Isogonic Formation with Connectivity Preservation for a Team of Holonomic Robots in a Cluttered Environment

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Abstract—A geometric approach to multi-robots group formation with connectivity preservation (from a graph-theoretic perspective) among group members has been presented. It has been proven that such formation approach would result into a unique shortest connectivity link among group members which would not only guarantee the communication preservation among the robotic agents but also facilitate the message passing and routing among the group members once needed.

Keywords—Multi-Robot Systems, Group Formation, Holonomic Robots, Inter-robot communication

I. INTRODUCTION

Whether it is a search and rescue, agricultural coverage, security and surveillance, or a game scenario for entertainment purpose, formation coordination among agents plays a crucial role. Improved system performance, distributed action at a distance and fault tolerance are few benefits that may result from such formation. Area of group formation and formation coordination have been subject to an intensive research and study during last decade. [1] identifies two main methods for formation control problems: Optimization-based method (e.g. [2]) and potential fields method (e.g. [3]). A leader-following formation approach is presented in [4] in which every individual is required to maintain a specific position relative to the leader. Local sensing is used in [5] for achieving objectives such as coverage and formation maintenance. Global grouping through local communication is the approach presented in [6]. In [7] a set of behaviors such as avoidance, aggregation and dispersion are combined to achieve a global flocking behavior. Assuming the availability of global knowledge to every robot, algorithm in [8] achieves formation by fitting the virtual structure (VS) to current robots’ positions. A behavior-based formation, based on three formation control principles namely, unit-centering, leader-referencing and neighbor-referencing is presented in [9]. In their work, authors demonstrate how an interaction law at sub-system level may lead to emergence of collective behavior. Some approaches like those of [7], [10], [11] and [12] provide no guarantee on achieving desired formation. Whereas, approaches presented in [8] and [9] require availability of global knowledge to every individual robots.

Proposed formation method presented in this article addresses the issue of the group formation and the connectivity preservation among the robotic members. In order to instruct robots to fall into required formation and/or switching among different formation configurations, the controller only needs position information of one single robot (referred to as group leader here after). All other group members’ locations would be then computed with regards to the group leader’s positioning information. Further more, no global knowledge of all robots’ positioning information is required for maintaining the formation.

The remaining of the paper is organized as follows: Section II introduces the nomenclature adapted in this work. The methodology, along with series of proofs will be demonstrated in section III. Simulation result will be presented in section IV. Section V provides conclusion and some insights on future work and direction.

II. NOMENCLATURE ADAPTED

Referring to Fig. 1, following naming convention has been adapted throughout the paper while making references to the group’s individuals:

- **Group Leader,** $R$: Other group members’ location information as well as their respective positions within the group will be calculated and maintained based on location information of $R$. $(x_R, y_R)$ has been used while referring to position information of group leader, $R$.
- **Side Robots,** $r_1$ and $r_2$: Robots that form left and right wings of the formation with respect to position of the group leader, $R$.
- **Isogonic**$^1$ **Robot,** $r_{iso}$: Robot whose position in the group adhere the isogonic points of the formation.
- **Central formation Angle,** $\gamma$: Angle formed between side robots, $r_1$, $r_2$ and the leader robot $R$. $\angle R r_1 r_2$ and $\gamma$ are used interchangeably throughout the text while referring to the central formation angle.
- **Formation base length** i.e. $\| r_1 r_2 \| = 2x$: The distance between side robots $r_1$ and $r_2$.

\(^1\text{Isogonic points of a triangle are the points that minimize the cumulative sum of the distances of the triangle’s vertices to it. There are two such points associated with every triangle.}\)
III. ISOGONIC FORMATION: THE METHODOLOGY

Group formation might be achieved in a top-down fashion. Another word, we may consider the final, desired formation as a single object with certain level of flexibility for reshaping and resizing, and try to ascertain individual’s location in one such formation. Fig. 1 illustrates one such formation in which group of four robots are fallen into an isosceles triangle. \( r_{iso} \) is associated with the location of isogonic point (first or second isogonic point) of \( \triangle r_1 R r_2 \). Considering the formation in Fig. 1, and assuming zero degree of rotation for initial condition, location of every robot involved can be calculated based on group leader, \( R \). Considering \( \triangle r_1 R r_2 \), we have:

\[
h = a \cos\left(\frac{\gamma}{2}\right), \quad x = a \sin\left(\frac{\gamma}{2}\right)
\]

Using (1), we get

\[
x_{r_1} = x_R - x, \quad y_{r_1} = y_R - h \\
x_{r_2} = x_R + x, \quad y_{r_2} = y_R - h
\]

Before calculating the position information of \( r_{iso} \) i.e. isogonic robot, we prove the following theorem.

**Theorem 1:** Location of isogonic robot \( r_{iso} \) would always lie along the normal from group leader, \( R \) to the base \( r_1 r_2 \).

