Parameter Estimation of Nonlinear Systems using Higher Order Sliding Modes

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Abstract- This paper presents a technique for parameter estimation of uncertain nonlinear systems based on the computation of an accurate and robust derivative using higher order sliding modes. The proposed scheme does not involve any observer design or numerical differentiation. The novelty of the method is to facilitate the determination of system parameters via calculating accurate derivatives of the measured system input-outputs. To validate the technique, simulations have been performed for an uncertain nonlinear Three Tank System. The convergence in this approach is shown to be faster than existing techniques by an order of magnitude even in the presence of measurement noise. The presented parameter estimation scheme is relatively simple and can be employed for the controller design, fault diagnosis leading to reconfigurable control of nonlinear systems.

Index Terms— higher order sliding modes, parameter estimation, uncertain nonlinear systems, robust exact differentiator.

I. INTRODUCTION

The behavior of real-life systems is represented by complex and highly nonlinear models. The model response is determined by the certain constants referred to as plant or model parameters. These parameters may be measured or determined using laws of physics, material properties etc. For most of the systems information about such parameters is not directly available and these parameters have to be deduced by observing the systems response to certain inputs. If these parameters do not change with time and the system is linear and stable, their values can be determined easily. For the applications where these unknown parameters change with time, or changes in operating conditions or aging of equipment, render the simple parameter estimation techniques ineffective. This becomes more difficult especially for the case of nonlinear systems. These complexities are further enhanced by coupled interactions between control elements and the elements of plant to be controlled. Parameter uncertainties arise due to imperfect knowledge of the physical parameter values, un-modeled system dynamics or parameter variations during operation. Examples of physical parameters include stiffness and damping coefficients in mechanical systems, aero-dynamical coefficients in flying objects, flow or discharge coefficients of valves/pipes in process industry, capacitors and inductors in electric circuits etc.

Model based parameter estimation methods for linear and nonlinear systems have been widely studied by the researchers [1], [5]–[7], [18] – [20], [22]. Most of these parameter estimation schemes provide means of updating system parameters online in real time for robust control, controller reconfiguration or effective fault diagnosis and isolation [5], [11], [12]. The nonlinear parameter estimation techniques do not require linearization of system equations and can account for the uncertainties in the estimated parameters, but most of these are difficult to implement online because of their complexities. Join et al [5], Flies and Sira-Ramirez [12] used algebraic estimation scheme for the parameter estimation of uncertain nonlinear three tank system. A novel concept of sliding mode control with the capability of controlling a nonlinear system in the presence of uncertainties associated with parameters and modeling errors has been proposed by Alwi and Edwards [11]. An observer is designed as an online filter to compute an estimate of state vector using limited measurements of some states of a commercial aircraft. A variable structure filter and smooth variable structure filter in conjunction with sliding mode control to estimate non-measurable states for the tracking controller were described in Habibi et al [10]. Khair et al [21] has developed two estimators based on unknown input observers, a high gain and 2-order sliding mode observers. The performance of both observers is compared for a gasoline engine, sliding mode observer based estimators performs better than high gain observer.

Higher order sliding modes (HOSM) are currently finding useful applications and have been developed actively by the researchers in the control and diagnosis community. Standard sliding modes provide finite time convergence and robustness with respect to internal and external disturbances. For this, the relative degree of the constraint has to be ‘one’ which leads to the possibility of chattering effect. Higher order sliding modes remove the chattering phenomena while preserving the main properties of standard sliding modes. Arbitrary order sliding controllers with finite time convergence have been demonstrated in [6], [7], [9], [13], [16], [17] and [23]. The real-time implementation of the arbitrary-order controllers needs robust estimation of higher order derivatives of inputs and outputs. This is achieved by the recently presented arbitrary-order robust exact differentiators with finite-time convergence [2]–[4], [14] and [17]. It has been proved that 3\(h\) order differentiator allows real time robust exact differentiation up to 3\(h\) order, provided that the next \((l+1)h\)th derivative is bounded. The performance of these differentiators is asymptotically optimal in the presence of small Lebesgue-measurable input noises.

The main difficulty in real-time differentiation is due to differentiation sensitivity to input noises. The high gain differentiators in [8], [13], [15] provide an exact derivative provided their gains tend to infinity which also leads to higher sensitivity to small high-frequency noises. Another drawback of the high-gain differentiators is their peaking effect, i.e., the maximal output values during the transient grows infinitely when gains tend to infinity. Join et al [5] has determined estimation of time derivative of an analytic function using the truncated series of Taylor expansion. This leads to a system of linear equation giving the estimated values of derivatives.

