Multi Objective Optimization for Vehicle Routing Problem Using Fitness Aggregated Genetic Algorithm (FAGA)

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Abstract

The vehicle routing problem is a combinatorial optimization problem and important one in the fields of transportation, distribution and logistics. Even though large number of previous research has used single objective for solving this problem, it has very rarely concentrated on multi objective optimization. In single-objective optimization fitness function comparisons among solutions are straightforward, but when dealing with multiple objectives, it becomes a difficult task. In this paper, we consider multi objective capacitated vehicle routing problem in which the total distance travelled by the vehicles and total number of vehicles used are minimized. Several multi-objective genetic algorithms like Weight-based genetic algorithm (WBGA), Nondominated sorting Genetic Algorithm (NSGA) etc. were developed in the recent years for multi objective optimization. Generally, multi-objective genetic algorithms differ based on their fitness assignment procedure, elitism, or diversification approaches. In this paper genetic algorithm with new fitness assignment procedure called Fitness Aggregated Genetic Algorithm (FAGA) is introduced for solving the problem. The proposed method is tested on standard benchmark problems for capacitated vehicle routing problem. Results show that the suggested new algorithm is highly competitive and quite effective for multi objective optimization of vehicle routing problem.

Keywords: Vehicle routing problem, Multi objective optimization, Genetic algorithm, Fitness evaluation for multi objectives.

1.0 INTRODUCTION

Transportation of goods and passengers is an important task in day to day life. Enormous amount of money is spent for transportation in the form of fuel, driver remuneration etc. Determination of effective way of using transportation resources is an important task in the field of operations research. The travelling salesman problem (TSP), shortest path problem (SPP), urban postman problem (UPP) and vehicle routing problem (VRP) are the some of different types of problems in the field of transportation. The VRP is a combinatorial optimization problem seeking to service a number of customers with a fleet of vehicles. VRP is an important problem in the fields of transportation, distribution and logistics. The VRP can be defined as the problem of designing routes for delivery vehicles which are operated from a single depot to supply a set of customers with known locations and known demands for a certain commodity. The Conditions for VRP are each vehicle must start and end at the central depot, each customer must be visited by exactly one vehicle and the sum of all customers demand in a route does not exceed the vehicle capacity [3]. The capacitated vehicle routing problem (CVRP) is a variant of VRP, which appears in a large number of practical situations, theoretically interesting and not at all easy to solve. Due to these reasons, CVRP has attracted a lot of attention in the academic research. In real case, particularly in logistics and distribution, the problems are multi-objective in nature. The most common objectives in CVRP are minimizing the total distance travelled, number of vehicles, driver remuneration, route balance and maximizing customer satisfaction, profit [8]. Though a lot of research has used single objective optimization for solving this CVRP problem, it has very rarely concentrated
on multi-objective optimization due to the following reason. In single-objective optimization, fitness function comparisons among solutions are straightforward, but when dealing with multiple objectives, it becomes a difficult task [4].

2.0 LITERATURE REVIEW

Prins [19] implemented genetic algorithm to solve CVRP and compared the results with tabu search. Their algorithm outperformed most of the published tabu search heuristics on the fourteen classical Christofieds instances. Mazzeo and Loiseau [17] considered minimization of cost in CVRP and reported that the performance of ant colony algorithm was better than simulated annealing and tabu search. Jozefowiez et al. [9] developed a model for bi-objective CVRP, where minimization of total length and route balance are the two objective functions and considered multi-objective genetic algorithm (MOGA) with target aiming pareto search to solve the problem. The results showed that the method was quite effective as compared to other heuristics. Tahakoli-Moghaddam et al. [23] developed a new model for CVRP with split deliveries, demand at nodes divided and serviced by different heterogeneous vehicles. The aim of the model was to minimize the distance travelled by using simulated annealing. A number of test problems in small and large sizes were solved and computational results showed that the implemented method was highly competitive. Wang and Lu [24] described a hybrid genetic algorithm, where sweep algorithm was incorporated into the genetic algorithm to minimize the distance for CVRP.

