Wireless Network Coding in Multi-Cell Networks: Analysis and Performance

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Abstract—Network coding has recently attracted a lot of interest from the research community, mainly due to its simplicity of application and the large number of applications it can be suited for. Research in network coding started on wired networks, but the researchers soon came to realize that the inherent properties of the wireless medium render it a fertile ground for the application of network coding, and the resulting efforts mainly targeted bidirectional traffic scenarios. In this work, we focus on practical implementation issues for network coding when applied to uplink cellular traffic. We provide our own relaying scheme based on decode-and-forward relays and network coding, with a specially designed method of decoding at the receiver. We complement this method with user grouping in order to make better use of the application of network coding. We measure the SINR and capacity performances of our proposed scheme and compare them to the classical decode-and-forward algorithm in a system level simulator. We show that user grouping should complement a network coding solution in order to translate the decrease in the number of transmissions offered by network coding into capacity gains.

Key Words: Network Coding, Relaying, System Performance, User Grouping, OFDM.

I. INTRODUCTION

The main limitation to the performance of wireless networks is the unreliability of the wireless channel. This shortcoming is usually overcome by introducing diversity in the system. One way to introduce diversity in the received signal is to exploit the spatial dimension when multiple antennas are used at the transmitter and/or receiver. The use of multiple antennas offers significant diversity and multiplexing gains relative to single antenna systems. Multiple-Input Multiple-Output (MIMO) wireless systems can thus improve the link reliability and the spectral efficiency relative to Single-Input Single-Output (SISO) systems. Another method that offers macro diversity is relaying or distributed systems such as distributed antenna systems (DAS) or cooperative systems. A relaying system is a conventional radio network that is complemented with relay nodes where these nodes communicate wirelessly with other network elements, e.g. a base station (BS), another relay or a user terminal (UT). A cooperative relaying system is a relaying system where the information sent to an intended destination is conveyed through various routes and combined at the destination. Each route can consist of one or more hops utilizing the relay nodes. In addition, the destination may receive the direct signal from the source. Cooperative relaying systems can be divided into numerous categories based on desired parameters. For instance, the way the signal is forwarded and encoded at the relay station can be classified into two categories: amplify-and-forward (AF) and decode-and-forward (DF). As its name indicates, in amplify-and-forward the relays simply amplify and forward the received signal. In the decode-and-forward case, the relays demodulate and decode the signal prior to re-encoding and retransmission.

The present-day communication networks, mentioned above, share the same fundamental principle of operation: the information or packet sent from one source, $O_1$, to a destination, $D$, is transported independently from other information sent from another source, $O_2$, to the same destination, $D$. Routers, repeaters or relays simply forward the data to the destination. In contrast to those communication networks, Network Coding (NC) is a new area of networking, in which data is manipulated inside the network (e.g. at an intermediate node, $N$, which we will refer to as the network coding node) to improve throughput, delay, and/or robustness. In particular, NC allows instead the nodes to recombine several input packets into one or several output packets. At the network coding node $N$, some linear coding could be performed on the packets present there, and the resulting encoded packet can be broadcasted for different recipients simultaneously instead of transmitting each packet separately.

The area of network coding was first introduced by Ahlswede et al. in [1]. In this paper, network coding was presented as a method that allows intermediate nodes to perform some processing (coding) on the packets they receive, in exchange for throughput gain. In [2], the authors showed that linear network coding is sufficient to achieve the maximum flow bounds between the source-destination pairs in wired networks. Ho et al. extended this result and showed in [3] that a random linear combination of packets is enough to achieve the capacity for multicast traffic. However, these works mainly targeted wired networks. Nevertheless, researchers soon came to realize that the broadcast property of the wireless medium makes it a natural environment for the application of network coding. Only few works, e.g. [4], [5], [6], [7], [8] have hitherto considered a network coding solution for wireless networks.

The majority of the previous works were dedicated towards bidirectional traffic. Very few works as [6] examined the case of network coding for the uplink channel, and none has tackled it in a multi-cell scenario. In [6] the outage performance of
network coding on a link level was studied. Further it was assumed that any users can be grouped together to perform network coding on. In reality, in a wireless network system there is a set of active users in a cell at a time that can be conveniently paired together. Hence the first obstacle that we are faced with at a network level is which set of users shall be selected and grouped to perform the network coding operation. Obviously a random selection will not yield the optimal capacity of the system.

