Distributed Multicast Algorithms for Lifetime Maximization in Wireless Ad Hoc Networks with Omni-directional and Directional Antennas

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Abstract—We consider the problem of maximizing the network lifetime for multicast connections in a wireless ad hoc network (WANET) that use omni-directional / directional antennas and have limited energy resources. Unlike most centralized multicast algorithms, we provide a globally optimal solution to this problem in a distributed manner in a WANET with omni-directional antennas. This graph theoretic approach provides us insights into more general case of using directional antennas, and inspires us to produce a group of distributed algorithms. Simulation results show that our distributed algorithms outperform other centralized multicast algorithms significantly in terms of network lifetime.

Keywords- Wireless Ad Hoc Network; Maximum Network Lifetime Routing; Distributed Multicast Algorithm

I. INTRODUCTION

Over the last few years, energy efficient communication in wireless ad hoc networks with directional antennas has received more and more attention, since the use of directional antennas can save transmission power by concentrating RF energy where it is needed. On the other hand, the broadcast / multicast communication is an important mechanism to communicate information in wireless ad hoc networks, since many routing protocols for wireless ad-hoc networks need a broadcast/multicast mechanism to update their states and maintain the routes between nodes. In this paper, we explore the energy conservation offered by the use of directional antennas for providing long-lived broadcasting / multicasting in wireless ad hoc networks.

The optimization metric for lifetime-aware broadcast / multicast routing algorithms is typically defined as the duration of the network operation time until the battery depletion of the first node in the network. Some work has considered maximizing the network lifetime in a WANET with omni-directional antennas for a single broadcast session, e.g. [1, 2, 3, 4], or a single multicast session, e.g. [4, 5, 6, 7]. Such optimization problem is of importance and interest, especially for the energy-constrained wireless broadcast / multicast networks [3]. The same problem with directional antennas has been studied in [8, 9, 10]. In [9, 10], the authors have extended the minimum-energy metric by incorporating residual battery energy based on the observation that long-lived multicast / broadcast trees should consume less energy and should avoid nodes with small residual energy as well.

Most of the existing solutions are centralized, meaning that at least one node needs global network information in order to construct an energy efficient multicast tree. This motivates us to design the distributed algorithms that can run on the wireless nodes with limited resources, e.g. memory, computational capacity, and bandwidth. The main contribution of this paper is that we present a globally optimal solution to this problem in a distributed manner for the special case of single multicast session in a WANET with omni-directional antennas. This graph theoretic approach provides us insights into more general case of using directional antennas, and inspires us to produce a group of distributed algorithms.

The rest of this paper is organized as follows. Section 2 develops the system model. Section 3 exploits some important properties of a min-max tree and proposes a group of distributed algorithms for both omni-directional and directional antenna scenarios. Section 4 demonstrates the performance of our algorithms through a simulation study. Section 5 gives the conclusion on the results.

II. SYSTEM MODEL

In this paper our focus is on establishing source-initiated maximum trees to maximize network operating time in energy-limited wireless ad hoc networks. Similar to previous research on similar problems [1-10], we only consider static networks because mobility adds a new dimension to the problem and it is out of the scope of this paper. We further assume an ideal MAC layer that provides bandwidth availability, i.e. frequency channels, time slots or CDMA orthogonal codes, depending on the access scheme.

We consider a wireless ad hoc network modeled by a simple directed graph G with a finite node set N and an arc set A corresponding to the unidirectional wireless communication links. Each node is equipped with a directional antenna, which concentrates RF transmission energy to where it is needed. Assuming the transmitted energy at node v to be uniformly distributed across the beamwidth $\theta_v$ ($\theta_{\min} \leq \theta_v \leq \theta_{\max}$), the
minimal transmitted power required by node $v$ to support a link between two nodes $v$ and $u$ separated by a distance $r_{vu}$ ($r_{vu} > 1$) is proportional to $r_{vu}^a$ and beamwidth $\theta_v$, where the propagation loss exponent $a$ typically takes on a value between 2 and 4. Without loss of generality, all receivers are assumed to have the same signal detection threshold, which is typically normalized to one. Then the transmission power $p_{vu}$ needed by node $v$ to reach node $u$ can be expressed as

$$p_{vu} = \frac{r_{vu}^a \theta_v}{360}. \tag{1}$$

We further assume that any node $v \in N$ can choose its transmission power level, not to exceed some maximum value $p_{max}$. In addition to RF propagation, energy may be also expended for transmission processing (on modulation, encoding, etc) and reception processing (on demodulation, decoding, etc). For simplicity, these quantities are the same for any node, denoted as $p_{trans}$ and $p_{recs}$, respectively.