**Proof:**

As shown in Fig. 1 and Fig. 2, isogonic points are the points that lie on the intersection of the lines, connecting each new vertex of the three equilateral triangles out of the three sides of the given triangle to the opposite triangle’s vertex. Such equilateral triangles would point outward in case of first isogonic point and inward for second isogonic point. Referring to Fig. 1 and considering \( \triangle r_1 S' r_2 \), we have

\[
\|r_1 S'\| = \|r_2 S'\| \quad \& \quad SS' \perp r_1 r_2 \Rightarrow \|r_1 S\| = \|r_2 S\|
\]

Similarly, in \( \triangle r_1 R r_2 \), we get

\[
\|r_1 R\| = \|r_2 R\| \quad \& \quad RS \perp r_1 r_2 \Rightarrow \|r_1 S\| = \|r_2 S\|
\]

Equations 4 and 5 imply that \( RS \) and \( SS' \) are aligned. Other word,

\[
RS' \perp r_1 r_2 \quad \& \quad \|r_1 S\| = \|r_2 S\|
\]

Same result can be obtained for second isogonic formation where \( \angle r_1 S' r_2 \) points upward and hence \( S' \) lies above \( R \) instead.

Using result obtained in Theorem 1, location of \( r_{iso} \) can be calculated as follow:

1) When \( \angle r_1 R r_2 < 120^\circ \): When all of \( \triangle r_1 R r_2 \) angles are less than \( 120^\circ \), the isogonic point, \( r_{iso} \) would lie within the convex of \( \triangle r_1 R r_2 \), with side angles of \( 120^\circ \) to every side of the triangle [13]. Fig. 1 illustrates one such scenario. In such a case, location of \( r_{iso} \), with the assumption of zero rotation for initial condition, would be:

\[
x_{r_{iso}} = x_R \\
y_{r_{iso}} = y_R - (h - h')
\]

2) When \( \angle r_1 R r_2 \geq 120^\circ \): Before calculating location of \( r_{iso} \) in such scenario, we prove the following theorem that demonstrates the necessity of introducing the second isogonic point into one such formation.

**Theorem 2:** Leader \( R \) and isogonic \( r_{iso} \) robots’ Locations would coincide if \( \angle R \geq 120^\circ \).

**Proof:** Let \( r_{iso} \) be the isogonic point of \( \triangle r_1 R r_2 \) with \( \angle R = 120^\circ \), \( \|r_1 R\| = \|r_2 R\| \). Further more, let

\[
\frac{r_{iso} r_1}{\|r_{iso} r_1\|} + \frac{r_{iso} r_2}{\|r_{iso} r_2\|} = \left( \frac{a_1}{a_2} \right)
\]

Where \((a_1, a_2)^{-1}\) represents summation result of left hand side normalized vectors. To ascertain \( r_{iso} \) so as to

![Figure 1. First Isogonic Formation. \( r_{iso} \) i.e. blue-colored circle is located within the convex of \( \triangle r_1 R r_2 \).](image)

![Figure 2. Second Isogonic Formation. \( r_{iso} \) i.e. blue-colored circle is located outside the convex of \( \triangle r_1 R r_2 \).](image)
have $\triangle r_1 R r_2$ as an isosceles triangle with $r_{iso}$ being its isogonic point, it is required to satisfy the following [14]:

$$\frac{r_{iso}^1}{\| r_{iso} r_1 \|} + \frac{r_{iso}^2}{\| r_{iso} r_2 \|} + \frac{r_{iso} R}{\| r_{iso} R \|} = 0 \quad (10)$$

Substituting (9) in (10), we get:

$$\begin{pmatrix} a_1 \\ a_2 \end{pmatrix} + \frac{r_{iso} R}{\| r_{iso} R \|} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \Rightarrow \begin{pmatrix} (1-a_1^2)(x_R-x_{r_{iso}})^2 - a_1^2(y_R-y_{r_{iso}})^2 = 0 \end{pmatrix}$$

$$\begin{pmatrix} (1-a_2^2)(y_R-y_{r_{iso}})^2 - a_2^2(x_R-x_{r_{iso}})^2 = 0 \end{pmatrix} \quad (11)$$

Solving (11) and (12) for $x_{r_{iso}}$ we get:

$$\begin{pmatrix} (1-a_1^2)(1-a_2^2)(x_R-x_{r_{iso}})^2 - a_2^2 a_2^2 \\ (x_R-x_{r_{iso}})^2 = 0 \end{pmatrix}$$

$$\Rightarrow x_{r_{iso}} = x_R \quad (13)$$

Substituting (13) in (12), we get:

$$\begin{pmatrix} (1-a_2^2)(y_R-y_{r_{iso}})^2 - a_2^2(2x_R-x_{r_{iso}})^2 = 0 \end{pmatrix}$$

$$\Rightarrow y_{r_{iso}} = y_R \quad (14)$$

Due to the result obtained in Theorem 2, it’s necessary to relocate the $r_{iso}$ so as to avoid collision while consistency of the approach is preserved. To do so, $r_{iso}$ will be relocated to second isogonic point of the $\triangle r_1 R r_2$, once the $\angle R \geq 120^\circ$. Fig. 2 illustrates one such situation. When $\angle R = 120^\circ$, the \|r_{iso} R\| would be equal to formation side length $a$. Considering $\triangle R r_{iso}$, we have:

$$\sin(\alpha) = \frac{\| R R \|}{\| r_{iso} R \|} = \frac{\Delta y}{a} \Rightarrow \Delta y = a \sin(\alpha)$$

$$\cos(\alpha) = \frac{\| r_{iso} R \|}{\| r_{iso} R \|} = \frac{\Delta x}{a} \Rightarrow \Delta x = a \cos(\alpha) \quad (15)$$