The main contributions of this paper can be summarized as follows:

- We present a novel network coding based relaying protocol complemented with a suitable decoding method at the receiver.
- We provide capacity results for NC when applied to a multi-cell OFDM system, and show through system level simulations that the random application of network coding\(^1\) is not sufficient to obtain the gains expected from lowering the number of transmissions.
- We propose the usage of user grouping whenever NC is performed in order to extract the capacity gains expected from the decrease in the number of transmissions.

The rest of this paper is organized as follows. In Section II, a brief overview of NC in wireless systems is given. Section III presents our user grouping algorithm. The signal to interference-and-noise ratio (SINR) equations and the sum-capacity equation based on our proposed decoding method are derived in Section IV. In Section V, our network deployment model is explained. The system-level simulation results are presented in Section VI. Lastly, conclusions are given in Section VII.

II. WIRELESS NETWORK CODING

In wireless communications, network coding can be divided into two generic schemes: digital and analog. Digital network coding refers to coding at the packet level, meaning that the network coding node will XOR (or alternatively perform other types of encoding on) the bits of the packets to be encoded. In order to do so, the network coding node needs to possess decoding capabilities, hence digital network coding can be performed only with DF relays. Digital NC tries to avoid interference between the sources encoded together. Figure 1 illustrates the transmission phases in a digital NC system. During the first slot (i.e. first phase) one of the sources (e.g. \(O_1\)) transmits its packet \(b_1\) using the entire available bandwidth (see Figure 1(a)). In the second slot (see Figure 1(b)), the other source (e.g. \(O_2\)) transmits its packet \(b_2\). During the third time phase (see Figure 1(c)), the network coding node \(N\) forwards the network coded packet (e.g. \(b_1 \oplus b_2\)) in a single slot instead of using 2 slots to separately transmit \(b_1\) and \(b_2\). In fine, digital NC requires only 3 transmissions in contrast to a classical relaying system where 4 transmissions are required\(^2\).

\(^1\)By random NC we mean that the users over which NC is performed are paired randomly.

\(^2\)The four transmission phases are:
- in the first slot \(O_1\) transmits \(b_1\),
- in the second slot \(N\) relays \(b_1\),
- in the third slot \(O_2\) transmits \(b_2\),
- and in the fourth slot \(N\) relays \(b_2\).

Analog network coding refers to coding at the signal level as is promoted in [9], [10]. This means that instead of encoding different packets by XOR-ing their bits as is promoted in digital network coding, analog network coding simply lets the analog signals add up through simultaneous transmissions (i.e. by letting two signals interfere with each other, intentionally). In short, analog network coding schemes consist of two transmission slots as opposed to four transmission slots when using the interference-free classical relaying. During the first time slot, both end sources transmit on the same band; whereas during the second slot, the relay forwards the interfering signals. Therefore, the trade-off is between the number of transmissions and the generated interference.

![Fig. 1. Transmission phases in a digital network coding system.](image-url)
users, a relay node and a BS are shown. The transmission of the data from the two users to the BS consists of three transmission phases. In the first phase, $T_1$, one of the users (e.g. $U_1$) will transmit the packet $b_1$. In the second phase, $T_2$, the other user (e.g. $U_2$) will transmit the packet $b_2$. Finally in the third transmission phase, $T_3$, the relay node will transmit a linear combination of $b_1$ and $b_2$ (e.g. $b_1 \oplus b_2$).

**Fig. 2.** A wireless relay network coding system.

III. USER GROUPING FOR NETWORK CODING

A common assumption in most previous works is that there are two users in the system, on which network coding is applied. In reality, in a wireless network system there is a set of active users in a cell which can be conveniently paired together. Hence the first obstacle that we are faced with at a network level is which set of users shall be selected and grouped to perform the network coding operation on. Obviously a random selection will not yield the optimal system capacity. In fact if we choose to pair users randomly then we could end up pairing users with *non-complementary* channel conditions to the relay and base station, and consequently losing the advantage provided by network coding. In other words, the proposed network coding scheme allows only one of the network coded pair to increase its SINR through the relay connection whereas the other user has to be decoded through its direct connection’s SINR$^3$ (see Section IV). Therefore, if both of the grouped users have a bad channel towards the base station, one of them will be decoded with a low SINR. Similarly, if both users have a good channel towards the base station, the capacity would decrease as compared to a direct transmission due to the time division among the users and the relay$^4$. Consequently, grouping users with complementary characteristics is essential in order to ensure a good performance of the network coding scheme. To illustrate how different groups of pairs can possibly be formed, consider the case where a total of 4 users $U_1$, $U_2$, $U_3$ and $U_4$ have data to transmit. Consequently, the following three groups of pairs can be formed:

$G_1 = \{(U_1, U_2); (U_3, U_4)\}$,

$G_2 = \{(U_1, U_3); (U_2, U_4)\}$,

$G_3 = \{(U_1, U_4); (U_2, U_3)\}$.

