I. INTRODUCTION

This paper presents a simulated process that tries to represent, in a simplified way and from a control point of view, the main characteristics of certain industrial-scale processes that mix continuous and batch units with product recirculation. In this way, different methods and algorithms developed for this benchmark could be scaled up later on to problems of industrial interest. This simulated process has been accepted as case study in the work package W4B in the network of excellence HYCON.

In these hybrid processes, the outflow of a continuous process is stored in a tank, which feeds several batch units. The batch units (for instance, reactors), share raw materials and power supply and unload discontinuously to other storage tanks and the final product is separated in a new continuous process (for instance, distillation columns) from other by-products that are sent back to the beginning of the process.

Normally, we can separate the control problem in two layers see Fig. 1: low-level control, it is associated with regulatory and sequential control of each batch unit. It is made by a local controller, like a PLC (Programmable Logic Controller) or a DCS (Distributed Control System). The other layer is a high level control, here, the timing and scheduling of the operation, combined with continuous control of other variables, is a key factor for the overall performance of the system. Obviously, both dynamics, continuous and batch ones are closely connected, so, a plant-wide control policy must be implemented. Plant-wide control is attracting considerable interest, both as a challenging research field and because of its practical importance. It is a topic characterized by complexity in terms of the number and type of equipments involved, diversity of aims, and lack of adequate models and control policies [1], [2] and [3].

This paper is focused in this high-level control. More specific, the control of the process has a double objective: First, to maintain a continuous flow of products avoiding bottlenecks. Second, to keep some variables like levels, compositions, etc. close to their desired set points, in spite of possible disturbances on flow of raw material, energy supply, etc. This kind of problem has received attention from the point of view of on-line re-scheduling of batch units, for instance [4] describes the problem of two autoclaves operating in parallel with shared resources, and [5] treats the case of a multiproduct batch reactor plant.

These processes that involves continuous as well as on/off variables and certain logic of operation can be formulated as hybrid systems where continuous variables are represented by real variables and the logic and on/off variables can be converted into binary variables in a set of inequalities. This leads, in the context of Model Predictive Control (MPC), to a mixed integer optimization problem that must be solved every sampling time, see, for instance [6], [7] and [8]. Nevertheless, very often, the computation times are not as short as desired, preventing its real-time implementation.

The approach followed in this paper to develop a plant-wide control is based on the non-linear model predictive control framework which implies an adequate representation of the continuous units as well as the scheduling and sequential rules of batch process, taken from the overall perspective of the behavior of the plant, so that the decision variables of the problem can be adjusted to maintain the performance of the process in spite of changes, disturbances, etc. We have formulated the control problem in terms of prescribed flow patterns and time of occurrence of events, besides the continuous variables. This focus leads to a
formulation where binary variables are avoided so a NLP optimization problem is solved on-line every sample time.

They are also four manipulated on/off variables: \( \delta_1 \) and \( \delta_2 \) start the operation of each batch unit. On the other hand, \( \delta_0 \) and \( \delta_3 \) are used for giving the order of unload each unit. The flows \( q_{11}, q_{12}, q_{23}, q_{22} \) are manipulated by the local control in each batch unit according to the stage of operation of each one. Also a security on/off valve \( V_0 \) exists, that allows to cut off the input flow of water \( q_0 \).

The main disturbances are flow and temperature of raw material \( (q_0, T_0) \) and temperature \( (T_{23}) \) of flow \( q_{23} \). For simplicity \( T_{23} \) is assume always equal to \( T_0 \).

The global operating objectives are to maintain a smooth operation of the whole process avoiding bottlenecks and to maintain desired temperatures in both the upper and lower tanks (1 and 3). The first objective is equivalent to maintain the levels in tanks 1 and 3 within pre-specified upper and lower limits and not closing security valve \( V_0 \).

On the other hand, disturbances affect strongly the operation of the plant. Notice, for instance, that the cycle time of each batch unit depends on the incoming temperature, and changes on the flow \( q_0 \) of raw material imply an adequate on-line re-scheduling of the batch units.