We consider a source-initiated multicast with multicast members $M = \{s\} \cup D$, where $s$ is the source node and $D$ are destination nodes. All the nodes involved in the multicast form a multicast tree rooted at the node $s$, i.e. a rooted tree $T_s$, with a tree node set $N(T_s)$ and a tree arc set $A(T_s)$. We define a rooted tree as a directed acyclic graph with a source node with no incoming arcs, and each other node $v$ has exactly one incoming arc. A node with no out-going arcs is called a leaf node, and all other nodes are internal nodes (also called relay nodes).

### III. DISTRIBUTED MIN-MAX TREE ALGORITHMS

Let the energy supply $\epsilon = \{\epsilon_v \mid u \in N\}$ be the initial energy level associated with each node in $G$ and $\tau_u$ be the residual lifetime of an arc $(v, u) \in A$. In a WANET with omni-directional antennas (i.e. $\theta_v = 360$), it has been shown in [8] that the maximum-lifetime multicast problem is equivalent to the following min-max tree problem if the arc weight in the graph $G$ is defined as $w_{vu} = 1/\tau_{vu}$, i.e.

$$w_{vu} = \begin{cases} \max\left(\frac{r_{vu}^a + p_{\text{trans}} + p_{\text{recs}}}{\epsilon_v}, \frac{p_{\text{recs}}}{\epsilon_u}\right) & v \neq s, \\
\max\left(\frac{r_{vu}^a + p_{\text{trans}} + p_{\text{recs}}}{\epsilon_v}, \frac{p_{\text{recs}}}{\epsilon_u}\right) & v = s. \end{cases} \tag{2}$$

The min-max tree problem is to determine a directed tree $T_s$ spanning all the multicast members (i.e. $M \in N(T_s)$) such that the maximum of the tree arc weight $\delta(T_s) = \max \{w_{vu} \mid (v, u) \in A(T_s)\}$ is minimized.

#### A. A Min-Max Tree Theorem

Let $T'_s$ be a min-max tree and $\Omega_M$ the family of the trees spanning all the nodes in $M$. We therefore have $\delta_{min} = \delta(T'_s) \leq \delta(T_s)$ for any $T_s \in \Omega_M$. A tree link $(v, u)$ is called the bottleneck link of the tree $T_s$ if $w_{vu} = \delta(T_s)$. A related theorem that may result in efficient distributed algorithms is presented as below.

**Theorem 1.** Let $T_s \in \Omega_M$ be a multicast tree. If there exists a node set $X, s \in X$ and $D \cap (N - X) \neq \phi$ such that $\delta(T_s) \leq w_{xy}$ for any $x \in X$ and $y \in N - X$, then $T_s$ is a min-max tree.

**Proof:** Let $T'_s \in \Omega_M$ be any multicast tree. Note that there is at least one destination node $z \in D$ belonging to $N - X$, i.e. $z \in D \cap (N - X)$, since otherwise it contradicts the fact $D \cap (N - X) \neq \phi$. Therefore, there must exist an arc $(x, y) \in A(T'_s)$ connecting $X$ and $N - X$ (i.e. $x \in X$ and $y \in N - X$) in order to guarantee that there exists a directed path from $s$ to reach node $z$. From the given condition in Theorem 1, we have $\delta(T'_s) \leq w_{xy}$. We can thus derive that $\delta(T'_s) \leq w_{xy} \leq \max \{w_{vu} \mid (v, u) \in A(T'_s)\} = \delta(T'_s)$ for any $T'_s \in \Omega_M$, i.e. $T'_s$ is a min-max tree. 

### B. MMT-OA: A Distributed Algorithm for WANETs with Omni-directional Antennas

The investigation of the min-max tree now would allow us to design distributed algorithm. Theorem 1 immediately suggests our MMT-OA (Min-Max Tree for Omni-directional Antennas) algorithm for the maximum lifetime multicast problem as follows.

Initially, the multicast tree $T_s$ only contains the source node $s$, i.e. $N(T) = \{s\}$ and $A(T) = \phi$. It then iteratively performs the following Search-and-Grow procedure until the tree contains all the nodes in $M$.

**Search-and-Grow:** Find the link $(v, u)$ connecting node sets $N(T_s)$ and $N - N(T_s)$ with minimum weight, i.e. $w_{vu} \leq w_{ab}$ for any $a \in N(T_s)$ and $b \in N - N(T_s)$, and then include it into the multicast tree. Consequently, the tree $T_s$ would grow by including as more links as possible into the multicast tree. All absorbed links, e.g. link $(x, y)$, must satisfy $w_{xy} \leq w_{vu}$ and still keep $T_s$ a tree structure. Such "grow" operation proceeds until no more such links can be found.