A possible group of pairs for 4 active users is exemplified in Figure 3.

**Fig. 3.** A possible group of pairs for a pool of 4 active users.

Based on the quality of the links of these users, to the relay and/or to the base station, the user grouping is carried out in order to optimize a certain cost function. This cost function can be in terms of sum-capacity, outage, interference or any other performance measure of interest.

In general, the optimization problem can be mathematically formulated as follows. Let $S$ be the set of all possible group of pairs of users belonging to a pool of $k$ active users. The number of possible group of pairs, $|S|$, is given by$^2$:

$$|S| = (k - 1) \times (k - 3) \times \ldots \times 3 \times 1.$$  \hspace{1cm} (1)

The user grouping algorithm determines which set of pairs should be network coded together so that the designed cost function of the considered pool of users is optimized. If we denote by $J(S)$ the cost function to be optimized, then the optimal user grouping should satisfy:

$$S^* = \arg \max_{S \in S} J(S),$$  \hspace{1cm} (2)

where $S^*$ is the user grouping satisfying the optimization problem, and $S \in S$.

Based on (1), the complexity of the proposed user grouping algorithm will increase exponentially. Consequently, we limit our study to a sub-optimal alternative while maintaining the complexity level within a reasonable range. As a result, we opt to divide the total number of simultaneously active users in a cell into smaller sub-groups and perform the grouping

$^3$Albeit both users would have a diversity order up to 2: the strong user through selection diversity, and the weak user through transmit diversity.

$^4$Even though the SINR would be higher than that of a direct transmission.

$^5$We assume that $k$ is an even number.
algorithm over each of them independently. For example, in the case of 40 simultaneously active users and a pool size (i.e. search window size) of 4, we will obtain 10 sub-groups over which the grouping algorithm will be run independently. Although this would provide sub-optimal performance as compared to running the grouping algorithm over all the active users in a cell, the complexity will be reduced considerably and large gains can still be obtained as will be shown subsequently. We study the cases of pools of size 2 (i.e. random application of network coding), 4 and 6.

In this work, the cost function is based on maximizing the sum-capacity $C_{\text{sum}}$ of a pool of active users at a certain time. Consequently, the algorithm implemented satisfies:

$$S^* = \arg \max_{S} C_{\text{sum}}(S).$$  \hfill (3)

IV. SINR AND SUM-CAPACITY DERIVATION

The transmission protocol is as described in the previous section: each radio node (i.e. the two users and the relay) transmits during its assigned time slot. The total transmission time is assumed to be divided equally among all radio nodes. The resultant received signals and SINR equations are presented below. They are derived assuming an OFDM/TDMA system: when a user is scheduled for transmission, it is assigned all the sub-carriers.

Let us assume that the users 1 and 2 are active in BS 1. During $T_1$, the first user transmits and its signal is received at the base station and the relay. Then the signal received after OFDM demodulation at the base station for the $k^{th}$ sub-carrier of user 1 is given by:

$$y_1^{(1)}(k) = H_{1,1}(k)s_{1,1}(k) + \sum_{i=2}^{6} H_{1,i}(k)s_{i,1}(k) + w_1(k),$$  \hfill (4)

where $H_{1,j}(k)$ is the radio channel between the transmitter of the $j^{th}$ user of BS 1 and the receiver of BS 1, $s_{i,j}(k)$ is the modulated signal of the $j^{th}$ user in BS $i$, $w_1(k)$ is the thermal noise, and $N_6$ is the number of base stations in the system. In the following we will omit the sub-carrier index $k$ for convenience.

The signal received at the relay after $T_1$ is given by:

$$y_{r,1}^{(1)} = H_{r,1,1}s_{1,1} + \sum_{i=2}^{6} H_{r,1,i}s_{i,1} + w_{r,1},$$  \hfill (5)

where $H_{r,i,j}$ is the radio channel between the transmitter of the $j^{th}$ user of BS $i$ and the receiver of the active relay node of BS 1.