Also loading and unloading batch unit has a strong impact on tank levels.

II. BENCHMARK DESCRIPTION

Fig. 2 shows the scheme of the proposed benchmark. As we can see, the plant is composed of: An upper tank (tank 1) where a flow \( q_0 \) of raw material (cold water) at temperature \( T_0 \) arrives. Two batch units (1 and 2), operating in parallel, and fed from tank 1, they try to simulate a chemical reactor and follow a prescribed loading-heating-reaction-unloading cycle. Finally, we have a lower tank (tank 3), where batch units unload its products and from which a flow \( q_{23} \), of the final product is obtained (cold water). This tank receives also an external flow \( q_{23} \) (cold water) and provides a recirculation flow \( q_{31} \) to tank 1.

Batch units work independently and are heated with electrical resistors of different power, so the operation is not symmetrical. Both batch units have a local PLC for performing its cycle, which is implemented also in the simulation. In order to simplify the process, no real reaction takes place in the batch units, because the only product is water, but they operate in a similar way to real reactors from the point of view of the overall control. Because of this simplification, the variable temperature has been taken as a substitute of composition, as a measure of quality.

There are two continuous manipulated variables: The recirculation flow \( (q_{31}) \), operating on a variable speed pump and the external flow \( (q_{23}) \), operating on a control valve. For simplicity, it is assumed that perfect flow control loops act on both variables, so that the set points of these flow control loops can be considered as the manipulated variables. Both flows are constrained between zero and a maximum value.

They are also four manipulated on/off variables: \( \delta_1 \) and \( \delta_2 \) start the operation of each batch unit. On the other hand, \( \delta_0 \) and \( \delta_3 \) are used for giving the order of unload each unit. The flows \( q_{11}, q_{12}, q_{23}, q_{22} \) are manipulated by the local control in each batch unit according to the stage of operation of each one. Also a security on/off valve \( V_0 \) exists, that allows to cut off the input flow of water \( q_0 \).

The main disturbances are flow and temperature of raw material \( (q_0, T_0) \) and temperature \( (T_{23}) \) of flow \( q_{23} \). For simplicity \( T_{23} \) is assume always equal to \( T_0 \).

The global operating objectives are to maintain a smooth operation of the whole process avoiding bottlenecks and to maintain desired temperatures in both the upper and lower tanks (1 and 3). The first objective is equivalent to maintain the levels in tanks 1 and 3 within pre-specified upper and lower limits and not closing security valve \( V_0 \).

On the other hand, disturbances affect strongly the operation of the plant. Notice, for instance, that the cycle time of each batch unit depends on the incoming temperature, and changes on the flow \( q_0 \) of raw material imply an adequate on-line re-scheduling of the batch units. Also loading and unloading batch unit has a strong impact on tank levels.

A. Batch unit operation

The operation of batch units follows a certain number of stages, which are executed cyclically, see Fig. 3. At the same time, different actions are carried out in each stage. Referring to batch unit 1, this sequencing has 6 stages:

1. **Waiting for loading**: In this stage, the unit is empty waiting until a signal of start loading is received.
2. **Loading**: In this stage the unit is loaded. The associated action at the beginning of this stage is to open pump \( B_{11} \). It finishes when a certain level has been reached.
3. **Heating**: Here water is heated with an electrical resistor \((r_{b1})\) until a desired temperature is reached \((T_{b1\text{max}})\).

4. **Reaction**: In this stage the temperature is maintained \((T_{b1}=T_{b1\text{max}})\) for a given period of time, as if a reaction took place.

5. **Waiting for unloading**: In this stage the unit is waiting until the signal of start unloading is received.

6. **Unloading**: In this stage the batch unit is unloading its content until it is empty (open pump \(B_{31}\)). When a minimum level of product has been reached, the unit moves to first stage “waiting for loading”.

