (15), it follows that the only connected components in \( T_1^* \) might contain cycles could be composed of either a simple cycle as shown in Figure 1b, or a simple cycle with sub-tree stubs as shown in Figure 1c. All these topologies are not feasible for the following reasons. Assume that the nodes \((n_1, n_2, \ldots, n_k) = (n_1), k > 1, \) form a simple cycle in \( T_1^* \). Then from Constraint (13), node \( s \) will never be included in such a cycle. Constraint in (18) implies that \( F_{n_1}^* \) could be positive if and only if \((v, u) \in A' \). Letting \( F_{n_1}^* \) be a constant \( f \), then from the Constraint (17) it follows that

\[
F_{n_1}^* = F_{n_1}^* - \sum_{r=1}^{k-1} Z_{n_1}^* = f - (r - 1) \text{ for } r = 1, \ldots, k.
\]

After substituting \( F_{n_1}^* = f - (k - 1) \) into Constraint (17), for \( u = n_1 \), we obtain

\[
\sum_{v \in N} F_{n_1}^* - \sum_{v \in N} F_{n_1}^* = f - (k - 1) - f = 1 - k < 0.
\]

On the other hand, \( \sum_{v \in N} F_{n_1}^* - \sum_{v \in N} F_{n_1}^* = \sum_{v \in N} Z_{n_1}^* \leq 0 \). Thus the Constraint (17) is violated, and therefore simple cycles are not possible in \( T_1^* \). Similar reasoning shows that the topology in Figure 1c also violates the Constraint (17) it follows that \( \phi_{n}^* = \phi_{n}^* - \theta/2 \) and \( \phi_{n}^* = \phi_{n}^* + \theta/2 \) indicated by the dotted lines as shown in Fig. 2.

\[
\phi_{n}^* = \phi_{n}^* - \theta/2 \quad \text{and} \quad \phi_{n}^* = \phi_{n}^* + \theta/2.
\]

\[
\text{Figure 2. Antenna beam coverage range}
\]

Let \( \alpha_u (0 \leq \alpha_u < 2\pi) \) be the angle measured counterclockwise from the horizontal axis to the vector \( \overrightarrow{vu} \) as shown in Fig. 2. Then the angle \( \alpha_u \) \((v, u) \in N\) can be obtained once their positions are given. In Fig. 2, the lighter shaded area is the space covered by the antenna beam of node \( v \) when it is about entering the position of node \( u \) (i.e., for \( v \) making contact with \( u \)), and the darker shaded area is the space just before the beam is leaving the position of node \( u \) (i.e., for \( v \) losing contact with \( u \)). Thus it is clear that the wireless link \((v, u) \) exists in the multicast tree \( T_1^* \), i.e., \( Z_{vu}^* \), only if the antenna orientation \( \phi \) is bounded by the two pointing directions \( \phi_1^* = \alpha_u - \theta/2 \) and \( \phi_2^* = \alpha_u + \theta/2 \) indicated by the dotted lines as shown in Fig. 2.

\[
\text{Figure 3. Illustration of linear constraint construction for property ACP}
\]

In order to simplify our analysis, we first extend the constraint \( \theta_{\text{min}} \leq \theta \leq \theta_{\text{max}} \) into \( 0 \leq \theta \leq 2\pi \). In the \( \phi_{n} - \theta_{n} \) plane as shown in Fig. 3a, the points \((\phi_{n}, \theta_{n})\) that satisfy the constraint \( \alpha_u^* - \theta/2 \leq \phi \leq \alpha_u^* + \theta/2 \) must be within the area bounded by line \( AB \), line \( AC \), and line \( BC \), where \( A = (0, \alpha_u^*) \), \( B = (2\pi, \alpha_u^* - \pi) \), and \( C = (2\pi, \alpha_u^* + \pi) \). When we consider \( 0 \leq \alpha_u^* \leq \pi \) and \( 0 \leq \phi < 2\pi \), this area must be mapped into the shaded area in Fig. 3a, since \( \phi_1 \) and \( \phi_2 + 2\pi \) denote the same physical direction. Let \( I \) denote the area covered by triangle \( CFG \), and \( II \) the area covered by quadrangle \( ADEC \), where \( D = (2\alpha_u^*, 0), E = (2\pi, 0), F = (2\pi, 2\phi), \) and \( G = (2\alpha_u^*, 2\pi) \). Recall that ACP requires node \( u \) to be located within the antenna beam of node \( v \), for any \((v, u) \) included in the multicast tree \( T_1^* \). Based on our analysis above, this property can be rewritten as \( Z_{vu}^* \) only if \((\phi_{n}, \theta_{n}) \in I \cup II \). Since I and II are disjoint sets, \( Z_{vu}^* \) can be decomposed into a summation of two new binary variables \( Z_{vu1} \) and \( Z_{vu2} \), where \( Z_{vu1} = 1 \) only if \((\phi_{n}, \theta_{n}) \in I \), and \( Z_{vu2} = 1 \) only if \((\phi_{n}, \theta_{n}) \in II \). Theorem 3 explains how property ACP can be satisfied by linear constraints.

