ABSTRACT
A new receiver structure for improving the performance of conventional Discrete Multitone (DMT) based Digital Subscriber Line (DSL) transceivers is described. In this method, the conventional demodulator consisting of an FFT followed by a scalar frequency-domain equalizer is augmented by an additional data-path utilizing windowing. Time-limited receiver windowing is done independently for each symbol over the orthogonality interval while the transmitter is unchanged. Windowing can be carried out efficiently in the time or frequency domain. A decision feedback equalizer is used to cancel the resulting inter-bin-interference. Computer simulation results are presented for ADSL with realistic interference conditions which show that this method is most effective against narrowband interference and other impairments which lead to diminished inter-bin orthogonality. The overall technique is also applicable to other multicarrier modems such as VDSL and wireless OFDM.

1. INTRODUCTION
The susceptibility of DMT based DSL modems to narrowband interference (NBI) and other impairments which cause inter-bin interference (IBI) is well known [6, 7]. The major source of NBI for Asymmetric DSL (ADSL) modems is ingress from AM broadcast and amateur radio. All frequency bands below 5 MHz are generally considered to be potential sources of interference to include aliasing effects. Terrestrial radio signals from these transmitters couple into twisted pair loops mounted on poles or buried underground. The extent of coupling depends on a number of characteristics of the cable (aerial/buried, length, orientation etc) and the radio transmitters (power, proximity, antenna orientation etc) [6]. The spectral occupation of ADSL is from approximately 30 KHz to 1.1 MHz (ADSL-Lite extends up to 550 KHz) while Very high-speed DSL (VDSL) spectrum generally extends from about 300 KHz to 10 MHz. Clearly, there is significant spectral overlap between existing AM and HAM radio stations and the spectral occupancy of ADSL and VDSL modems. Other impairments which lead to IBI are inadequate channel shortening, spectrally colored crosstalk and symbol timing offsets with jitter.

This paper presents a new receiver-end technique based on time-limited receiver windowing to combat IBI. Receiver windowing is simple to implement and can be done in the time or frequency domain. Due to its reduced sidelobe response, the windowed FFT greatly reduces the susceptibility of conventional DMT receivers to IBI causing impairments. The deterministic IBI created by windowing itself is cancelled by a frequency-domain decision feedback equalizer (DFE).

2. DMT SIGNAL MODEL
Consider a discrete-time DMT system model commonly used for DSL transceivers [6] in which N orthogonal sinusoids (tones/bins) are transmitted each symbol time using $2N$ samples per symbol. The $n$th time domain sample of the normalized transmitted signal for the $i$th bin is given by

$$s_i(n) = \frac{2}{\sqrt{2N}} [I_i \cos(w_i n) - Q_i \sin(w_i n)]$$

for $n \in [0, 2N - 1]$, $i \in [0, N - 1]$ and where $I_i$ and $Q_i$ are the real and imaginary parts of the $i$th complex frequency domain sub-symbol $a_i = I_i + jQ_i$, chosen from a QAM constellation; $j = \sqrt{-1}$; $w_i = 2\pi i / 2N$ is the angular frequency in radians/sample of the $i$th bin. Denoting the minimum and maximum active bin number by $b_{\text{min}}$ and $b_{\text{max}}$ respectively, the $n$th sample of the composite transmitted signal is given by, $x(n) = \sum_{i=b_{\text{min}}}^{b_{\text{max}}} s_i(n)$.

Let the sample rate be $f_s$ samples/sec. Each symbol is prefixed with a cyclic prefix (CP) of duration $T_{CP} = G/f_s$ seconds by setting $x(-n) = x(2N - n)$, for $n \in [1, G]$. Inter-symbol-interference is eliminated and orthogonality between bins is preserved when the CP duration exceeds the channel impulse response (CIR) i.e. when $G \geq U$ [7]. The effective discrete-time CIR encountered by the above signal includes the analog communication channel, transmitter D/A, receiver A/D, interpolation and front-end filtering in the transmitter and receiver. Thus, let the discrete-time composite CIR be denoted by $h(n)$ and assume that it spans a maximum duration $T_{CIR} = U/f_s$ seconds. Assum-
ing the DSL channel transfer function to be linear, time-invariant and causal, the discrete time received signal is given by
\[ r(n) = z(n) * h(n) + z(n) \]
where \( z(n) \) denotes the composite discrete-time AWGN and crosstalk samples and \( * \) denotes the linear convolution operation. After discarding the CP, a conventional receiver performs a Discrete Fourier Transform (DFT) operation via the FFT algorithm each symbol time. This is followed by a scalar (single-tap-per-bin) frequency domain equalizer (FEQ).