Batch unit 2 has 6 another similar stages. All sequences and actions executed in each stage come from a local control, a simulated PLC. Nevertheless, signals of start loading and start unloading come directly from a high-level control. The PLC also close automatically the electro-valve \(V_0\) when level of tank 1 increase over the permitted level, so, in this case, there is not flow \(q_0\) of raw material.

![Batch units cycle](image)

**III. Hybrid Model Predictive Control**

In our process, we are interested in the plant wide-control or high-level control. So, it has been developed a hybrid model predictive control which is responsible of maintaining a proper operation of the whole section taking decisions that have a global impact on the process dynamics: scheduling of the batch units (start load and unload) and values of the set points of the regulators that fix the flows of \(q_{31}\) and \(q_{23}\). From the point of view of this control, the regulatory and sequential controls can be seen as included in the process, operating in cascade over them, see Fig. 1.

A natural approach to a many decision problems is Model Predictive Control (MPC): An internal model of the process is used to predict its future behavior as a function of the present and future control actions, which are selected in order to minimize some performance index or cost function. The optimal control signals corresponding to the present time are applied to the process and the whole procedure is repeated in the next sampling period.

The performance index (according to the control objectives described in section II) is the function

\[
J = \int_0^T \left( \sum_{i=1}^{4} (h_i - h_{i,\text{ref}})^2 + a_3 (T_i - T_{i,\text{ref}})^2 \right) dt
\]

where \(h_{i,\text{ref}}, T_{i,\text{ref}}\) and \(T_{3,\text{ref}}\) are the set points of the controlled variables (levels and temperatures of tanks 1 and 3) and \(a_i\) \((i=1,2,3,4)\) are appropriate weight factors. In principle, the only variable that should be associated to a set point in (1) is the temperature of tank 3 \((T_3)\) that is to say, the quality of the final product. However, in industrial process with this same scheme, it is important to regulate the quality of the feed, because the operation time and final quality of the product obtained in a batch unit depend on this quality. Normally the quality of feed is fixed according with a certain productive scheme. For this reason we have included the setpoint of \(T_i\) in the control objectives.

Moreover, several kinds of constraints can be taken into account: First, constraints imposed by the range of the controlled and manipulated variables.

\[
T_{\text{imin}} \leq T_i \leq T_{i,\text{max}} \quad h_{\text{imin}} \leq h_i \leq h_{i,\text{max}} \quad i = 1,3
\]
\[
0 \leq q_{31} \leq q_{31,\text{max}} \quad 0 \leq q_{23} \leq q_{23,\text{max}}
\]

Second, constraints imposed by the logic of the operation and finally constraints that correspond to the dynamic internal model, which allows to link controlled and decision variables.

It is very important that this internal model being as simple as possible while still being a good representation of the process. On the other hand, it must correspond to the view and purpose of the plant-wide control, including only those variables and phenomena relevant. Internal model of this benchmark combines dynamic mass and energy balances in the continuous units (tanks 1 and 3) with abstractions and simplifications of the other parts of the process, batch units 1 and 2.

A. **The internal model**

The behavior of the continuous units is described by mass and energy balances (all tanks are considered well isolated, so, there are not heat losses):

\[
A_i \frac{dh_i(t)}{dt} = q_0 + q_{31} - q_{31} - q_{32}
\]
\[
A_i \frac{dT_i(t)}{dt} = q_0 (T_0 - T_i) + q_{31} (T_{31} - T_i)
\]

(3)

\[
A_i \frac{dh_i(t)}{dt} = q_{23} + q_{32} + q_{33} - q_{31} - q_{32}
\]
\[
A_i \frac{dT_i(t)}{dt} = q_{22} (T_{22\text{max}} - T_i) + q_{32} (T_{22\text{max}} - T_i) + q_{23} (T_0 - T_i)
\]

\(h_i\) represent the level in the tank \((j=1,3)\), \(T_i\) the temperature and \(A_i\) is the cross section. The densities and specific heat
have been assumed constants. Variables $q_0$ and $T_0$ are, respectively, the flow and temperature of the raw material and can be considered as disturbances. $T_{\text{3max}}$ ($i=1,2$) correspond to the batch reaction temperatures that are parameters known in advance. It is assumed that the sequential control of every batch units performs well. The recirculation and mixing flows, $q_3$ and $q_2$, are decision variables. Finally, out flow $q_2$ from tank 3 is proportional to square root of level $h_3$.