\[
\text{Theorem 3. For any } (v, u) \text{ included in the multicast tree } T_1^* \text{ and } 0 \leq \alpha_u \leq \pi, \text{ node } u \text{ must locate within the antenna beam of node } v \text{ if the following constraints hold.}
\]

\[
2\phi + \phi_{n} - (4\pi + 2\alpha_u)_{Z_{vu1}} \geq 0 \quad (19)
\]

\[
2\phi - \phi_{n} + (4\pi - 2\alpha_u)_{Z_{vu2}} \leq 4\pi \quad (20)
\]

\[
2\phi + \theta_{n} - 2\alpha_u_{Z_{vu1}} \geq 0 \quad (21)
\]

\[
\text{Proof: We only need to prove that the statement “} Z_{vu1} = 1 \text{ only if } \phi_{n} \in I \cup II \text{” is equivalent to the Constraints (19) to (21).}
\]

Since \( Z_{vu1} = 1 + Z_{vu2} \), and they are all binary variables, \( Z_{vu1} = 1 \) if and only if just one of the Boolean expressions \( Z_{vu1} = 1 \) and \( Z_{vu2} = 0 \) and \( Z_{vu1} = 0 \) and \( Z_{vu2} = 1 \) is true. We first consider the case \( Z_{vu1} = 1 \) and \( Z_{vu2} = 0 \). Thus Constraints (19) – (21) become \( \phi_{n} \geq -\theta/2 + 2\pi + \alpha_u, \phi_{n} \leq \theta/2 + 2\pi, \alpha_u \leq -\theta/2 \) respectively. Considering the boundary conditions \( 0 \leq \theta \leq 2\pi \) and \( 0 \leq \phi < 2\pi \), we observe that these constraints just define the area I as shown in Fig. 3a. Similarly, after substituting \( Z_{vu1} = 0 \) and \( Z_{vu2} = 1 \) into Constraints (19) – (21), we can easily examine that the resulting constraints \( \phi_{n} \geq -\theta/2, \phi_{n} \leq \alpha_u + \theta/2, \alpha_u \leq \alpha_u - \theta/2 \) define the area II. Combining the two...
cases, we conclude that Constraints (19) – (21) characterize the statement “\( Z_{vu} = 1 \) only if \((\varphi_v, \theta_u) \in I \cup II \)” correctly. ■

So far, we only consider the case \( 0 \leq \alpha_u \leq \pi \). A similar constraint construction for property ACP can be made under condition \( \pi < \alpha_u < 2\pi \). Fig. 3b shows the shaded area, only in which the value of \( Z_{vu} \) can be equal to 1. The corresponding linear constraints that characterize property ACP for \( \pi < \alpha_u < 2\pi \) are summarized in the theorem below.

**Theorem 4.** For any \((v, u)\) included in the multicast tree \( T_u^* \) and \( \pi \leq \alpha_u \leq 2\pi \), node \( u \) must locate within the antenna beam of node \( v \) if the following constraints hold.

\[
\begin{align*}
2\varphi_v - \theta_u + (8\pi - 2\alpha_u)Z_{vu1} & \leq 4\pi \\
2\varphi_v - \theta_u + (4\pi - 2\alpha_u)Z_{vu2} & \leq 4\pi \\
2\varphi_v + \theta_u - 2\alpha_uZ_{vu2} & \geq 0
\end{align*}
\]

(D. MILP Model)

