Nonlinear Control of Large Scale Complex Systems Using Convex Optimization Tools and Self-Adaptation

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Abstract—The inability of existing theoretical and practical tools to scaleably and efficiently deal with the control of complex, uncertain and time-changing large-scale systems, not only leads to a effort-, time- and cost-consuming deployment of Large-Scale Control Systems (LSCSs), but also prohibits the wide application of LSCS in areas and applications where LSCSs could potentially have a tremendous effect in improving system efficiency and Quality of Services (QoS), reducing energy consumption and emissions, and improving the day-to-day quality of life.

Based on recent advances on convex design for LSCSs and robust and efficient LSCS self-tuning, the AGILE methodology aims at providing an integrated LSCS-design, applicable to large-scale systems of arbitrary scale, heterogeneity and complexity and capable of:

- Providing pro-active, arbitrarily-close-to-optimal LSCS performance;
- Being intrinsically self-tunable, able to rapidly and efficiently optimize LSCS performance when short- medium- and long-time variations affect the large-scale system; and, achieving all the above, while being scalable and modular.

The purpose of the present paper is to briefly describe the main features of the AGILE control design methodology.

I. INTRODUCTION

The inability of existing Large-Scale Control System (LSCS) designs to enjoy a broad applicability in real-life large-scale applications and, in essence, to convince large-scale system authorities and operators to adopt them, is due to the lack of theoretical and practical design tools to efficiently deal with large-scale applications involving complex systems that are characterized by:

- highly nonlinear phenomena,
- system uncertainties and variations,
- faults, atypical system behavior, the need for major changes in the large-scale systems performance, as well as,
- constraints related to performance, safety, QoS, resource use, computational and communication requirements, etc.

In general, there is lack of theoretical and practical design methodologies that can provide rapid and modular LSCS design to deal with major faults and incidents, frequent changes in the performance requirements, addition/removal of control nodes, etc. Such unpredictable changes require a quick response which may involve major re-design (re-configuration) of the LSCS in real time while maintaining performance and optimal operation.

This paper presents a new LSCS design approach that attempts to remedy most of the above-mentioned shortcomings of existing systems. The proposed LSCS design approach – abbreviated as AGILE from the name of the European Commission research project AGILE [3] whose main objectives are to (a) render this methodology practically implementable and (b) test and evaluate the methodology using real-life large-scale systems – consists of three basic “ingredients” developed to address different issues in LSCS design:

1. A Convex Control Design (ConvCD) methodology [8]-[12] that transforms the problem of – approximate – optimal control of generic nonlinear systems (whose dynamics are explicitly known) into a convex optimization problem (linear or quadratic subject to semidefinite constraints).

2. An Adaptive Self-Tuning (AdaptST) tool [4]-[7], [11], [12] that provides a method for the efficient and “free-of-singularities” self-tuning of LSCS in order to compensate short-, medium- and long-term variations of the system dynamics and/or the exogenous factors affecting the system.

3. A Multi-Model Adaptive Control with Mixing (MMACM) – see [14]-[18] and the references therein – that provides a scalable LSCS design approach that smoothly “mixes” a set of linear – and thus fully-scalable – controllers that are designed using well established robust control tools and adaptive techniques capable of coping with system uncertainties.

The key idea of AGILE is based on the observation that although the above three design methodologies address different open problems in LSCS design, their structure allows for their easy and straightforward integration into a single integrated LSCS design that maintains all the nice features...
of each of the three methodologies, while overcoming their limitations and drawbacks.

The aim of the paper is to describe the main features and characteristics of the proposed AGILE methodology.