The interactions with the batch units in (3) are given through the loading and unloading flows $q_{1i}$ and $q_{2i}$ that correspond to the orders of start load and start unload. According to logic of the operation, see Fig. 4 a), the loading flow $q_{1i}$ is zero except for the loading stages, where it maintain a constant value $q_{1\text{max}}$ while the unloading flow $q_{2i}$ is zero all times except for the unloading period, where it has a constant value $q_{2\text{max}}$ after a discharge order takes place. On the other hand, the model use a continuous time descriptions, so, $T_{li}$ and $T_{di}$ denote the duration of wait for loading and wait for unloading stages, where at the end of these stages start loading and unloading process for batch unit $i$ with known values $q_{1\text{max}}$, $q_{2\text{max}}$ respectively. The times $T_{li}$ and $T_{di}$ are unknown variables that must be estimated. Only two of them per cycle $k$ and per unit $i$ are enough to represent the interacting flows to and from a batch unit.

Next, in order to represent adequately the flow patterns of Fig. 4 a), it is necessary known all periods of all stages of every cycle ($k=1,2,3,…$) and for every batch unit ($i=1,2$). Times $T_{li}$ and $T_{di}$ are calculated by the optimization algorithm. The rest times are $P_{li}$ and $P_{di}$: duration of loading and unloading stages, they are fixed values (every batch unit is always loaded and unloaded completely and with same flows). $P_{ri}$: duration of operation period in which, are included heating $P_{hi}$ and reaction $P_{ri}$ stages ($P_{r}=P_{h}+P_{r}$). Then, loading, unloading and reaction periods are known and fixed along the prediction, but the heating period $P_{hi}$ depends on the temperature of the feed $T_1$ and the available Power$_{hi}$ and must be estimated along the prediction with the follow equation:

$$V_{li}T_{hi} = \left( q_{1i}(T_1-T_{hi}) + \frac{\text{Power}_{hi}}{\rho C_p} \right) P_{hi}$$

where $V_{hi}$ and $T_{hi}$ are the volume and temperature of batch unit $i$, $C_p$ the calorific capacity of water and $\rho$ the water density. Can be defined the duration $C^k_i$ of a certain cycle $k$ of every batch unit $i$ like the sum of duration of all stages:

$$C^0_i = 0$$

$$C^k_i = T_{li}^k + P_{li}^k + P_{ri}^k + T_{di}^k + P_{di}^k \quad k=1,2,3,…$$

Finally, Fig. 4 a) can be translated into the internal model in a set of equations to calculate flows to and from a batch unit $i$ ($i=1,2$) for different cycles $k$ ($k=1,2,3,…$). So, $q_{1i}$ is equal to $q_{1\text{max}}$ when the time of prediction ($t_{\text{pred}}$) verifies:

$$t_{\text{pred}} \leq \sum \left( C_{1i}^{k-1} \right) + T_{ci}^k + P_{ci}^k \quad \Rightarrow \quad q_{1i} = q_{1\text{max}}$$

and $q_{1i}$ is zero in the rest of cases. $q_{2i}$ is $q_{2\text{max}}$ when,

$$t_{\text{pred}} \leq \sum \left( C_{1i}^{k-1} \right) + T_{ci}^k + P_{ci}^k + P_{ri}^k + T_{di}^k + P_{di}^k \quad \Rightarrow \quad q_{2i} = q_{2\text{max}}$$

and zero in the rest of cases. Notice that, as the optimization is performed every sampling time ($T_s$), the state of the cycle at current time $t=kT_s$ will change continuously. For example, if the state of a cycle at time $t$ is loading, see Fig. 4 b), then $q_{1i}$ is $q_{1\text{max}}$ when time of prediction ($t_{\text{pred}}$) verifies:

$$t_{\text{pred}} \leq \sum \left( C_{1i}^{k-1} \right) + T_{ci}^k + P_{ci}^k - P_r \quad \Rightarrow \quad q_{1i} = q_{1\text{max}}$$

or

$$t_{\text{pred}} \leq \sum \left( C_{1i}^{k-1} \right) + P_{hi}^k - P_r + P_{ri}^k + T_{di}^k + P_{di}^k \quad \Rightarrow \quad q_{1i} = q_{1\text{max}}$$

and $q_{2i}$ is $q_{2\text{max}}$ when:

$$t_{\text{pred}} \leq \sum \left( C_{1i}^{k-1} \right) + P_{hi}^k - P_r + P_{ri}^k + T_{di}^k + P_{di}^k \quad \Rightarrow \quad q_{2i} = q_{2\text{max}}$$

where $P_{ri}$ is the executed time of actual stage in the process. A set of similar inequalities (8) and (9) is necessary to calculate $q_{1i}$ and $q_{2i}$, when the state of the time is the stages of heating, operation, waiting for load and unloading, Fig 4 c), d) and e). In this way, all variables in the internal model (3)-(9) are continuous while, at the same time the scheduling of the batch units has been integrated in the plant-wide model. This means that the associated optimization is not a mixed integer one, so that NLP algorithms can be used instead, saving computation time.

### B. Prediction and control horizons

It is necessary to adapt some concepts used in standard continuous MPC to the context of mix continuous-batch processes, that is, the prediction and control horizons. The prediction horizon ($N_2$) corresponds to the future time interval used to compute predictions with the internal model. In standard MPC, it is chosen longer than the process settling time. In our context, it will be translated into $N_p$, minimum number of full cycles performed for all batch units, and will be selected longer than the number of cycles assumed to be
required for obtaining a stable operational pattern. This means that, in order to compute the cost function (1), the internal model will be integrated until the slowest batch unit completes $N_p$ batches. Notice that the capacity of the batch units can be different, so, each one can perform a different number of batches in the same period of time $T_p$.

Fig. 5 displays an example with two batch units and $N_p=3$ where the prediction horizon is the time needed by batch number two to implement three full cycles. Obviously, batch unit 1 makes more cycles (3 and a half). Predictions of the continuous variables will also be made up to this time.

The concept of control horizon ($N_u$) in continuous MPC corresponds to the time interval where present and future control actions are computed. After $N_u$ sampling times the controller will use the last control signal computed at $N_u$ until the end of the prediction horizon in order to compute its output predictions. In the batch-continuous environment, the control horizon is split into batch control horizon $N_b$ and continuous control horizon $N_c$. The first one is applied to every batch unit $i$ ($i=1,2$). The controller will compute the decision variables $T^k_{ci}$ and $T^k_{di}$, loading and unloading periods of every cycle ($k=1,2,3,...$), needed to schedule a number of cycles equal to $N_b$. From the future cycle number $N_b$ until the end of the prediction horizon (cycle number $N_p$), the schedule of batch unit $i$ will follow the pattern of the $N_b$ cycle, in a similar way to how the future control signals are treated in continuous MPC. In the example considered in Fig. 5, batch unit number 1 has $N_b=1$, so, only the times $T^1_{ci}$ and $T^1_{di}$, that correspond to the first cycle will be computed and in the following ones the same pattern will be applied, that is to say, $T^2_{ci} = T^0_{ci} = T^1_{ci}$ and $T^2_{di} = T^1_{di} = T^2_{di}$ (for cycles 2, 3 and 4). Batch unit number two has been assigned $N_b=2$, so, the controller compute $T^1_{ci}$, $T^1_{di}$, $T^2_{ci}$, and $T^1_{di}$ and from the second cycle on, will follow the load/unload pattern defined by the last $T^2_{ci}$ and $T^2_{di}$ times computed, so $T^3_{ci} = T^2_{ci}$ and $T^3_{di} = T^2_{di}$ (for cycle 3).