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Preprint submitted to 7th International Conference on Control and Automation.
Received May 23, 2009.
This derivative estimation technique does not require the precise statistical knowledge of the process or measurement noise as is required in the case of Extended Kalman Filters. This technique involves higher order time polynomial approximation, the iterative integration process to low pass filter high-frequency perturbation or noise, which is computationally time consuming and also can have numerical issues in online implementation, in the mentioned numeric technique the derivative approximations become invalid which needs periodic resetting the calculations [12]. HOSM observer has been used to estimate simultaneously throttle discharge coefficient, indicated torque parameter and load torque of a gasoline engine by Butt and Bhatti [1]. The exact derivatives have been calculated by successive implementation of robust exact first-order differentiator with finite-time convergence. This differentiator is based on 2-sliding mode and is proved to feature the best possible asymptotically converging derivative provided the second time derivative of the base signal is bounded [2], [3], [7], [14].

As per the author’s best knowledge, the proposed HOSM differentiator based technique is used perhaps for the first time for parameters estimation of nonlinear systems. This paper presents a generic framework for the robust estimation of uncertain parameters in nonlinear systems using robust exact differentiator. The presented framework does not need the development of an observer or numeric differentiator and no precise knowledge of statistical properties of the nonlinear system is required. The remaining of the paper is organized as follows: In Section II generalized problem formulation for the estimation of unknown uncertain parameters of nonlinear systems is briefly described. It also contains description of HOSM robust exact differentiator being used for the estimation of time derivatives. An example of nonlinear uncertain Three Tank System for parameter estimation is discussed in Section III. Section IV presents the simulation results. Section V contains the concluding remarks.

II. PROBLEM FORMULATION

Consider a system of the form:

\[
\begin{align*}
\dot{x} &= f(x,p,t) + g(x,t)\Phi(u) \\
y &= h(x,t)
\end{align*}
\]

(1)

where, \(x \in \mathbb{R}^n\) is a measurable state vector, \(u \in \mathbb{R}^m\) is a control input vector, \(g(x,t)\) is a known nonlinear function with \(g(x,t) \neq 0\). \(p \in \mathbb{R}^q\) is the unknown/uncertain parameter vector in parameter space \(P \ni [p_{imin}, p_{imax}], i = 1,2,\ldots,q\). \(y \in \mathbb{R}^p\). \(\Phi(u)\) can be a nonlinear continuous function. \(f(x,p,t)\) is a smooth nonlinear function, and functions \(f, \Phi\) satisfy the following assumptions [24]:

Assumption 1:
\[
\begin{align*}
&f(x,p,t) = \alpha^T(p)\xi(x,t) & &\alpha^T = [\alpha_1,\alpha_2,\ldots,\alpha_q] \\
&\xi^T = [\xi_1,\xi_2,\ldots,\xi_q]
\end{align*}
\]

(2)

where \(\xi_i = \xi_i(x,t)\) are known nonlinear functions and linearly independent. \(\alpha_i = \alpha_i(p)\) are the combinations of \(p\); \(q\) denotes the dimension of \(\alpha\).

Assumption 2:
\[
u \Phi(u) \geq hu^2
\]

(3)

where \(h\) is a positive and non-zero constant. \(\Phi(u)\) is a measurable and \(\Phi(0) = 0\).

As the bounds of \(p\) are given, the bounds of \(\alpha(p)\) can also be obtained and the following assumption can be introduced.

Assumption 3:
\[
\alpha_i \in [\alpha_{imin}, \alpha_{imax}] \forall p \in P
\]

(4)

Suppose \(\hat{\alpha}\) is the uncertain parameter vector of the estimator and \(\hat{\alpha}_0(t)\) is a time varying parameter vector which is calculated in terms uncertain parameter and its bounds:

\[
\hat{\alpha}_0(t) = \begin{cases} 
\alpha_{imin} & \text{if } \hat{\alpha}_i < \alpha_{imin} \\
\hat{\alpha}_i(t) & \text{if } \alpha_{imin} \leq \hat{\alpha}_i \leq \alpha_{imax} \\
\alpha_{imax} & \text{if } \hat{\alpha}_i > \alpha_{imax}
\end{cases}
\]

(5)

In the following paragraphs, we will omit the elements of some functions for simplicity.