II. THE AGILE INGREDIENTS

A. Convex Control Design for LSCS

ConvCD assumes that the system dynamics evolve according to the standard in the nonlinear optimal control literature differential equation of the form

\[ \dot{x} = f(x) + g(x)u \]  

(1)

The objective is to design a state-feedback control law \( u = k(x) \) so that the closed-system is stable and the following optimization criterion is optimized

\[ J = \int_0^\infty z^T(x(s))Qz(x(s))ds \]  

(2)

In (1) and (2), \( x, u \) are the vectors of system states and controls, respectively, \( f, g \) are nonlinear smooth vector functions, \( z(x) \) is a vector of monomials of order \( L \) and \( Q \) is a user-defined positive semi-definite (psd) matrix. The reason for \( z(x) \) to be a vector of monomials will be made clear in the sequel. The optimization criterion (2) can model or approximate with arbitrary accuracy any optimization criterion that depends on the system states. Moreover, it can model or approximate optimization criteria that also depend on the control inputs \( u \). This can be done by adding the pre-compensator \( \hat{u} = v \), where \( v \) is the new — "fictitious" — control input, and augment the system dynamics (1) appropriately.

Application of the well-known Hamilton-Jacobi-Bellman (HJB) equation to the above problem results in the following equation

\[ \frac{\partial V}{\partial x}^T (f(x) + g(x)u^*)(x) = -z^T(x)Qz(x) \]  

(3)

where \( V \) is the optimal-cost-to-go function and \( u^* \) is the optimal control.

The idea behind ConvCD is simple: as a first step, \( V, f, u^* \) are approximated by polynomial approximators:

\[ V(x) \approx z^T(x)Pz(x), f(x) = \Phi z(x), u^*(x) \approx \Theta z(x) \]  

(4)

where \( P, \Phi, \Theta \) are constant matrices of appropriate dimensions and \( P \) being a positive definite (pd) matrix. The crucial point here is that the polynomial used for approximating the optimal-cost-to-go function \( V \) is a Sum-of-Squares (SOS) polynomial.

Replacing the approximations (4) into the HJB equation (3) we obtain:

\[ z^T \left[ (\Phi + g\Theta)^T M^T P + PM [\Phi + g\Theta] + Q \right] z = \nu \]  

(5)

where \( M \equiv M(x) \) denotes the polynomial matrix whose \((i,j)\)th entry is given by \( M_{ij}(x) = \partial z_i(x)/\partial x_j \) and \( \nu \equiv \nu(x) \) is the approximation error term [this term is inversely proportional to the order \( L \) of the monomial vector \( z(x) \)]. From equation (5) we have that

\[ z^T \left[ (\Phi + g\Theta)^T M^T P + PM [\Phi + g\Theta] + Q - \nu \right] z = 0 \]  

(6)

where \( z^T \nu z = \nu \). Working similar to [21], we multiply the terms inside the parenthesis of (6) from the left and the right by \( P^{-1} \) to obtain

\[ \mathcal{F}_{P,\Theta,\nu}(x) \triangleq z^T \left[ (\Phi \nu^T + \Theta \nu P - \nu) M^T + M [\Phi \nu P + g\Theta P] + Q \right] z = \nu(x) \]  

(7)

where \( \nu = z^T P^{-1} \nu P^{-1} z \) and

\[ P \triangleq P^{-1}, \Theta \nu \triangleq \Theta \nu P^{-1} = \Theta P, Q \triangleq \nu PQ \equiv P^{-1}QP \]  

Equation (7) depends on the unknown approximation-error-dependent term \( \nu(x) \) and thus cannot be solved. Instead of solving (7), the solution of the following optimization problem is pursued within ConvCD:

\[ \min_{(P, \Theta, \nu, Q)} \frac{1}{2} ||\mathcal{F}_{P,\Theta,\nu}(x)||^2 \]  