Using 15, coordinates of $r_{iso}$ within the group and with regards to the group leader $R$ can be calculated as:

$$x_{r_{iso}} = x_R - \Delta x \Rightarrow x_{r_{iso}} = x_R - a \cos(\alpha)$$

$$y_{r_{iso}} = y_R - \Delta y \Rightarrow y_{r_{iso}} = y_R - a \sin(\alpha) \quad (16)$$

With $\alpha$ and $a$ being formation heading angle and formation side length, respectively.

Theorem 3: Isogonic formations of group of robots $r_i$, $i \leq n$, $n = 4$ would always result into a unique, shortest connectivity link among group members.

Proof: Referencing to fig. 1 and fig. 2, $r_{iso}$ is the isogonic point of $\triangle r_1 R r_2$. This implies that $r_{iso}$ is the point that minimizes the cumulative sum of the Euclidean distances to other members of formation. Other word, it satisfies

$$\min(\| R - r_{iso} \| + \sum_{i=1}^{2} \| r_i - r_{iso} \|) \quad (17)$$

A. Formation Preservation During the Rotation

Once all robots are in their designated locations, the entire flock might be treated as a single isosceles triangle whose rotational maneuvering can be delivered via the application of transformational matrix to its reference point. In our case, the reference point of the formation is in fact the group leader i.e. $R$. Having the rotational matrix around Z-axis of group leader been presented by

$$R_z(\alpha) = \begin{pmatrix} \cos(\alpha) & -\sin(\alpha) & 0 \\ \sin(\alpha) & \cos(\alpha) & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad (18)$$

with $\alpha$ being the heading angle i.e. the orientation angle of the group. The position information of $r_1$ and $r_2$ with regards to $R_z(\alpha)$ is given by:

$$\begin{pmatrix} -1/2 \cos(\alpha) \times x + \sin(\alpha) \times h \\ -1/2 \sin(\alpha) \times x - \cos(\alpha) \times h \\ 1 \end{pmatrix} \quad (19)$$

B. Computing Isogonic Position Information During the Rotation

There are two scenarios to be addressed:

1) When $\angle R < 120^\circ$: Recalling 7 and 8, and using 18, position information of $r_{iso}$, during the rotation is given by:

$$\begin{pmatrix} -\sin(\alpha) \times (-h + 1/3 \times x \times 3^1(1/2)) \\ \cos(\alpha) \times (-h + 1/3 \times x \times 3^1(1/2)) \\ 1 \end{pmatrix} \quad (20)$$

2) When $\angle R \geq 120^\circ$: Using 16 and 18, position information of $r_{iso}$ is computed as:

$$\begin{pmatrix} 2a \sin(\alpha) \cos(\alpha) \\ a(\sin(\alpha)^2 - \cos(\alpha)^2) \\ 1 \end{pmatrix} \quad (21)$$

IV. SIMULATION RESULT

While performing the simulation and in order to provide the group with a collision-free path, the controller is coupled with a non-parametric navigation controller [15]. Fig. 3 (parts 1 through 6) demonstrates the formation preservation process by the controller during the simulation. In the event of any obstacle encounter, the controller instructs the group to shrink into the formation size that is suitable for obstacle avoidance. Once the obstacle is avoided, the controller would decide if enough space is available for the flock to expand. If positive, the expansion of the formation would take place.
Figure 3. Robotic group and their corresponding formation. The collision-free path is depicted in red. The cyan-color line shows the start-to-goal path in an obstacle-free environment. The obstacles are shown as brown-color boxes.

(part 3 of Fig. 3), otherwise the flock would maintain their shrunk formation style until the field is safe for switching to expansion. Once all the obstacles are avoided and the environment is safe, the robotic group will be instructed to switch back into second isogonic formation (part 6 of Fig. 3).

V. CONCLUSION

A geometric approach to group formation and connectivity preservation for a group of four holonomic, circular robots has been presented. Despite the convincing results obtained during the experiences conducted in simulation environment, there are still several important factors that are required to be addressed in the future. The current implementation of the controller is incapable of instructing the robotic group to perform line and column formation. In addition, introduction of moving obstacles into the field of operation is an important issue that has not been addressed in the current implementation. Centralized property of the controller is another limitation of the present approach that requires special attention since its distributed-ness, would provide the resulting formation with higher degree of flexibility and fault-tolerance.

REFERENCES