**Lemma 1.** At least one bottleneck link must be added into the tree in a Search operation of the MMT-OA algorithm.

**Proof:** We prove it by contradiction. Suppose that each bottleneck link, e.g. $(x, y)$, of the tree constructed by MMT-OA is added into the tree in a Grow operation, and the link $(v, u)$ is included into the tree just in the preceding Search operation. From the Search-and-Grow procedure, we have $w_{vu} \leq w_{xy}$. On the other hand, $w_{xy} \leq w_{vu}$ because $(x, y)$ is a bottleneck link of the tree. Therefore, we derive $w_{vu} = w_{xy}$, i.e. $(v, u)$ is also a bottleneck link, which contradicts the above assumption that all bottleneck links are included in Grow operations.

**Theorem 2.** MMT-OA constructs a min-max tree.

**Proof:** From the conclusion of Lemma 1, there exists a bottleneck link that is added into the tree in a Search operation. Let $T_s$ be the partially constructed multicast tree before entering such Search operation. At this situation, the node set $X = N(T_s)$ satisfies the conditions in Theorem 1 and therefore we conclude that the final tree obtained from the MMT-OA algorithm is a min-max tree.

For each Search operation of the MMT-OA algorithm, let $T_s$ be the partially constructed tree obtained at the beginning of this operation, and $(v, u)$ be the link found in this operation. From the above analysis, we have the following observations.
(1) The weight \( w_{uv} \), denoted as \( \delta_{lb} \), must be a lower bound of \( \delta_{\text{min}} \) and it is given by
\[
\delta_{lb} = \min \{ w_{uv} \mid (x, y) \in A, x \in N(T), y \in N-N(T) \}.
\] (3)

(2) The sequence of the weight \( w_{uv} \) in the min-max tree formation is in an increasing order and the final one in the sequence is equal to \( \delta_{\text{min}} \).

Our MMT-OA algorithm uses Search-and-Grow cycles to discover a min-max tree. Such feature is beneficial to implement it in a distributed fashion. We have formulated a data structure to maintain locally the multicast forwarding state at each tree node \( v \): a membership status and the neighborhood table \( N_v \). The membership status indicates if this node is a source, receiver, or forwarder. A node can be both a receiver and forwarder. The neighborhood table \( N_v \) contains one entry for each neighbor \( u \) within its maximum transmission range. Each entry in the table includes a flag to indicate if the node \( u \) is a tree node or a non-tree node. More specifically, if \( u \) is a tree node, the relationship to node \( v \) is further indicated as parent, child, or other (neither parent nor child). All tree nodes within \( N_v \) are denoted as \( T_N \).

The distributed algorithm assumes an underlying beaconing protocol which allows each node to be aware of the existence of all its neighbors and the information \( w_{uv} \) between any two neighbor nodes \( x \) and \( y \). After the beacon discovery, any node \( v \) can create an entry for each neighbor \( u \) within its maximum transmission range. Each entry in the table includes a flag to indicate if the node \( u \) is a tree node or a non-tree node. More specifically, if \( u \) is a tree node, the relationship to node \( v \) is further indicated as parent, child, or other (neither parent nor child). All tree nodes within \( N_v \) are denoted as \( T_N \).

In a Search operation, each tree node \( v \) (initially only source node \( s \)) first calculates its lower bound locally as
\[
\delta_{lb} = \min \{ w_{uv} \mid u \in N_v - TN_v \}.
\] (4)

It would unicast a MULTICAST-JOIN-REPLY (MJREP) message back to its parent with the parameter \( \delta_{lb} \) if \( v \) is a leaf node, or with the parameter \( \min \{ \delta_{lb}, \delta_{lb}' \} \mid x \) is a child node of \( v \) after collecting all MJREPs from its children if \( v \) is a relay node. Note that node \( v \) does not send this message if the parent flag is not set yet. Furthermore, if \( v \) is a multicast member, it also attaches its own address in the MJREP message, which shall be propagated to the source to notify its attendance to the multicast.

The multicast membership status at each tree node \( v \) is set when it is forwarding the MJREP message. If node \( v \) is a destination, it will set it as receiver. In addition, if node \( v \) is a relay node (i.e., there is at least one entry with a child flag in its neighborhood table), it will set its membership status as forwarder.