(8)

where \( P_u \succ P_l \succ 0, \Theta_l \succ 0 \) are user-defined matrices. To understand the reasoning behind choosing to solve the optimization problem (8), consider the case where the design matrices \( P_l, P_u, \Theta_l, Q_l \) are chosen according to \( P_l = \epsilon_1 I, P_u = \epsilon_2 I, Q_l = \epsilon_3 I, \) with \( \epsilon_i, i = 1, 2, 3 \) being positive design constants. Let also \( \mathcal{X}_0 \) denote the bounded subset of all admissible initial state systems \( x(0) \) and \( \mathcal{X} \) denote a subset containing \( \mathcal{X}_0 \) and whose diameter is sufficiently larger than that of \( \mathcal{X}_0 \). Moreover, let

\[ \mu(x) = \frac{\partial (z^T (P_u)^{-1} z(x))}{\partial x} \left( f(x) + g(x)\Theta^T (P_u)^{-1} z(x) \right) \]  

According to [8], under the appropriate selection of constants \( \epsilon_i, i = 1, 2, 3 \), there exists a lower bound \( L^* \) on the order of the monomial vector \( z(x) \) such that for all choices of the monomial order \( L \) that satisfy \( L \geq L^* \) then

\[ \mu(x) < 0, \forall x : x \in \mathcal{X} \quad \text{and} \quad x \notin \mathcal{E} \]  

(9)

for some subset \( \mathcal{E} \subset \mathcal{X}_0 \), and moreover, the closed-loop system (1) admits solution under the feedback \( u = \Theta^*_P (P_u)^{-1} z(x) \) that satisfy

\[ |z(x(t))| \leq \alpha_1 \exp^{-\alpha_2 t} |z(x(0))| + \alpha_3 \]  

(10)

\[ \alpha_1 = \sqrt{\frac{\epsilon_2}{\epsilon_1}}, \alpha_2 = \left( \frac{\epsilon_3}{2\epsilon_1} - \frac{\epsilon_1 \cdot \text{const.}}{2 \epsilon_2^2} \right), \alpha_3 = \sqrt{\epsilon_2 \sup_{x \in \mathcal{E}} |\mu(x)|} \]  

Furthermore, the set \( \mathcal{E} \) as well as the term \( \sup_{x \in \mathcal{E}} |\mu(x)| \) in (10) shrink to zero as the order \( L \) of the vector of monomials \( z \) decreases. The significance of (9) and (10) is two-fold:

- First of all, (9) and (10) do not require that the approximation error \( \nu(x) \) is negligible; instead, they require the existence of the subset \( \mathcal{E} \) so that (9) holds, which is significantly less restrictive than requiring \( \nu(x) \) to be negligible. For instance, in many practical examples — like the ones presented in the end of this section — it suffices to
choose a monomial vector of second order (i.e. \( L \geq L^* = 2 \)) in order for (9) to hold.

- Most importantly, inequality (10) provides a straightforward way for choosing the design constants \( \epsilon_i, i = 1, 2, 3 \): based on (10) the overshoot, rate of convergence and steady-state performance of the closed-loop system can be tuned in a similar way it is performed in the Linear-Quadratic control design.

In a nutshell, according to (9) and (10), ConvCD does not have to employ a monomial order \( L \) that is “very large” or, even, “extremely large” as it is typical in other approximation-based approaches for solving optimal control problems. It suffices to choose \( L \) large enough so that (9) holds and then “play” with the constants \( \epsilon_i, i = 1, 2, 3 \) to get the desired closed-loop performance. If the desired closed-loop performance is not achieved then \( L \) should be increased, a new choice for \( \epsilon_i, i = 1, 2, 3 \) should be made and so on.

We finally notice that the above analysis holds in the case the ConvCD design matrices \( P_i, P_a, Q_i \) are selected to be non-diagonal matrices as assumed above. In that case the above analysis holds by replacing the constants \( \epsilon_1, \epsilon_2, \epsilon_3 \) with \( \lambda_{\min}(P_i), \lambda_{\max}(P_a), \lambda_{\min}(Q_i) \), respectively.