Finally the $N_c$ control horizon is applied to every continuous manipulated variable $u_j$ but need to be redefined because the classical meaning of $N_u$ is not very useful in this context where, in order to obtain a stable pattern of behavior, control actions are needed along the whole prediction horizon. Taking into account this need and in
correspondence with the continuous time framework of the controller, the control horizon \( N_{cj} \) will mean the number of changes allowed to the manipulated variable \( u_j \) within the prediction horizon. Notice that this definition implies that not only the value of the manipulated variable \( u_{j\ell} \) (\( \ell = 1, ..., N_{cj} \)), but the time duration of these changes will be considered as new decision variables \( T_{j\ell} \) (\( \ell = 1, ..., N_{cj} \)). On the bottom side of Fig. 5, it is shown a manipulated variable \( u_j \) with \( N_{cj} = 4 \) so, we have four future changes \( u_{j1}, u_{j2}, u_{j3} \) and \( u_{j4} \), and three time intervals \( T_{j1}, T_{j2} \) and \( T_{j3} \), that is to say, every future value \( u_{j\ell} \) (\( \ell = 1, ..., N_{cj} \)) has a duration of \( T_{j\ell} \) (\( \ell = 1, ..., N_{cj} - 1 \)).

![Fig. 5. Prediction and control horizons in hybrid MPC.](image)

### C. NMPC controller

The aim is to minimize (1) under the dynamics and static constraints (2)-(9), and the decision variables correspond to the set points of flow control loops of batch units (\( i = 1, 2, ..., N_b \)). To simplify the resulting problem (only in terms on continuous decision variables) is solved periodically (see Fig. 6), with a given sampling period, using a NLP optimization algorithm.

![Fig. 6. Non-linear MPC controller implementation.](image)

The simulation package integrates the internal model equations along the prediction horizon \( N_p \) taking as initial conditions the current process state and evaluating the formulated cost function \( J \) (1) at the end of the integration. Path constraints (2) are implemented as penalty functions in (1). Resuming, our optimization problem involves the following decision variables: \( T_{ci}^k \) and \( T_{di}^k \) (\( k = 1, ..., N_b \)) for batch units (\( i = 1, 2, ..., N_b \)) and \( T_{cj}^k \) (\( \ell = 1, ..., N_{cj} - 1 \)) for continuous manipulated variables (\( j = 1, 2, ..., N_c \)).

### IV. Simulations Results

The model of the benchmark is based in first principles; so, we have a detailed model of the process in terms of mass and energy balances for all tanks and a set of logic equations and constraints to describe the batch units operation.

The benchmark has been simulated and tested using the simulation language EcosimPro [9]. It belongs to the category of the so called “modelling languages” that are object oriented and allows an easy re-use of the components, as well as combine DAE equations with events, while performing a correct integration in spite of the model discontinuities. It generates automatically C++ code corresponding to the simulation. Also, it is available an OPC server of the simulation of this process. Moreover, there is a pilot plant with the same structure in the laboratory of the Department of Systems Engineering and Automatic Control, at University of Valladolid, available for experiments.

About the controller, it was programmed in C++ and the simulation package used to integrate internal model was also EcosimPro, the optimization algorithm was a SQP implemented in a commercial library NAG for C. The methodology to develop and design non linear predictive controllers like the one presented here, easy to maintain and to prove in different environments (from simulation to real process) is explained in [10].

An experiments of 5 hours (18000 sec.) will be presented, under strong changes in the inflow \( q_0 \) and several disturbances in temperature \( T_0 \); see Fig. 7. The nominal values for these disturbances are \( q_0 = 30 \text{ cm}^3/\text{s} \) and \( T_0 = 15.5^\circ \text{C} \). The internal model uses always these constant values to make the predictions.