A. Observability & Identifiability of Nonlinear System

Identifiability is the possibility to identify the parameters of a control system from its input-output map. The identifiability property, a prerequisite for parameter estimation guarantees that the model parameters can be determined uniquely from measured data. The identifiability property is well characterized and algorithms for ordinary differential equation exist. In literature different methods are used to test the observability of nonlinear systems, the differential geometry and algebraic approach. Rank test condition is being used for both the approaches, where observability of a system is determined by calculating the dimension of the space spanned by gradients of the Lie-derivatives of its output functions. For this, one has to find the rank of the following Jacobian matrix [25]:

\[
O = \begin{bmatrix}
\frac{\partial f_{i1}h_1}{\partial x_1} & \cdots & \frac{\partial f_{i1}h_n}{\partial x_n} \\
\vdots & \ddots & \vdots \\
\frac{\partial f_{ih_m}h_1}{\partial x_1} & \cdots & \frac{\partial f_{ih_m}h_n}{\partial x_n}
\end{bmatrix}
\]

If this Jacobian matrix is a full rank (\(\text{Rank}(O) = n\)), then system is algebraically observable. The problem of parameter identifiability can be treated as special case of observability problem by considering parameters as state variables with time derivative zero i.e., \(p = 0\), so the observability rank test can used to determine parameter identifiability. For nonlinear system (1) with assumption \(p = 0\), \(x\) and \(p\) are considered as the same type of variables. Without initial conditions for \(x\), the non-observable variables can be both in \(x\) and in \(p\). Suppose that we are given a full set of initial conditions on \(x\) i.e., \(x(0) = x^0\). Then the problem of observability of the \(x\) variables disappears [25], what is left is exactly the problem of identifiability for the parameters, the set of parameters that realizes a given input-output map, at least locally. This can be determined by the rank test using differential geometry and algebraic approaches. For analytical system (1) the rank test amounts to calculating the rank of the following Jacobian for \(i = 1,2,\ldots,p\), \(j = 1,2,\ldots,q\) and \(k = 1,2,\ldots,n - 1\):
If the rank of this matrix is \( n \), then the system (1) is identifiable. If not, the non-identifiable parameters can be found by removing associated columns in this matrix as described in [25].

### B. Estimation of HOSM Based Robust Derivatives:

The sliding mode control (SMC) methods are developed to design a stable system with high rate of convergence to the desired state and low sensitivity to plant parameter variations. The sliding mode control is based on keeping exactly a properly chosen constraint by means of high frequency control switching. The biggest advantage of SMC is its insensitivity to variation in system parameters, external disturbances and modeling errors. The main idea of higher order sliding modes is to act on higher order derivatives of sliding variables as compared to the 1st order derivative in standard sliding mode technique. This gives all the advantages of standard sliding modes with additional advantage leading to removal of chattering effect. The r-sliding mode is determined by \( \sigma = \sigma = \cdots = \sigma^{r-1} = 0 \), which gives an r-dimensional condition on state of the dynamic system. The sliding order is a measure of the smoothness of the sliding variable in the vicinity of the sliding mode in general any r-sliding controller that keep \( \sigma = 0 \), needs, \( \sigma = \cdots = \sigma^{r-1} \) to be made available [13], [17]. Sliding mode techniques are known to be robust with a straight forward design formulation and thus provide a robust solution to parameter estimation, control and fault diagnosis. A robust exact differentiator developed by [7], [14], [17], as a part of the arbitrary order sliding mode controllers, with finite-time convergence has been adapted for the purpose of parameter estimation and is presented here for completeness of proposed scheme.

Consider an input signal \( f(t) \) a function defined on \([0, \infty)\), consisting of a bounded Lebesgue-measurable noise with unknown features and unknown base signal \( f_0(t) \) with \( n \)-th derivative having a known Lipschitz constant \( L > 0 \) resulting from a real time noisy measurement [3], [14]. The unknown sampling noise \( f(t) - f_0(t) \) is assumed to be bounded, assume that the associated differential inclusion discussed in [14] exist as a solution to (1), which causes boundedness of \( \sigma \), allowing the implementation of \( 1 \)st order robust differentiator with finite-time convergence. The task is to find real estimates of \( \dot{f}_0, \ddot{f}_0 \) using only values of \( f(t) \) and number \( L \). The estimates are to be exact in the absence of noise, when \( f(t) = f_0(t) \). Let the noises be absent. Introduce an auxiliary dynamic system \( \ddot{z}_0 = u, \ \sigma(t, z_0) = z_0 - f(t) \). The task is to make \( \sigma \) and \( \sigma \) vanish in finite time by means of continuous control using only measurements of \( \sigma \) i.e. to establish a 2-sliding mode. The modified version of the super-twisting controller is applied, producing the close-loop system.