Despite the above nice features of a design that is based on the optimization problem (8), its solution requires discretization of the state-space as it is an infinite-dimensional, state-dependent problem. Fortunately, due to the particular form of the optimization problem (8), the number of discretization points does not have to be as large as it is required in a typical state-dependent optimization problem: as it was seen in [8], [11] the number of discretization points can be as few as the total number of free variables in the matrices \( P, \Theta, Q \). This is in contrast to alternative formulations to the optimization problem (8): for instance, an equivalent formulation that involves instead of a quadratic optimization criterion, a linear one subject to state-dependent semi-definite constraints (state-dependent LMIs) requires an extremely larger number of discretization points.

Table I briefly presents the procedure proposed in [8], [11] for solving the optimization problem (8) and, thus, for realizing the ConvCD approach. Due to the convex nature of (11), this optimization problem, which is the core element within ConvCD admits a unique and global optimum. Summarizing the approach presented in Table I and assuming that the large-scale dynamics are perfectly known, the ConvCD approach constructs a polynomial AOC that assumes the following form

\[
\Theta(z) \approx \text{MMCM}(\vartheta, x)
\]

and applying to the actual plant the controller \( u = \text{MMCM}(\vartheta, x) \), where \( \text{MMCM}(\vartheta, x) \) is a controller that smoothly switches between linear control elements. The mathematical form of the controller \( u = \text{MMCM}(\vartheta, x) \) is as follows:

\[
u = \text{MMCM}(\vartheta, x) = \sum_{i=1}^{M} \beta_i(x) L_i x
\]

where \( M \) denotes the total number of linear control elements of the MMCM, \( L_i \) are the linear controller matrices and \( \beta_i(x) \) are the mixing signals that are responsible for smoothly switching from one linear element to another; The mixing signals are designed to satisfy \( \beta_i(x) \geq 0, \sum_i \beta_i(x) = 1 \) and, moreover, \( \beta_i(x) \) needs to be continuously differentiable so as to ensure smooth switching between controllers.

Using (16), the computational requirements are similar to those of a linear controller while the addition/removal of control nodes, sensors, etc in the control design is straightforward. Additionally, the use of MMCM controllers enables the use of its adaptive version MMACM; See the section on MMCM and MMACM for more details.

**Comments.**

- As ConvCD is based on optimal control principles, it can be extended to incorporate constraints of various types (e.g., state/control input constraints).
- Similar – but more complicated – arguments as the ones briefly presented in Table I can be used in order to develop LSCS for output feedback control, i.e., in cases where the output instead of the states are available for measurement; See e.g. [1], [9] for more details.

**B. Adaptive Self-Tuning**

As ConvCD assumes perfect knowledge of the system dynamics it may become inefficient in cases of system uncertainties or variations or, even worse, in cases where minor or major faults or anomalies affect the system dynamics. Thus in order for ConvCD to be practically efficient, an adaptive self-tuning/re-design tool is required that will take care of all the above-mentioned factors that may affect the efficiency of ConvCD. Unfortunately, the current state-of-the-art methods for providing adaptation, self-tuning or control re-design face the problems of poor transient performance as well as loss-of-controllability. AdaptST has been developed in order to address the above two shortcomings of existing approaches. It was first introduced to address the problem of Automated Fine-Tuning (AFT) of LSCSs originally designed using conventional control design methods [4], [5], [6]. Later it was extended so that it can be combined with the ConvCD method [11], [12]. To overcome the poor transient and loss-of-controllability problems, AdaptST makes sure that the control actions are calculated so that both of the following properties are satisfied:
Choose randomly $\mathcal{N}$ vectors $\vec{x}^{|i|} \in \mathcal{X}$ and solve the following convex optimization problem:

$$
\min_{(\vec{P}, \Theta, \bar{Q})} \sum_{i=1}^{\mathcal{N}} \| \mathcal{F}_{\vec{P}, \Theta, \bar{Q}} \left( \vec{x}^{|i|} \right) \|^2 \\
\text{s.t.} \\
\vec{P}_u \succeq \vec{P} \succeq \bar{P} \\
\bar{Q} \succeq \bar{Q}_l 
$$