**Objective Function:** minimize \( w \)

**Subject to:**

\[
\begin{align*}
w - \left(\frac{\varphi_v}{2\pi} - \frac{\theta_u}{2\pi} + \frac{q_u}{e_v}\right) & \geq \gamma(Z_{vu1} + Z_{vu2} - 1); \quad \forall (v, u) \in A \\
\sum_{v \in N} (Z_{vu1} + Z_{vu2}) &= 0; \quad \forall u \in M \setminus \{s\} \\
\sum_{v \in N} (Z_{vu1} + Z_{vu2}) &= 1; \quad \forall u \in M \setminus M \\
\sum_{v \in N} (Z_{vu1} + Z_{vu2}) & \leq (n-1) \sum_{v \in N} (Z_{vu1} + Z_{vu2}); \quad \forall u \in N \setminus M \\
\sum_{v \in N} F_v - \sum_{v \in N} F_u &= \sum_{v \in N} (Z_{vu1} + Z_{vu2}); \quad \forall u \in N \setminus \{s\}, \forall v \in N \\
(Z_{vu1} + Z_{vu2}) & \leq F_u - (n-1) (Z_{vu1} + Z_{vu2}); \quad \forall u \in N \setminus \{s\}, \forall v \in N \\
A_{vu} \varphi_v - \theta_u + B_{vu} Z_{vu1} & \leq C_{vu}; \quad \forall v, u \in N \\
2\varphi_v - \theta_u + (4\pi - 2\alpha_u)Z_{vu2} & \leq 4\pi; \quad \forall v, u \in N \\
2\varphi_v + \theta_u - 2\alpha_uZ_{vu2} & \geq 0; \quad \forall v, u \in N \\
0 & \leq \varphi_v < 2\pi, \quad \forall v \in N \\
0 & \leq \theta_u \\n\theta_{\text{max}} & \leq \theta_u \leq \theta_{\text{max}}; \quad \forall v \in N \\
Z_{vu1} & \in \{0, 1\}, \quad Z_{vu2} \in \{0, 1\}; \quad \forall v, u \in N
\end{align*}
\]

Figure 4. MILP model for Problem MLM

Our previous derivation on the linear constraints can now help us to rewrite the problem formulation as an MILP model. This is shown in Fig. 4, in which the coefficients \( A_{vu}, B_{vu}, \) and \( C_{vu} \) are given in Table 1. In this formulation, \( Z_{vu1} \) and \( Z_{vu2} \) are binary variables; \( w, F_v, \theta_u, \) and \( \varphi_v \) are continuous variables. The number of variables in the formulation is approximately \( 3n^2 + 2n + 1 \), and the number of constraints is of the order of \( O(n^2) \).

<table>
<thead>
<tr>
<th>TABLE I. COEFFICIENT VALUES</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A_{vu} )</td>
</tr>
<tr>
<td>( B_{vu} )</td>
</tr>
<tr>
<td>( C_{vu} )</td>
</tr>
<tr>
<td>( 0 \leq \alpha_u &lt; \pi ), ( \pi \leq \alpha_u &lt; 2\pi )</td>
</tr>
</tbody>
</table>

V. PERFORMANCE EVALUATION

After the valid problem formulation, the optimal solution can be always obtained by CPLEX [6] for small and medium size wireless ad-hoc networks. We have also evaluated the performance of a set of heuristic algorithms [RB-MIP-\(\beta\), D-MIP-\(\beta\)], where \( \beta \) is a parameter that reflects the importance assigned to the impact of residual energy [4] and we use RB-MIP-\(\beta\) and D-MIP-\(\beta\) to denote algorithms RB-MIP and D-MIP with different values of \( \beta \) respectively. We have only considered \( \beta = 0, 1, \) and \( 2 \).

We have the following observations based on many network examples with 50 nodes: (1) the minimal-energy multicast algorithms RB-MIP and D-MIP (when \( \beta = 0 \)) do not guarantee maximum lifetime either for broadcast or multicast, only with a lifetime around 20-30% of the optimal solutions; (2) by incorporating residual energy into the cost metric, the revised minimum-energy multicast algorithms, like RB-MIP-\(1\)/RB-MIP-\(2\) and D-MIP-\(1\)/D-MIP-\(2\), could provide longer lifetime that is between 30-80% of the optimal solutions. Due to the length limit, the extensive simulation results can not be included but are now included in [7].

VI. CONCLUSION

In this paper we present a constraint formulation for the multicast lifetime maximization problem in wireless ad hoc networks with adaptive antennas. Based on the analysis on the properties of multicast tree, the problem can be characterized in a form of mixed integer linear programming problem. A major challenge, and a topic of continued research, is to extend our analytical model to large-scale networks. Currently, we are working on a distributed algorithm of MLM to cope with the dynamic topology changes.

ACKNOWLEDGEMENT

This work is supported in part by an NSERC Grant under #RGPIN42878.

REFERENCES


\( E(t) \) is the residual energy at Node \( v \) at time \( t \).