3. PROPOSED EQUALIZER STRUCTURE

Most of the attention in the literature on windowing in Multicarrier transceivers has been confined to the case when the window or pulse shaping function is done at the transmitter and extends over one or more symbols. They are generally also designed to preserve inter-bin orthogonality to eliminate or greatly reduce any resulting IBI. However, this greatly restricts the choice of shaping functions and leads to significantly higher complexity at the receiver \[4\].

A time-domain receiver windowing approach was described in \[5\] for VDSL modems. This approach requires the window to extend into a symbol prefix and suffix beyond the orthogonality interval. This is done in order to preserve inter-bin orthogonality but also results in a higher level of ISI since the prefix and suffix overlap with the guard time.

A key aspect of the proposed method is that windowing is done such that it introduces a deterministic amount of IBI which can be canceled out. Thus, the distinguishing feature of the proposed technique compared to the conventional configuration (FFT followed by FEQ) described above is the use of a parallel windowing stage followed by a DFE along frequency to cancel the window-induced IBI. Windowing can be implemented equivalently in the time- or frequency-domains depending on the window type, computational constraints and convenience of implementation (Figure 1 and 2). The goal of receiver windowing is to enable suppression of the side-lobes of the DFT frequency response and consequently obtain better performance against crosstalk and NBI, while keeping the transmitter unchanged.

Time-domain windowing (TDW) is performed by a sample-by-sample multiplication of \( 2N \) received signal samples each symbol time with the window coefficients. For this purpose, a variety of windows may be used, such as the normalized Hanning window given by \( w(n) = (1 - \cos \omega_n) \) for \( n \in [0, 2N - 1] \). Consider now the receiver architecture of Figure 1 or 2. The upper box marked as Data-Path 1 (DP1) is the conventional demodulator described in Section 2. The lower box marked as Data-Path 2 (DP2) is the new portion. Incoming samples simultaneously pass through both DP1 and DP2. Let the \( i \)-th demodulated sub-symbol (after FFT) in DP1 be denoted by \( \hat{a}_i \) and in DP2 by \( \hat{b}_i \). Then,

\[
\hat{b}_i = \frac{1}{\sqrt{2N}} \sum_{n=0}^{2N-1} r(n)w(n)e^{-j\omega_i n} = \beta_i a_i + u_i + \mu_i
\]

\[
\hat{a}_i = \frac{1}{\sqrt{2N}} \sum_{n=0}^{2N-1} r(n)e^{-j\omega_i n} = \alpha_i a_i + u_i
\]

where

\[
\alpha_i = \sum_{u=0}^{U-1} h(u)e^{-j\omega_i u}, \quad u_i = \frac{1}{\sqrt{2N}} \sum_{n=0}^{2N-1} z(n)e^{-j\omega_i n}
\]

\[
\beta_i = c_0 \alpha_i, \quad u_i = \frac{1}{\sqrt{2N}} \sum_{n=0}^{2N-1} z(n)w(n)e^{-j\omega_i n}
\]

\[
c_k = \frac{1}{2N} \sum_{n=0}^{2N-1} w(n)e^{-j\omega_k n}, \quad \text{and} \quad \mu_i = \sum_{l=0}^{b_{\text{max}}} a_l a_{l-i}
\]

In the above, \( \mu_i \) denotes the window-induced IBI experienced by the \( i \)-th bin from all other active bins. The window coefficients may be normalized such that \( c_0 = 1 \) (assumed to be used throughout). For example, for a Hanning window, \( c_0 = 1, c_{-1} = c_{+1} = -0.5 \). Note that \( \mu_i \) vanishes for the special case of rectangular windowing.
Figure 3: Magnitude response of different Frequency-domain windows.