In this manner, the source will eventually obtain the lower bound \( \delta_{lb} \) just as given in Equ. (3) after all MJREPs are received from its children. If not all multicast members are included in the tree, the source will initiate the Grow operation by propagating the MULTICAST-JOIN-REQUEST (MJREQ) messages with the parameter \( \delta_{lb} \) all over the network.

When receiving the first MJREQ message, each intermediate node \( v \) would first set the transmitting node (from which MJREQ is received) as parent in its neighborhood table, then forward MJREQ to the non-tree node \( u \) only if \( w_{uv} \leq \delta_{lb} \), and finally set node \( u \) as child. All subsequent duplicate MJREPs (with the same request ID) from other nodes are simply dropped, while the corresponding relationship flag is set as other for each of these nodes in node \( v \)'s neighborhood table. After a short period of time, no more MJREPs would be received at node \( v \). This means that the Grow operation completes around node \( v \), and it then goes to the Search operation again as described earlier.

Finally, a forwarding tree is created in these Search-and-Grow cycles until all members join the tree. After that, a min-max multicast tree is obtained by pruning all the unnecessary links in a distributed fashion from the non-member leaf nodes.

The above distributed MMT-OA algorithm for the omnidirectional antenna networks can be straightforward applied to the directional antenna networks by reducing the antennas beamwidth of each internal node \( v \) to the smallest possible value that provides coverage of the node’s downstream neighbors in the tree, subject to the constraint \( \theta_{\text{min}} \leq \theta \leq \theta_{\text{max}} \).

C. MMT-DA: A Distributed Algorithm for WANETs with Directional Antennas

The MMT-DA is similar in principle to MMT-OA algorithm for the formation of min-max tree, in the sense that new nodes are added into the tree in Search-and-Grow cycles. We must first incorporate the antenna beamwidth into the weight on each arc as follows.

\[
w_{uv} = \max \left( \frac{r_{uv}^\alpha \cdot \theta(C_v)}{360 \cdot e_v} + \frac{p_{\text{man}} + p_{\text{rec}}}{e_v}, \frac{r_{uv}^\alpha \cdot \theta(C_v)}{360 \cdot e_u} + \frac{p_{\text{man}} + p_{\text{rec}}}{e_u} \right) \quad v \neq s; \tag{5}
\]

where \( \theta(C_v) \in [\theta_{\text{min}}, \theta_{\text{max}}] \) is the minimum possible antenna beamwidth applied by node \( v \) to cover all its children \( C_v \) in the tree.

In a Search operation, let \( T_s \) be the partially constructed tree obtained at the beginning of this operation. In order to obtain the lower bound given in Equ. (4), each tree node \( v \) needs to calculate the weight \( w_{uv} \) first using Equ. (5), in which the children set \( C_v \) is given as follows.

\[
C_v = \{ x \mid (v, x) \in A(T_s) \} \cup \{ u \}.
\] (6)

In a Grow operation, the new children, e.g., node \( x \), of each tree node \( v \) should be included into the tree as many as possible if a tree structure is still maintained and \( w_{uv} \) is not greater than the lower bound \( \delta_{lb} \) that is obtained from the previous Search operation, i.e.

\[
C_v = \arg \max_{C_v} \{ x \mid x \in N_v - TN_v \land w_{uv} \leq \delta_{lb} \}.
\] (7)

Finally, we use a ten-node network as a simple example to illustrate the basic tree construction steps in MMT-DA. All nodes are multicast members and Node 0 is the source. Each node has the same initial energy supply in a 10x10 square as shown in Fig. 1. The propagation-loss exponent is \( \alpha = 2 \) and the maximum transmission power is \( p_{\text{max}} = 25 \).
Step 0: Initially, the tree consists of only the source node 0.

Step 1: The link (0, 4) is found in the Search operation and then added into the tree with minimum beamwidth \( \theta_b(\{4\}) = 30 \) as shown as the shaded sector in Fig. 1a. There is no other links included in the tree in the following Grow operation.

Step 2: The link (4, 1) is found and added into the tree with minimum beamwidth \( \theta_b(\{1\}) = 30 \) in the Search operation. The tree then grows by including links (1, 3), (1, 6) (3, 8), and (6, 2) as shown in Fig. 1b, where \( \theta_b(\{3, 6\}) = 316^\circ, \theta_b(\{8\}) = 30 \), and \( \theta_b(\{2\}) = 30 \), since the weights \( w_{13}, w_{16}, w_{38}, \) and \( w_{62} \) are all less than \( w_{04} \).

Step 3: The link (8, 5) is found and added into the tree with minimum beamwidth \( \theta_b(\{5\}) = 30 \). The tree then grows by including links (5, 9) and (9, 7). The min-max tree is eventually obtained as shown in Fig. 1c with the bottleneck link (8, 5) that is found in the last iteration.