The SINR at the base station after $T_1$, $\Gamma_1$, is given by:

$$\Gamma_1 = \frac{|H_{1,1}|^2 p_{1,1}}{\sum_{i=2}^{6} |H_{1,i}|^2 p_{i,1} + \sigma_1^2},$$  \hfill (6)

where $p_{i,j}$ is the transmitted power of $j^{th}$ user of BS $i$. The SINR for user 1 at the relay after $T_1$ is denoted as $\Gamma_{r,1}$ and is given by:

$$\Gamma_{r,1} = \frac{|H_{r,1,1}|^2 p_{1,1}}{\sum_{i=2}^{6} |H_{r,1,i}|^2 p_{i,1} + \sigma_1^2}.$$  \hfill (7)

Similarly, the signal received at the base station after user 2 has transmitted is given by:

$$y_{1}^{(2)} = H_{1,2}s_{1,2} + \sum_{i=2}^{6} H_{1,2}(k)s_{i,2} + w_1.$$  \hfill (8)

The signal received at the relay after $T_2$ is given by:

$$y_{r,1}^{(1)} = H_{r,1,1}s_{1,2} + \sum_{i=2}^{6} H_{r,1,2}s_{i,2} + w_{r,2}.$$  \hfill (9)

The SINR at the base station after $T_2$, $\Gamma_2$, is given by:

$$\Gamma_2 = \frac{|H_{1,2}|^2 p_{1,2}}{\sum_{i=2}^{6} |H_{1,2}|^2 p_{i,2} + \sigma_2^2}.$$  \hfill (10)

The SINR for user 2 at the relay after $T_2$ is denoted as $\Gamma_{r,2}$ and is given by:

$$\Gamma_{r,2} = \frac{|H_{r,1,2}|^2 p_{1,2}}{\sum_{i=2}^{6} |H_{r,1,2}|^2 p_{i,2} + \sigma_2^2}.$$  \hfill (11)

During the third hop, only the relay transmits the network-coded signal. The signal received at the base station after $T_3$ is given by:

$$y_1^{(3)} = G_{r,1}d_1 + \sum_{i=2}^{6} G_{r,i}d_i + w_3,$$  \hfill (12)

where $G_{r,i}$ is the channel between the transmitter of the active relay of BS $i$ and the receiver of BS 1, and $d_i$ denotes the network-coded transmitted signal from the active relay of BS $i$. The resulting SINR, $\Gamma_3$, is then given by:

$$\Gamma_3 = \frac{|G_{r,1}|^2 p_{r,1}}{\sum_{i=2}^{6} |G_{r,i}|^2 p_{r,i} + \sigma_3^2},$$  \hfill (13)

where $p_{r,i}$ is the transmitted power of the active relay in BS $i$.

Next we present our proposed decoding method that would dictate how the equivalent SINR of each user is obtained. Without loss of generality, let us assume that user 1 has a better link to the base station than user 2, i.e. $\Gamma_1 > \Gamma_2$. Then the data of user 1 (i.e. the ‘strong’ user) will be decoded based on its SINR through the direct link to the base station. However, we still require this data to be transmitted at a rate so that it would be decoded at the relay node as well. This is needed for a successful network encoding at the relay. The resulting equivalent SINR of user 1 would then be given by:

$$\Gamma_1' = \min(\Gamma_1; \Gamma_{r,1}).$$  \hfill (14)

After being decoded against the transmission of user 1, the relayed signal will then be used in order to improve the equivalent SINR of the weak user in the pair, i.e. user 2. Assuming maximum ratio combining (MRC), the equivalent SINR of user 2 will then be given by:

$$\Gamma_2' = \min\{\Gamma_2 + \Gamma_3; \Gamma_{r,2}\}.$$  \hfill (15)
TABLE I
SIMULATION PARAMETERS

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of subcarriers</td>
<td>128</td>
</tr>
<tr>
<td>Number of sites</td>
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</tr>
<tr>
<td>Sectors (i.e. cells) per site</td>
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</tr>
<tr>
<td>Cell radius</td>
<td>500m</td>
</tr>
<tr>
<td>Shadow fading standard deviation</td>
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<tr>
<td>Carrier frequency</td>
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<tr>
<td>RN TX Power</td>
<td>34 dBm</td>
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<tr>
<td>UE TX Power</td>
<td>24 dBm</td>
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<tr>
<td>Frequency reuse</td>
<td>1</td>
</tr>
<tr>
<td>Path loss exponent</td>
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</tr>
</tbody>
</table>

Consequently, the strong user in the network coded pair will sacrifice additional transmit diversity (i.e. the link from the relay to the BS) in return of one less transmission required to deliver the data of the pair (3 transmissions instead of 4) as compared to the DF protocol. However, being the ‘strong’ user in the pair means it intrinsically benefits from selection diversity.