\( w(n) = 1 \). Observe that the multiplication of the window with the DMT symbol samples in the time-domain followed by an FFT is equivalent to a frequency-domain convolution. In other words, windowing can be carried out efficiently in the frequency domain as,

\[
\hat{b}_i = \sum_{l=b_{\text{min}}}^{b_{\text{max}}} \hat{a}_{i+l} c_l \quad \text{and} \quad \hat{v}_i = \sum_{l=b_{\text{min}}}^{b_{\text{max}}} u_{i+l} c_l \tag{4}
\]

A few properties of the post-FFT noise sequences \( \{u_i\} \) and \( \{v_i\} \) are apparent.

1. If \( \{z(n)\} \) is zero-mean and Gaussian, \( u_i \) and \( v_i \) are also zero-mean and Gaussian. This follows from (3) where \( u_i \) and \( v_i \) are obtained as linear combinations of the noise samples \( z(n) \).

2. Assume \( \{z(n)\} \) is a realization of a wide-sense stationary random process. If \( r_z(n) \) and \( P_z(w) \) denote the discrete time \( n \)th auto-correlation coefficient and Power Spectral Density (PSD) of \( \{z(n)\} \) respectively, the variance of \( u_i \) is given as

\[
\sigma_u^2(i) = E[|u_i|^2] = \sum_{n=-(2N-1)}^{2N-1} (1 - |n|/2N) r_z(n) e^{-jwn} = P_z(w) \frac{\sin^2(2Nw/2)}{2N\sin^2(w/2)} |w=w_i|
\]

3. If \( \{z(n)\} \) is white \( (r_z(n) = \sigma^2 \delta(n), P_z(w) = \sigma^2 / 2\pi) \), then the auto-correlation functions of \( u_i \) and \( v_i \) from bin-to-bin are given by,

\[
r_u(i) = \sigma^2 \delta(i), \quad \text{and} \quad r_v(i, k) = \sum_m \sum_l r_u(k+m-i-l)c_l^* c_m \tag{5}
\]

Thus, \( u_i \) is a white sequence from bin-to-bin while \( v_i \) is spectrally colored as per the windowing function. Their respective variances \( \sigma_u^2 \) and \( \sigma_v^2 \) (for any bin) are given by,

\[
\sigma_u^2 = \sigma^2, \quad \text{and} \quad \sigma_v^2 = \sigma^2 \left( \frac{1}{2N} \sum_{n=0}^{2N-1} w(n)^2 \right) = \sigma^2 \sum_{l=b_{\text{min}}}^{b_{\text{max}}} |c_l|^2 \tag{6}
\]

Equations (3) and (4) suggest that windowing can be looked upon as a form of partial response signaling (PRS) along frequency with coefficients \( c_l \) [3]. The Hanning window can be effectively viewed as a class V PRS response [2]. An important difference between conventional PRS and receiver windowing in the problem at hand is that the PRS function is carried out at the transmitter while windowing is carried out at the receiver-end including the effect of channel attenuation coefficients and additive noise.

This suggests the structure of Figure 2 for Frequency-domain windowing (FDW) using equation (4). This formulation is significantly more computationally efficient when the window-DFT response has only a few non-zero coefficients \( c_l \).

The operation of the DFE is now described. First only DP1 is active to estimate the FEQ coefficients for each bin via conventional procedures. Denoting the FEQ coefficients by \( f_i \), the equalized sub-symbols in DP1 are given by

\[
\hat{a}_i = f_i a_i = f_i a_i a_1 + f_i u_i \tag{7}
\]