IV. PERFORMANCE EVALUATION

RB-MIP and D-MIP [10] are well-known long-lifetime multicast algorithms for WANETs with directional antennas. Both algorithms incorporate a parameter \( \beta \) that reflects the importance assigned to the impact of residual energy\(^2\) into the link cost function. We use RB-MIP-\( \beta \) and D-MIP-\( \beta \) to denote algorithms RB-MIP and D-MIP with different values of \( \beta \) respectively. We have evaluated the performance of heuristic algorithms {MMT-OA, MMT-DA, RB-MIP-\( \beta \), D-MIP-\( \beta \)} (\( \beta = 0, 1, \) and 2) for many network examples. In each network example, a number of nodes are randomly generated within a square region \( 10 \times 10 \). The values of parameters used in simulation are given in Table 1.

We use the metric normalized network lifetime to evaluate and compare algorithm performance. It is defined as the ratio of actual network lifetime obtained using heuristic algorithm to the best solution obtained by choosing the maximum lifetime from all heuristic algorithms. Such metric provides a measure of how close each algorithm comes to provide the longest-lifetime tree. Thus allows us to facilitate the comparison of different algorithms over a wide range of network examples.

Table 1. Parameter values for simulation

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Description</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>( m )</td>
<td>network size</td>
<td>100</td>
</tr>
<tr>
<td>( \theta_{min} )</td>
<td>multicast group size</td>
<td>5, 25, 50, and 100</td>
</tr>
<tr>
<td>( \theta_{max} )</td>
<td>minimum antenna beamwidth</td>
<td>10, 30, 60, 90, 180, and 360</td>
</tr>
<tr>
<td>( p_{trans} )</td>
<td>maximum antenna beamwidth</td>
<td>360</td>
</tr>
<tr>
<td>( p_{recv} )</td>
<td>maximum RF power level</td>
<td>100</td>
</tr>
<tr>
<td>( p_{trans} )</td>
<td>minimum power needed for transmission processing</td>
<td>0.1(^3)</td>
</tr>
<tr>
<td>( p_{recv} )</td>
<td>minimum power needed for reception processing</td>
<td>1</td>
</tr>
<tr>
<td>( E_i(\epsilon) )</td>
<td>mean of the initial energy</td>
<td>500(^4)</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>variance of the initial energy</td>
<td>200</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>propagation loss exponent</td>
<td>2</td>
</tr>
</tbody>
</table>

In our experiments, multicast groups of a specified size \( m \) are chosen randomly from the overall set of nodes. One of the nodes is randomly chosen to be the source. We randomly generated 100 different network examples, and we present here the average over those examples for all cases.

Figure 2 illustrates the mean normalized network lifetime as a function of multicast group size and minimal antenna beamwidth for all algorithms. In all cases, MMT-DA provides much better performance than other algorithms, and its superiority is even greater in network examples with larger \( \theta_{min} \), e.g. always within 5% close to the best solution when \( \theta_{min} \geq 30^\circ \). In fact, as guaranteed by the Theorem 2, MMT-DA degenerates into MMT-OA and therefore both achieve the globally optimal solutions for the case of using omnidirectional antennas. We also observe that the minimal total energy consumption does not guarantee maximum lifetime and the revised minimum-energy multicast algorithms, like RB-MIP-\( \beta \) / D-MIP-\( \beta \) (\( \beta = 1 \) and 2), by incorporating residual energy into the cost metric could provide longer lifetime compared to RB-MIP-0 / D-MIP-0 as shown in Fig. 2.

\(^1\) The symbol \( \angle abc \) indicates the degree of angle between arcs (b, a) and (b, c).

\(^2\) The cost of a link (v, a) is defined as \( c_{va} = \exp \left( E_i(0)/E_i(t) \right)^2 \), where \( E_i(t) \) is the residual energy at Node v at time t.

\(^3\) We have also used other values of \( (p_{trans}, p_{recv}) = (0, 0) \) and \((0.01, 0.1)\), and have observed similar simulation results.

\(^4\) It can be arbitrary units that are consistent with the units of distance.
Figure 2. Performance comparison based on normalized network lifetime for 100-node networks with single multicast session.

V. CONCLUSION

We have presented a group of distributed multicast algorithms for static WANETs. The correctness of our algorithm in providing the optimal maximum-lifetime multicast tree has been proved for WANETs with omni-directional antennas. The superior performance of our algorithms for WANETs with directional antennas has been also validated using the simulations over a large number of network examples.

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