![Fig. 7. Non measured disturbances \( q_0 \) and \( T_0 \).](image)

Sampling time is 30 sec. The prediction horizon was fixed in \( N_p = 3 \), which corresponds to a predictions of around half an hour (1500 sec.) and the control horizons for every batch unit were selected as \( N_{cj} = 2 \), while the continuous control horizons were chosen as \( N_{cj} = 4 \). So the dimension of optimization problem is 19 decision variables \( 2 \times N_{c1} + 2 \times N_{c2} + N_{c1} + N_{c2} + (N_{cj} - 1) \). Notice that, we have used the same durations of changes for the manipulated variables \( q_{31} \) and \( q_{23} \). So we have increased the decision variables in \( (N_{cj} - 1) \). Finally, the control objectives (setpoints, maximum, and minimum values permitted for controlled variables) and weights in cost function (1) are:
Fig. 8 and Fig. 9 show the time evolution of levels (in cm) and temperatures (in ºC) in the upper tank and lower tank besides their allowed operating ranges and set points.

<table>
<thead>
<tr>
<th></th>
<th>$h_1$ (cm)</th>
<th>$T_1$ (ºC)</th>
<th>$h_3$ (cm)</th>
<th>$T_3$ (ºC)</th>
</tr>
</thead>
<tbody>
<tr>
<td>maximum</td>
<td>22</td>
<td>-</td>
<td>22</td>
<td>-</td>
</tr>
<tr>
<td>minimum</td>
<td>6</td>
<td>6</td>
<td>6</td>
<td>6</td>
</tr>
<tr>
<td>setpoint</td>
<td>15.5</td>
<td>17</td>
<td>15.5</td>
<td>25</td>
</tr>
<tr>
<td>weight ($\alpha_i$)</td>
<td>0.1</td>
<td>1</td>
<td>0.1</td>
<td>2</td>
</tr>
</tbody>
</table>

Hybrid controller is able of maintaining the operation of the process adapting the scheduling to the different conditions (non measured disturbances). Temperatures are maintained close of its set points and levels are within permitted range. Only in one case (around 14000 sec.), the processing capacity of the plant is overflow and security on/off valve $V_0$ is closed.

In Fig. 10, on the top, it is shown the time distribution of the stages: waiting to load, loading, heating, reaction, waiting to unload and unloading, with numbers 1, 2, 3, 4, 5 and 6 respectively for batch unit 2. On the bottom, the same variable, but now for batch unit 1. Notice that both units show different patterns due to the differences of available power (3000 w. in unit 1 and 1500 w. in unit 2).

Fig. 11 shows the time evolution of the continuous manipulated variables the recycle $q_{31}$ and the supply $q_{23}$ (in cm$^3$/sec.), and Fig. 12 shows the outflow of the process $q_{32}$ (in cm$^3$/sec.).
A new simulation has been carried out, in a similar way, Figs. 13 to 15 show the same variables in the same experimental conditions, except, now, we increase to 1 the weight factors of levels in cost function (1), so \( \alpha_1 = \alpha_3 = 1 \), \( \alpha_2 = 1 \) and \( \alpha_4 = 2 \).

For this second simulation, the evolution of levels of both tanks has improved: the amplitude of the oscillations has decreased, levels are more centered on its set points and the inflow \( q_0 \) is never cut off. However, temperatures in the same tanks are worse than in the previous results. Compare Fig. 8 and Fig. 9 with Fig. 13 and Fig. 14. This result is the expected one, because to maintain the upper level if there is not enough raw material \( q_0 \) it is necessary to recirculate more flow \( q_3 \) with a high temperature, and of course to introduce an extra inflow \( q_{23} \) to the lower tank, see Figs. 13, 14 and 15 at time 6000 sec. These actions imply an increase of upper tank temperature and decrease the lower tank temperature. The opposite effect occurs if the inflow \( q_0 \) is large, see the same figures but at time 14000 sec.

The strategy followed in this paper to control processes that combine continuous and batch units performs well in simulation and opens the door to practical implementations at industrial scale.

REFERENCES