Jozefowiez et al. [10] proposed a bi-objective CVRP in which the total length of routes as well as the route balance was minimized. Elitist diversification method was incorporated into the evolutionary algorithm to improve the results. Lin et al. [15] applied hybrid algorithm of simulated annealing and tabu search to solve CVRP. The algorithm found eight best solutions of classical fourteen instances. Ai and Kachitvichyanukul [1] constructed a model for CVRP to minimize the distance travelled by the vehicles and used particle swarm optimization to solve the problem. The computational result showed that the method was better than other methods for solving CVRP. Yurtkuran and Emel [26] introduced hybrid electromagnetism-like algorithm to solve CVRP, where minimization of distance was the objective function. It was reported that the algorithm gave promising results within acceptable computational times when compared to other novel meta-heuristics. Juan et al. [11] proposed a hybrid algorithm that combines a CVRP classical heuristic with Monte Carlo simulation to solve the problem. Lee et al. [14] considered minimization of total travelling cost as objective function and proposed enhanced ant colony optimization (EACO) to solve the CVRP. The proposed algorithm was found to be superior to the original ant colony optimization and simulated annealing. Chen et al. [2] introduced iterated variable neighborhood descent algorithm (IVND) and IVND performed well and was quite competitive with other state-of-the-art heuristics.

Guimarans et al. [6] presented original hybrid approach to solve CVRP. The approach combines a probabilistic algorithm with constraint programming and lagrangian relaxation. The efficiency of the suggested approach was higher for some well-known CVRP benchmarks. Szeto et al. [22] introduced artificial bee colony algorithm for the CVRP. The method produced good solutions when compared with existing heuristics. Kim and Son [12] improved results of the CVRP by introducing probability matrix as the main device for particle encoding and decoding in particle swarm optimization. Ribeiro and Laporte [20] developed a model for cumulative capacitated vehicle routing problem (CCVRP) which is an extension of the delivery man problem and proposed an adaptive large neighborhood search heuristic to solve CCVRP. Nazif and Lee [18] minimized travelling cost for CVRP by using genetic algorithm with optimized crossover operator. The results showed that the proposed algorithm was competitive in terms of the quality of the solutions found. Jin et al. [7] solved the CVRP by a parallel multi-neighborhood cooperative tabu search. Yousefkhoshbakht and Khorraram [25] presented hybrid meta-heuristic algorithm to solve CVRP. The approach combines a sweep algorithm and ant colony systems. Extensive computational tests on standard instances confirmed the effectiveness of the presented approach. Marinakis [16] modified the greedy randomized adaptive search procedure (GRASP) to multi phase neighborhood search-GRASP (MPNS-GRASP) for solving CVRP. The computational time of the algorithm decreased significantly compared to other heuristic and meta-heuristic algorithms. Stanojevic et al. [21] proposed extended savings algorithm (ESA) to solve CVRP. The proposed algorithm was found to be superior to other algorithms.

In this paper genetic algorithm with new fitness assignment procedure called Fitness Aggregated Genetic Algorithm (FAGA) is introduced for solving the capacitated vehicle routing problem.

3.0 PROBLEM BACKGROUND

Though CVRP has attracted more attention in the academic research, it has very rarely concentrated on multi-objective optimization. Many of the real world vehicle routing problems are generally characterized by the presence of multi conflicting objectives. The principle of multi-objective optimization is different from single-objective optimization. Many researchers convert the original problem with multiple objectives into a single objective optimization problem by adding weights to generate only a single Pareto optimal solution. For different weight combinations, the model has to be executed several times. This is called a scalarized problem. If scalarization is done carefully, Pareto optimality of the solutions obtained can be
4.0 MATHEMATICAL FORMULATION

In CVRP, there is N number of customers. Each customer is geographically located at coordinates (x, y), and has a demand q. The demand in each customer should be greater than zero. Central depot denoted by the symbol 0, from which all customers are serviced using homogeneous vehicles. In depot the demand is zero.

The problem is to design a set of K routes such that each route begins and ends at the depot, and each customer is serviced by exactly one vehicle. So each vehicle is assigned a set of customers that it has to supply, with sum of their demands not exceeding the vehicle capacity C. The number of customers in each route is denoted by the symbol Nk. The travel between two customers has associated costs such as the distance (relating to fuel cost) and driver remuneration (relating to driver cost). The distances between two customers are simply taken to be the Euclidean distances. In order to formulate the model, following notations are defined:

- D = Total distance traveled by all vehicles
- K = Number of vehicles or routes
- dk = Distance traveled by vehicle k
- Nk = Number of customers visited by vehicle k
- du(i,k), u(i+1,k) = Distance traveled for vehicle k from node i to i+1
- qu(i,k) = Demand at node i for vehicle k
- C = Vehicle capacity

\[
\text{Min } D = \sum_{k=1}^{K} d_k \quad (1)
\]

where

\[
d_k = \sum_{i=0}^{N_k} d_{u(i,k), u(i+1,k)}
\]

where

\[
d_{u(i,k), u(i+1,k)} = \left[ (x_i - x_{i+1})^2 + (y_i - y_{i+1})^2 \right]^{1/2}
\]

\[
\text{Min } K \quad (2)
\]

\[
\sum_{i=1}^{N_k} q_{u(i,k)} \leq C \quad \forall k = 1, 2, ...., K \quad (3)
\]

Eq. (1) is the objective function for minimization of total distance travelled by all vehicles, Eq. (2) is the objective function for minimization of total number of vehicles used and Eq. (3) is the vehicle capacity constraint.

5.0 PROPOSED METHODOLOGY

Since Genetic Algorithm (GA) is a population-based approach, it is well suited to solve multi-objective optimization problems. The ability of GA to simultaneously search different regions of a solution space makes it possible to find a diverse set of solutions for difficult problems. Therefore, GA has been the most popular heuristic approach to multi-objective design and optimization problems. Several multi-objective genetic algorithms were developed in the recent years including Vector evaluated genetic algorithm (VEGA), Multi-objective genetic algorithm (MOGA), Niched pareto genetic algorithm (NPGA), Weight-based genetic algorithm (WBGA), Random weighted genetic algorithm (RWGA), Nondominated sorting Genetic Algorithm (NSGA), Strength pareto evolutionary algorithm (SPEA), improved Strength pareto evolutionary algorithm (SPEA2) , Pareto-archived evolution strategy (PAES), Pareto envelope-based selection algorithm (PESA), Fast nondominated Sorting genetic algorithm (NSGA-II), Multi-objective evolutionary algorithm (MEA), Micro-genetic algorithm (MGA), Rank-density based genetic algorithm (RDGA), and Dynamic multi-objective evolutionary algorithm (DMOEA) [13]. Generally, multi-objective genetic algorithms differ based on their fitness assignment procedure, elitism, or diversification approaches. In this paper a new fitness assignment procedure with genetic algorithm called fitness aggregated genetic algorithm (FAGA) is introduced for solving CVRP. Fig. 2. shows the flowchart for FAGA. The stages involved in the proposed methodology are explained below.

5.1 Creation of Initial Population

Initial population is created randomly as follows: First any one customer is randomly selected and placed in the first location on the first route. Then a different customer is randomly selected. If the capacity constraint would be met, it is placed in the current route after the previous customer. If the constraint is not met, then the second route is created and this customer is placed in the first location on the new route. Then again different customer is randomly selected. If the constraint is met, it is placed in the first route after the previous customer, else the same procedure is repeated in the second route, else if the new route is created and this customer is placed in the first location on the new route. This procedure is repeated till all customers in a problem are assigned to routes. Fig. 1. Shows sample solution representation for VRP. The entire procedure is repeated for predefined population size value.
5.2 Fitness Aggregation Method
The aim of the fitness aggregation method is to evaluate fitness function value for multi objectives. First both Objective values that is travelled distance and number of vehicles is determined for all solutions in a population. Then the fitness for the travelled distance alone for all solutions in a population is calculated by the following formula

\[
F(D)_i = \frac{(D)_{\text{max}} - (D)_i}{(D)_{\text{max}} - (D)_{\text{min}}} \quad \forall i = 1,2,\ldots,S
\]  

(4)

Where \(F(D)_i\) is the fitness function value of distance travelled for \(i^{\text{th}}\) solution in a population, \((D)_{\text{max}}\) is the maximum distance travelled in a population, \((D)_{\text{min}}\) is the minimum distance travelled in a population, \((D)_i\) is the distance travelled for the \(i^{\text{th}}\) solution in a population and \(S\) is the size of the population. \(F(D)\) is calculated for all solutions in a population. Similarly fitness for the number of vehicles used for all solutions in a population is calculated by the following formula

\[
F(K)_i = \frac{(K)_{\text{max}} - (K)_i}{(K)_{\text{max}} - (K)_{\text{min}}} \quad \forall i = 1,2,\ldots,S
\]  

(5)

Where \(F(K)_i\) is the fitness function value of number of vehicles used for \(i^{\text{th}}\) solution in a population, \((K)_{\text{max}}\) is the maximum number of vehicles used in a population, \((K)_{\text{min}}\) is the minimum number of vehicles used in a population, \((K)_i\) is the number of vehicles used for the \(i^{\text{th}}\) solution in a population. \(F(K)\) is calculated for all solutions in a population. Then the aggregate fitness for a solution is determined by the following formula
Table 1: Example for Fitness Aggregation for a Population of size 10.