The IBI contribution from the \( i \)th bin into the \( k \)th bin is characterized by the IBI coefficients \( c_{k,i} \). These interfering terms can be cancelled by feeding back sub-symbols which have already been decoded. Thus, assume that decisions are fed back from the "right", \( i.e. \) the equalization of bin \( k \) utilizes decisions made upon bin \( (k+1) \), bin \( (k+2) \) and so on. The same procedure may be carried out using decisions from the "left". In the rest of this paper, the only right-feedback is shown for convenience. Assuming \( L \) feedback taps \( (b_{\text{min}} = 0, b_{\text{max}} = L), (L+1) \times 1 \) DFE weight vector for the \( i \)th bin in DP2, \( w_i \), is given as,

\[
w_i = f_i [1, \ldots, -c_L]^T \tag{8}
\]

where \( c_l \) are the window IBI coefficients obtained from equation (3). This solution for \( w_i \) is directly obtained after step
1 and does not require any additional training symbols. Note that the coefficients \( c_l \) for \( l < 0 \) are merely shifted to the right such that the reference sub-symbol (sub-symbol at output of equalizer) is multiplied by \( c_0 \). For example, a frequency shifted version of the Hanning window has coefficients \( c_0 = -0.5, c_1 = 1 \) and \( c_2 = -0.5 \). Normalizing the main tap to unity, this becomes \( c_0 = 1, c_1 = -2 \) and \( c_2 = 1 \) for use by the DFE. The above DFE coefficients are used in conjunction with the input vectors \( x_i \) given as

\[
x_i^\text{right} = \left[ \hat{b}_i, \ldots, \hat{a}_{i+L}\hat{a}_{i+L} \right]^T
\]

where \( \hat{a} \) denotes decision feedback sub-symbol values. After the DFE coefficients have been initialized, equalized sub-symbols from DP2 are given by

\[
\hat{b}_i = w_i^T x_i^\text{right} = f_i \alpha_i a_i + f_i v_i + \sum_{l=1}^{L} f_i c_l (\alpha_{i+l} a_{i+l} - \hat{a}_{i+l} \hat{a}_{i+l})
\]

The last term arises from any discrepancies in channel estimation and feedback decision errors. In comparison, the output of DP1 is given by equation (7) above. Figure 4 depicts the above described DFE structure with \( L = 2 \) feedback taps.

Figure 3 plots four different window responses – Rectangular (only \( c_0 = 1 \)), Hanning (\( c_0 = 1, c_1 = -2, c_2 = 1 \)), Asymmetric FDW with non-zero coefficients (\( c_0 = 1, c_1 = -0.5 \)), and with coefficients (\( c_0 = 1, c_1 = -1 \)). There are, of course, a variety of other windows that may be used as well. It is seen that among the above windows the Hanning window displays the best side-lobe suppression. However, it also requires 2 feedback taps and has the largest SNR degradation in white noise of 10 log10 \( 6 = 7.78 \) dB. In comparison, the last two windows have higher side-lobes but require only a single feedback tap and have white-noise SNR degradations of 10 log10 \( 1.25 = 0.97 \) dB and 10 log10 \( 2 = 3 \) dB respectively.

During regular operation, \( \hat{a} \) elements in \( x_i^\text{right} \) (or \( x_i^\text{left} \)) are obtained as reconstructed sub-symbol values obtained from the slicer. The bin-select logic (Figures 1 and 2) determines the data-path (DP1 or DP2) used to provide the slicer input for any given sub-symbol. (Figure 4 shows slicer operating on \( \hat{b}_i \). The bin-select logic may select all sub-symbols from DP1 only (i.e., no windowing is used), all from DP2 only, or select from DP1 or DP2 on a bin by bin basis. In Section 4, the simple criterion of selecting the path which yields higher number of bits in a given bin is used. Once a SNR-per-bin profile is computed for DP1 and DP2, the bit-loading algorithm [6] is run twice, once on each profile. This ensures that the bit-loading algorithm being used need not be modified and so that the combined bit-allocation is no worse than a conventional receiver (DP1 only).