\[
\dot{z}_0 = -\lambda_0(z_0 - f(t))^{1/2}\text{sign}(z_0 - f(t)) + z_1,
\]

\[
\dot{z}_1 = -\lambda_1\text{sign}(z_0 - f).
\]

Here \( \lambda_1 > L \) and \( \lambda_0 \) is chosen sufficiently large with respect to \( \lambda_1 \). The 2-sliding mode \( \sigma(z_0 - f(t)) = 0 \), \( \dot{\sigma} = \lambda_0|\sigma|^{1/2}\text{sign} \sigma + z_1 - f = z_1 - f = 0 \) is established in finite time. Thus in the presence of noise \( z_0 \) and \( z_1 \) are considered as estimates \( f_0 \) and \( \ddot{f}_0 \) respectively.

### C. Estimation of Uncertain Nonlinear System Parameters:

The \( n \)-th order robust exact differentiator is utilized to estimate the derivatives of input and outputs using the measured system data. The technique used for the estimation of parameters exploits the main features of the higher order sliding modes. Assume that the unknown parameters \( \alpha^T = [\alpha_1, \alpha_2, \cdots, \alpha_q] \) satisfy some natural identifiability conditions (6), then for \( j = 1, \cdots, q \),

\[
\alpha_j = Y_j(t, L_0^2h, L_1^3h, \cdots, L_p^ph, u, \dot{u}, \cdots, u^{(m)})
\]

where \( Y_j \) is a nonlinear function of its arguments. An accurate estimate \( [\alpha]_e \) of \( \alpha_j \) is obtained by replacing the derivatives of control input and output variables in equation (8) with their estimates obtained by the robust exact differentiator estimator (7):

\[
[\alpha]_e = Y_j(t, L_0^2h, L_1^3h, \cdots, L_p^ph, u, \dot{u}, \cdots, u^{(m)})
\]

This gives a relationship for the estimates of the parameters in terms of input, output and their derivatives. The required derivatives are obtained through estimation of higher order sliding modes (HOSM) based robust exact derivative technique discussed below.

### III. Example: Three Tank System Description

The three tank system represents a typical process system in process industry, fuel management system of airplanes in aircrafts and flight vehicles. Parameters like viscosity coefficients are uncertain [5], [12] because of change in liquid characteristics, aging effects and change operating environments.
A schematic diagram is shown in Figure 1. Here $q_{ij}$ represents the water flow rates from tank $i$ to $j$, which is given by $q_{ij} = \mu_i S_p \text{sign}(L_i - L_j) \sqrt{2gL_i - L_j}$. $i, j = 1, 2, 3$ and $q_{20} = \mu_2 S_p \sqrt{2gL_2}$. $L_i, S_p$ are the flow coefficients and cross sectional areas of interconnecting pipes, $L_i$ are water levels in tanks, $q_{ij}$ and $q_{20}$ are flow rates into tank 1 and tank 2 respectively. The full system model is then obtained as follows:

$$
\begin{align*}
\dot{x}_1 &= -C_1 \text{sign}(x_1 - x_2) \sqrt{|x_1 - x_2|} + \frac{(u_1 + u_2)}{s} \\
\dot{x}_2 &= C_2 \text{sign}(x_2 - x_3) \sqrt{|x_2 - x_3|} - C_2 \text{sign}(x_2) \sqrt{|x_2|} + \frac{(u_2 + u_3)}{s} \\
\dot{x}_3 &= C_3 \text{sign}(x_3 - x_2) \sqrt{|x_3 - x_2|} - C_3 \text{sign}(x_3 - x_2) \sqrt{|x_3 - x_2|} \\
y_1 &= x_1, y_2 = x_2, y_3 = x_3
\end{align*}
$$

where $x_i(t)$ is the liquid level in tank $i$ and $C_i = \frac{1}{s} \mu_i S_p \sqrt{2gL_i}$. The control signals $u_1(t)$ and $u_2(t)$ are input flow rates respectively, $u_3$ and $u_2$ are actuator faults/disturbances which perturb the behavior of the system. The parameters $C_i$ includes flow coefficient $\mu_i$ which are assumed unknown/uncertain and are to be estimated, $[\alpha_1, \alpha_2, \alpha_3] = [\mu_1, \mu_2, \mu_3]$. The typical parameters values of benchmark three tank system are given in Table 1.