<table>
<thead>
<tr>
<th>S.No</th>
<th>D</th>
<th>K</th>
<th>F(D)</th>
<th>F(K)</th>
<th>F(D,K)</th>
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<td>1</td>
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<td>5</td>
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<td>0</td>
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<td>0.5</td>
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<td>0.207</td>
<td>0.5</td>
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</table>

Fig. 3: Example for BCRC with 9 Customers

\[
F(D,K)_i = \frac{F(D)_i + F(K)_i}{2} \quad \forall i = 1,2,\ldots,S \tag{6}
\]

F(D,K) is calculated for all solutions in a population. Since it is a minimization problem a solution with high aggregate fitness function value is an optimal solution for the problem is shown in Table 1 by bolded line. The next stage in the proposed methodology is tournament selection which is done based on the aggregate fitness function value. The remaining things in the selection operator phase are same as in the traditional genetic algorithm.

5.3 Best Cost Route Crossover

Crossover or recombination is one of the unique and important operator of the genetic algorithms. Ghoseiri and Ghannadpour [5] stated that standard genetic operators like One-point crossover, two-point ordered crossover, uniform ordered crossover, partially mapped crossover and route crossover may generate infeasible solutions for CVRP. Due to the above reason a specialized genetic operator like best
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5.4 Swap Mutation

This operator is used after the completion of crossover phase. Fig. 4. Shows the procedure of Swap Mutation (SM).

![Swap Mutation Example](image)

In SM each one gene from any two route of a solution is randomly selected and their position is exchanged if the demand constraint is satisfied. Thereafter, any gene from any route of the same solution is selected randomly and it is removed from its original position and reinserted into rest of any one route if it satisfies the demand constraint.

6.0 COMPUTATIONAL RESULTS

This section explains computational experiments carried out to investigate the performance of the proposed fitness aggregation method with genetic algorithm. The algorithm was coded in JAVA and run on a PC with 2.93GHz CPU and 3GB RAM. The results presented below are based on the following parameters:

- Population Size = 50.
- Number of Generations = 500.
- Crossover Probability = 0.9.
- Mutation Probability = 0.2.

The experiments use the standard Augerats benchmark problem instances for CVRP. The problems vary with number of customers, geographical locations of customers and vehicle capacity for different test cases. The customers details are given in the sequence of customer index, location in x and y coordinates and demand for load. Tables 2, 3 and 4 presents a summary of results for few instances of Augerat A set, B set and P set benchmark problems for CVRP respectively and compare the findings with the best known solutions (BK) that are reported in the literature.

The column labeled ‘BK’ gives the best known published solutions so far, column labeled ‘proposed method’ gives the results that are obtained by the proposed method and

<table>
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<th>S.No</th>
<th>Instance Name</th>
<th>BK</th>
<th>Proposed Method</th>
<th>% deviation</th>
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Table 3: Testing results for few instances of Augerat B set.

<table>
<thead>
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<th>Instance Name</th>
<th>BK</th>
<th>Proposed Method</th>
<th>% deviation</th>
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Table 4: Testing results for few instances of Augerat P set.

<table>
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<th>Proposed Method</th>
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column labeled ‘% deviation’ gives closeness of the results produced by the proposed method to the BK. Bolded lines in Tables 2, 3 and 4 indicates the solutions obtained by the proposed method, which are exactly equal to the BK reported in the literature.

Out of twenty seven benchmark instances, the proposed methodology gives fifteen problems with same results as compared to BK. Hence it is reported that the proposed fitness aggregation method with genetic algorithm is highly competitive with BK and quite effective.

7.0 CONCLUSIONS

This paper introduces a fitness aggregated genetic algorithm (FAGA) for multi objective optimization model for capacitated vehicle routing problem. This model considers the minimization of total distance travelled by all vehicles and total number of vehicles used as multi objective functions, subject to vehicle capacity constraint. Best cost route crossover and swap mutation are implemented. The proposed method is tested on few instances of standard benchmarks for capacitated vehicle routing problem and is found to be quite effective and competitive. Further improvements might be possible by testing different variants of the vehicle routing problem and by considering many objective optimization.

REFERENCES


[27] http://www.branchandcut.org/VRP/data/