### 3.1. Error propagation in DFE

For DP1, the probability of symbol error per dimension for \( M^2 \)-ary QAM with AWGN is given by

\[
P_e = 1 - [1 - \left( 1 - \frac{1}{M} \right) \text{Prob}(|u_i| > \frac{d_{\text{min},i}}{2})]^2
\]

\[
\approx 4\left( 1 - \frac{1}{M} \right) Q\left( \frac{d_{\text{min},i}}{2\sigma_u} \right)
\]

where \( d_{\text{min},i} = |\alpha_i|/d \) denotes the minimum distance between QAM constellation points at the channel output, \( d \) equals the distance between uncoded input QAM constellation points and the Q-function is defined as \( Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty \exp(-\frac{x^2}{2})dx \). For ADSL modems, the probability of symbol error per dimension \((P_e/2)\) is generally specified \((10^{-7})\) for ADSL). Similarly, for DP2, using (10) and (11),

\[
P_{e,\text{DFE}} \approx 2\left( 1 - \frac{1}{M} \right) \text{Prob}(|u_i| + \sum_{l=1}^{L} c_l (\alpha_{i+l} a_{i+l} - \hat{a}_{i+l} \hat{a}_{i+l}) > \frac{d_{\text{min},i}}{2})
\]

This expression is highly intractable to evaluate for general cases when there are feedback decision errors. If error propagation is ignored and \( \alpha_i \) estimates are exact, (12) reduces to (11) with \( \sigma_u \) replaced by \( \sigma_v \). Clearly, DFE performance will be degraded by feedback errors and it is important to take its effect into account in devising the overall equalization structure [2]. This can be compensated by providing an additional SNR margin while doing bit-loading. The method used to compute this additional margin is based on probability of error calculations in QAM with error propagation [2], but is omitted here for lack of space. Any errors are also likely to propagate through multiple bins because of the typically large values of feedback coefficients and increase the BER from the specified value. However, owing to symbol-by-symbol detection in DMT, propagating errors are always confined to the same DMT symbol.

### 4. SIMULATION RESULTS

Figure 5 shows the PSD's of the transmit, receive and cumulative additive noise signals for parameters chosen according to the ITU-T G.992.1 (ADSL) standard [1]. This figure corresponds to the downstream ADSL transmission over...
Figure 5: PSD of Downstream transmit signal, received Signal and additive noise with ITU G.992.1 (ADSL) test case 7 using loop T1.601\# 9 with 24 ISDN NEXT, -140 dBm/Hz AWGN and -53 dBm AM Ingress centered at 540 KHz.

loop T1.601\# 9 with 24 ISDN near-end crosstalk (NEXT), AWGN of PSD -140 dBm/Hz and NBI. The NBI is modeled as AM radio signals centered at 540 KHz with a total power of -53 dBm distributed over a 10 KHz bandwidth (-93 dBm/Hz peak). Based on experimental field measurements, around 50% of households in North America are expected to encounter this level and type of AM ingress. Frequency division duplexing is used with a transmit PSD of -40 dBm/Hz over the downstream passband and a fixed channel-shortening receiver TEQ. The roll-off of the transmit filter is seen on the left side. In this example, a 2-tap FDW using the pilot bin 64 as the reference bin. For bin indices greater than 64, the DFE input vector is $x^\text{right}_i$ with window coefficients $c_0 = 1, c_1 = -1$. For bins index below 64, $x^\text{left}_i$ is used with $c_0 = 1, c_{-1} = -1$. This window provides a good trade-off between having fewer feedback taps, good side-lobe suppression and low SNR degradation in white noise.

Figure 6 plots the achieved SNR per bin at the slicer input for the conventional method (from DP1) and with windowing (DP2). The severe degradation in the conventional SNR is apparent while the windowed datapath is able to provide substantial gains in SNR. These SNR profiles are used to carry out bit-loading using any one of many known algorithms [6]. Bit-loading for DP2 also includes an additional SNR margin of 1.1 dB to combat any error propagation in the DFE (see Section 3.1). In this example, for the specified bit error rate of $10^{-7}$ and 6 dB margin, the combined bit-rate is approximately double of the conventional bit-rate achievable from DP1 alone. The bin-select logic for selecting DP1 or DP2 for any given bin is simply to select the path which yields higher bits per bin after bit-loading is complete. The difference between received signal PSD and the cumulative noise PSD is also plotted for comparison. This SNR difference can be used to compute the Shannon capacity for the particular loop and noise conditions.

5. REFERENCES