<table>
<thead>
<tr>
<th>Table 1</th>
<th>Typical Parameter Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S$, Area of the Tanks</td>
<td>0.0154 m²</td>
</tr>
<tr>
<td>$S_p$, Area of pipes, $p=1,2,3$</td>
<td>5x10⁻⁵ m²</td>
</tr>
<tr>
<td>$u_{\max}$, $u_{2\max}$ (input flow rates)</td>
<td>100 ml/s</td>
</tr>
<tr>
<td>$L_{i,\max}$, $x_i$, Level in Tanks, $i=1,2,3$</td>
<td>0.62 m</td>
</tr>
<tr>
<td>$\mu_1$, $\mu_2$, $\mu_3$</td>
<td>0.5, 0.675, 0.5</td>
</tr>
<tr>
<td>$\mu_{1\min}$, $\mu_{2\max}$, $\mu_{3\max}$</td>
<td>1.0, 1.0, 1.0</td>
</tr>
<tr>
<td>$\mu_{1\min}$, $\mu_{2\max}$, $\mu_{3\min}$</td>
<td>$\eta_1, \eta_2, \eta_3 &gt; 0$</td>
</tr>
</tbody>
</table>

Assumptions 1 and 2 are analyzed and validated for the system (10), $\eta_1, \eta_2, \eta_3$ are very small constants taken as minimum bounds of viscosity coefficients, maximum bounds of viscosity coefficients and control inputs are given in Table 1. In order to carry out identifiability analysis of the system we have:

$$
\begin{align*}
 f(x, t) &= \begin{bmatrix}
 -C_1 \text{sign}(x_1 - x_2) \sqrt{|x_1 - x_2|} \\
 C_2 \text{sign}(x_2 - x_3) \sqrt{|x_2 - x_3|} - C_2 \text{sign}(x_2) \sqrt{|x_2|} \\
 C_3 \text{sign}(x_3 - x_2) \sqrt{|x_3 - x_2|} - C_3 \text{sign}(x_3 - x_2) \sqrt{|x_3 - x_2|}
\end{bmatrix} \\
 g(x, t) &= \begin{bmatrix}
 1/s \\
 1/s \\
 0
\end{bmatrix} \\
 h(x, t) &= \begin{bmatrix}
 y_1 \\
 y_2 \\
 y_3
\end{bmatrix} = \begin{bmatrix}
 x_1 \\
 x_2 \\
 x_3
\end{bmatrix}
\end{align*}
$$

The function $f(x, p, t)$ can be easily represented in terms of $\alpha$'s and $\xi$'s in form of equation (2). The Jacobean of Lie-derivatives (6) of output for system (10) is:

$$
J = \begin{bmatrix}
\frac{\partial}{\partial p_1} L_f h_1 \\
\frac{\partial}{\partial p_2} L_f h_1 \\
\frac{\partial}{\partial p_3} L_f h_1
\end{bmatrix}, i = j = 1, 2, 3
$$

In Jacobian matrix form we can write it as:

$$
J = \begin{bmatrix}
\frac{\partial}{\partial p_1} L_f h_1 & \frac{\partial}{\partial p_2} L_f h_1 & \frac{\partial}{\partial p_3} L_f h_1 \\
\frac{\partial}{\partial p_1} L_f h_2 & \frac{\partial}{\partial p_2} L_f h_2 & \frac{\partial}{\partial p_3} L_f h_2 \\
\frac{\partial}{\partial p_1} L_f h_3 & \frac{\partial}{\partial p_2} L_f h_3 & \frac{\partial}{\partial p_3} L_f h_3
\end{bmatrix}
$$

Where $\frac{\partial}{\partial p_1} L_f h_1 = \frac{\partial}{\partial p_2} L_f h_1 = \frac{\partial}{\partial p_3} L_f h_2 = \frac{\partial}{\partial p_3} L_f h_3 = 0$. The determinant of the Jacobian matrix comes out to be:

$$
\det(J) = -\frac{\partial}{\partial p_1} L_f h_2 \frac{\partial}{\partial p_2} L_f h_3 \frac{\partial}{\partial p_3} L_f h_3
$$