Assuming all transmitting nodes have equal access to the channel, the resulting sum-capacity equation would then be given by:

\[ C_{\text{sum}} = \frac{1}{3} [\log_2(1 + \Gamma_1) + \log_2(1 + \Gamma_2)] \]  \hspace{1cm} (16)

V. NETWORK DEPLOYMENT MODEL

A network deployment with seven sites where each site comprises one sector is considered. The number of BS antennas per sector is one. BS antennas are placed above rooftop. The network is assumed to operate at a carrier frequency of 2 GHz and OFDM with 128 sub-carriers is used within the 5 MHz transmission bandwidth. Table 1 provides a summary of the assumed system parameters.

A. Radio Channel Model

The C2 metropolitan area pathloss and channel model from [11] is used in the evaluations. The model is applicable to a scenario with macro BS installed above rooftops and UTs located outdoors on street level. Six relays are deployed per cell, and are placed symmetrically around the base stations, at half distance between the base station and the edge of the cell. Non line-of-sight propagation is assumed between the BS antennas and the UTs. Shadow fading is modeled as a lognormally distributed random variable with a standard deviation of 8 dB.

B. Radio Network Algorithms

UTs connect to the sector with the lowest path-loss, and shadowing gain. An OFDM/TDMA system with frequency-adaptive transmission is assumed.

C. Link-to-System Interface

The model used for the link to system interface is based on the Shannon capacity model.

VI. SYSTEM-LEVEL PERFORMANCE RESULTS

We first measure the performance of applying random network coding (i.e. pool of size 2) against the DF protocol. As evidenced by the simulation results, the random application of network coding would provide a lower performance than DF for most of the cases as shown in Fig. 4 and Fig. 5. The random application of network coding is the main cause of its poor capacity and SINR performances. Whereas DF clearly outperforms NC in the SINR measure, the one less transmission for NC would contribute to an improvement in its capacity performance, but this is not enough to outperform DF.
and 6. The SINR and capacity results for different group sizes are shown in Fig. 7 and Fig. 6, respectively. The performance of DF is plotted for the purpose of comparison. One can notice that as the group size increases, a better performance is achieved by user grouping as evidenced by the simulation results. This is because a larger group size would allow a better matching among the users. The DF scheme provides a normalized mean capacity of 1.47 [b/s/Hz], as opposed to 1.27 [b/s/Hz] for random network coding, 1.52 [b/s/Hz] for a pool of size 4, and 1.70 [b/s/Hz] for a pool of size 6. Consequently, mean capacity gains of 34% and 16% can be achieved by the application of user grouping on a search window of 6 users as compared to random NC and DF, respectively. Increasing the search window size (i.e. larger pools of users) might further increase the capacity gains on the expense of complexity as the number of possible pairings increases; however, an elaborate analysis of this issue is outside the scope of this paper. Furthermore, one can notice that the performance of DF is better at lower percentiles; this is mainly because the cost function used aims at maximizing the sum-capacity of active users, and typically only small gains can be obtained by increasing the SINR of very low users as opposed to increasing the SINRs around 0 dB, due to the logarithmic behavior of the capacity function. This is why users with very low SINR are not enhanced by the application of NC. Other cost functions can be designed in order to overcome this shortcoming.

Fig. 6. Normalized capacity of NC with user grouping.

VII. CONCLUSION

In this paper, we presented a novel network coding based relaying technique with a complementing decoding strategy at the receiver. We provided performance measures based on SINR and capacity for a multi-cell environment using a system level simulator. We showed that the random application of network coding does not achieve the capacity gains expected from the decreased number of transmissions. As a solution, we introduced a user grouping strategy that is based on a cost function, and showed that this method is necessary to implement with network coding in order to exploit the decrease in the number of transmissions. When applied with a window size of 6, the user grouping algorithm provided mean capacity gains of 34% and 16% as compared to random network coding and decode-and-forward relaying, respectively.

REFERENCES