(11)

Then, the resulting controller takes the form

$$
u = \Theta z(x) \equiv \Theta_{\vec{P}} \bar{P}^{-1} z(x)$$

(18)

**Important note**: Using similar arguments as in e.g., the proof of Theorem 1 of [11], it can be seen that it suffices for $\mathcal{N}$ to be equal to the number of elements of the matrices $(\bar{P}, \Theta_{\vec{P}}, \bar{Q})$. In practice, $\mathcal{N}$ is chosen 2 or 3 times the number of elements of $(\bar{P}, \Theta_{\vec{P}}, \bar{Q})$ to avoid numerical problems.

[(P1)] The control actions are as close as possible to the optimal ones by making sure that they satisfy the approximate equation (5);

[(P2)] The control actions provide what is known in the parameter estimation and adaptive control literature as Persistence of Excitation (PE), see e.g., [2]: PE is a necessary and sufficient condition for the rapid convergence of the estimators employed within the identification scheme, which in turn, is sufficient - for schemes employing the ConvCD approach - to guarantee that no poor transient performance or loss-of-controllability problems occur.

AdaptST employs a simple reasoning [11], [12] to calculate the control actions so that (P1) and (P2) are concurrently satisfied:

[(AD1)] Use a standard identification scheme\(^1\) to continuously estimate the system dynamics;

[(AD2)] At each controller time-step a set of many different candidate perturbations of the controller parameters is randomly generated;

[(AD3)] The perturbation that best satisfies (5) – where in (5) the actual system dynamics are replaced by the estimated ones as generated in (AD1) – is chosen in order to update the controller parameters. The resulting control signal is applied to the system;

[(AD4)] ConvCD is re-designed using an estimate for the system dynamics that is generated by the system identification scheme in (AD1).

Stability, convergence and robustness of the scheme (AD1)-(AD4) have been established in [5], [11], [12]. Under some mild controllability assumptions on the controlled system, the adaptive scheme (AD1)-(AD4) guarantees that the closed-loop system performance converges exponentially fast to the same performance as the one of a non-adaptive ConvCD (that assumes perfect knowledge of the system dynamics). Practically speaking, the poor transients that are present in the majority of adaptive control designs are avoided using scheme (AD1)-(AD4). Moreover, whenever there are large deviations in the system dynamics (due to e.g., faults, incidents, atypical exogenous factors), the scheme (AD1)-(AD4) guarantees fast recovery and controller re-configuration.

C. MMCM and MMACM

The main drawback of AdaptST is that it abolishes the scalability nature of the replacement (16) in case many linear switching elements are required. Since AdaptST requires ConCD re-design at every controller time-step, all control parameters of MMCM have to be re-calculated at each time-step which may result in a highly non-scalable control design. To better understand this issue, let us take a closer look at the MMCM design and its properties: in the case where a large number $M$ of linear elements is required, the design of the mixing signals $\beta_i(x)$ should be such that each time (or, equivalently, for each state $x$) only a small number of $\beta_i(x)$ is active (i.e., different than zero). This can be achieved by reducing the number of overlaps in the parameter space where possible and/or by allowing some discontinuity in the switching accompanied with a hysteresis element. Even if the MMCM is designed so that only a small number of mixing signals is active each time, the AdaptST re-design Step (AD4) requires that all of the $M$ linear elements are being re-designed at each time-step. To avoid such a situation, the AdaptST scheme (AD1)-(AD4) is combined – in a hybrid fashion – with an adaptive MMCM (Multi-Model Adaptive Control with Mixing–MMACM) which employs standard parameter adjusting rules for updating the controller matrices $L_i$. One possible way to adjust $L_i$ is by making use of the following adaptive rule:

$$
\dot{L}_i = -Pr \left\{ \gamma \left( \nabla^T \bar{V}(x) \bar{g}(x) \right)^T \beta_i(x) x^T \right\} 
$$