So the $\text{Rank}(J) = n = 3$, thus the system (10) does not lose rank, so the system parameters are identifiable. Since, identifiability is a special case of observability implying the observability of the system equation (10). The linearized version system (10) about an equilibrium point is also tested for observability and found observable [26]. The uncertain/unknown flow coefficients are estimated using the following equation [5, 12]:

$$
\begin{align*}
[\mu_1] &= \frac{-S(y_1)_e - u_1}{S_p \text{sign}(y_1 - y_3) \sqrt{2g}|y_1 - y_3|} \\
[\mu_2] &= \frac{-S(y_1)_e + S(y_2)_e + S(y_3)_e - u_1 - u_2}{S_p \text{sign}(y_2 - y_3) \sqrt{2g}|y_2 - y_3|} \\
[\mu_3] &= \frac{-S(y_1)_e + S(y_2)_e - u_1}{S_p \text{sign}(y_3 - y_2) \sqrt{2g}|y_3 - y_2|}
\end{align*}
$$

Subject to the conditions: $|y_1 - y_3| \neq 0$, $|y_2 - y_3| \neq 0$ and $|y_3| \neq 0$. The equations (13) give the estimated parameters in terms of inputs, outputs and their estimated derivatives. The next section covers the simulations for estimation of these parameters.

IV. SIMULATION RESULTS

The proposed parameter estimation framework developed in Section III has been used for the estimation of viscosity coefficients of the uncertain three tank system. The targeted viscosity coefficients for estimation are $\mu_1$, $\mu_2$ and $\mu_3$ of interconnecting pipes between the tanks. The nominal values of these viscosity coefficients along with parameters used for the simulations are given in Table 1. The system is working in feedback configuration with nonlinear PI controller adapted from Flies and Sira-Ramirez [12]. A special case study for the simulations is considered i.e. operating conditions with system states $x_1 > x_2 > x_3$ to satisfy the full rank conditions for parameter identifiability. The simulation setup is detailed in Figure 2. Simulated measurements of inputs and outputs (Figure 3) of the close-loop system are fed to the robust exact differentiator based parameter estimator (7). The estimated parameters $[\hat{\mu}_1, \hat{\mu}_2, \hat{\mu}_3]_e$ are obtained using equation (13). The desired steady state water levels (states) to be maintained by controller are $x_1 = 0.60, x_2 = 0.4$. It may be noted that the desired steady state levels are achieved in approximately 20 seconds. The estimated viscosity coefficients are shown in Figure 4. Initially the estimates are very large due to different initial conditions; the estimates approach their nominal values as shown in Figure 4 and Figure 5. The estimates con-
verge asymptotically to their nominal values. The convergence time is about one second, which is reasonably fast considering the slow dynamics of the three tank system which shorter than the estimate times of the order of 100 seconds as compared with algebraic techniques of Join et al [5] and Flies and Sira-Ramirez [12]. The convergence time is also faster by orders of magnitude as compared with Sensitivity-Model-Based Adaptive Filters by C. Bhon [27]. Figure 6 shows the estimated values of viscosity coefficients in the presence of noise with zero mean and variance of 0.01. The results shows the estimates are reasonably good in the presence of noise. However the convergence time has increased, still it is very good as compared to the overall system response and above quoted reference. 

V. CONCLUSIONS

A general framework for parameter estimation of dynamic systems using robust exact differentiator is presented. Unlike numerical derivatives the differentiation method used is not prone to measurement noises and other arithmetic issues. The estimation method based on efficient robust exact differentiator is validated by the online estimation of viscosity coefficients of an industry standard benchmark: three tank system. The estimated parameter approaches to their nominal

Figure 2. Schematic diagram of parameter estimation

Figure 3. Plant output history with state feedback PI Controller

Figure 4. Estimates of viscosity coefficients during initial phase

Figure 5. Estimates of viscosity coefficients in steady state

Figure 6. Estimates of viscosity coefficients during steady state in the presence of noise with variance 0.01
values asymptotically. The performance is estimator is also very good in the presence of noise. The convergence of parameter estimation based on higher order sliding mode technique is exact and insensitive to high frequency noise. The estimated parameters can be effectively employed in systems modeling, controller design and fault diagnostics. The concept of arbitrary order differentiator can be used to increase the number of parameters estimated using a single nonlinear dynamical equation.

ACKNOWLEDGMENT

This work was partially supported by Higher Education Commission (HEC) of Pakistan. The authors are also thankful to the research fellows at Control and Signal Processing Research (CASPR) group, M. A. Jinnah University, Islamabad for their useful help and suggestions.

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Figure 7. Estimation error of viscosity coefficient as compared with their nominal values of 0.5, 0.675 and 0.5 respectively. The estimation error is less than 1% of the nominal values.