(18)
where $\gamma$ is a positive user-defined constant, $\hat{V}(x) = z^T(x)P^{-1}z(x)$ is the most recent estimate of the optimal-cost-to-go function as generated at Step (AD4) of AdaptST, $\hat{g}(x)$ is the estimate of the input control vector-field as generated by the identifier used in (AD1) and $Pr\{\cdot\}$ is the projection operator used to keep $L_i$ bounded [2].

Using the adaptive law (18) only the control matrices $L_i$ of the linear elements that are currently active are adjusted. However, the use of adaptive scheme (18) cannot guarantee – in a stand alone mode – efficient performance as it may face poor transients or even controller failure due to loss-of-controllability.

D. The AdaptST/MMACM Hybrid Scheme

AdaptST and MMACM are combined within AGILE into a hybrid scheme that takes advantage of both schemes while avoiding their shortcomings mentioned before: in such a hybrid scheme, a Performance Evaluation Module (PEM) monitors on-line the LSCS performance and if the LSCS under MMACM performance is satisfactory then it keeps running MMACM while AdaptST runs in an off-line mode; Whenever the PEM detects that the LSCS performance under MMACM exceeds certain alarm thresholds (i.e., it is about to become unsatisfactory), the hybrid scheme switches to AdaptST. There are many advantages for developing and employing such a hybrid scheme:

1. It overcomes the problem of requiring the computationally demanding implementation of AdaptST at each time-step.
2. By switching to AdaptST whenever poor performance or loss-of-controllability are about to occur, efficient performance of the overall LSCS scheme is maintained. Consequently the poor transient performance or loss of controllability problem associated with MMACM are avoided.
3. As the AdaptST scheme runs off-line while MMACM is active, the vast majority of computations required for its implementation takes place off-line. As a result, whenever AdaptST is activated by PEM only a small amount of computations is required.

The design of AGILE’s Performance Evaluation Module (PEM) is based on the approximate Hamilton-Jacobi-Bellman (HJB) equation (5); by replacing in (5) the polynomial controller by its MMACM approximation and the actual system dynamics by the estimated ones, the LHS of (5) reads as follows:

$$V(x) = \left( \hat{f}(x) + \hat{g}(x) \sum_{i=1}^{M} \beta_i(x)L_i\varphi \right)^T M^*(x)\bar{P}^{-1}z(x)$$

\[ + z^T(x)\bar{P}^{-1}M(x)\left( \hat{f}(x) + \hat{f}(x) \sum_{i=1}^{M} \beta_i(x)L_i\varphi \right) \]

(19)

where $\hat{f}, \hat{g}$ denote the estimates of $f, g$ as produced by the identification scheme [see (A1) in AdaptST]. The term $V(x)$ in the above equation serves two purposes: (a) firstly, by comparing its value with the term $-z^TQz$ [cf. RHS of (5)] it provides a measure of how close the MMACM performance is to the optimal, and (b) second it is the time-derivative of the positive definite function $z^T(x)\bar{P}^{-1}z(x)$ which is an approximation/estimation of a Control Lyapunov Function–(CLF) for the system and, as a result, its time-derivative $V(x)$ has to be negative definite in order to ensure closed-loop stability. Based on these observations, AGILE’s PEM keeps MMACM active as long as (a) the difference between $V(x)$ and $-z^TQz$ remains within some pre-specified thresholds and (b) the term $V(x)$ is negative. If one of the above conditions is not satisfied then PEM activates AdaptST and implements it for one time-step.

III. THE INTEGRATED AGILE SCHEME

Based on the concise analysis of the previous section, the overall – integrated – AGILE scheme can be summarized according to Table III. A preliminary (off-line) step is required in order to define the mixing signals. Such a preliminary step may be executed either heuristically (based on intuition or experience from the past) or by solving a non-convex optimization problem involving the nominal system dynamics. Since the solution to this problem needs to be provided off-line, its non-convexity does not affect the efficiency of the AGILE scheme.

After the preliminary Step 0, the AGILE system is ready to be implemented: note that Step 0 provides an initial set for the matrices $P$ and $L_i$ necessary for the execution of ConvCD, AdaptST and MMACM. A standard linear-in-the-parameters identification scheme is employed within the scheme; Such a scheme may be directly obtained from a physical model of the large-scale system dynamics (in case such a model is linear-in-the-parameters) or based on linear-in-the-parameters nonlinear approximation models. Using the estimates provided by the identification scheme and two simple conditions on the term $V$, AGILE switches among AdaptST and MMACM. Note that a low-pass pre-compensator (used to filter the random-like control decisions of AdaptST) will also have a positive effect in filtering out the discontinuities due to switching among AdaptST and MMACM.

The next Theorem summarizes the properties of the integrated AGILE scheme.

Theorem 1: Suppose that the Integrated AGILE system is

- either applied for the first time to the large-scale system
- or there is a minor or major constant change in the large-scale system dynamics. Here the term “constant” change means that the form of the system (1) functions $f, g$ changes instantaneously but remains fixed thereafter.

Moreover, suppose that the Integrated AGILE design parameters $L$ and design matrices $\bar{P}, \bar{p}, \bar{q}_1$ are selected so that (9) is satisfied and, the design parameters $t_1, t_2$ are appropriately chosen so that MMACM remains non-singular – in Step I(c) – while ConvCD is being re-designed. There exists an upper bound $\bar{T}$ such that if $T_{ConvCD} \leq \bar{T}$, then the following hold:

- The closed-loop system is stable;
- Moreover, the system state trajectories satisfy

$$|z(x(t))| \leq \alpha_1 \exp^{-\alpha_2 t} (|z(x(0))| + \delta_1) + \alpha_3 + \delta_2$$
**Table III: The Integrated AGILE Scheme**

<table>
<thead>
<tr>
<th>Step</th>
<th>Description</th>
</tr>
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<tbody>
<tr>
<td>0</td>
<td>Augment – if necessary – the system dynamics using a low-pass pre-compensator. Fix the functions for the mixing signals $\beta_i(x)$ either based on the particular large-scale system dynamics (i.e., by intuition) or by employing some type of – non-convex – optimization that is based on the system dynamics. For instance, the mixing signals can be designed by solving the following nonlinear optimal control problem</td>
</tr>
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|      | \[
|      | \begin{align*}
|      | \min_{\theta, \hat{\hat{\beta}}} & \sup_{x \in \mathcal{X}_0} J \\
|      | \text{s.t.} & \dot{x} = f_{\text{nom}}(x) + g_{\text{nom}}(x) \sum_{i=1}^{M} \beta_i(x)L_i x
|      | \end{align*}
|      | \] where $\theta$ denotes the vector of all tunable parameters for the mixing signals $\beta_i(x)$ and $f_{\text{nom}}, g_{\text{nom}}$ correspond to an estimate (nominal value) for the system dynamics. Choose the monomial order $L$ and the ConvCD design matrices $P, Q, \mathcal{R}$. Make sure that the design for the mixing signals $\beta_i(x)$ is such that only a small number of them is activated (i.e., $\beta_i(x) \neq 0$) for each $x$. After designing the mixing signals $\beta_i(x)$ apply the ConvCD procedure and the approximation (16) – again by employing the nominal system dynamics – to get the initial $P$ and $L_i$. |
| 1.a  | Get on-line estimates $\hat{f}, \hat{g}$ of the $f, g$ using standard identification schemes. |
| 1.b  | If the term $V$ defined in (19) satisfies both $V(x) \leq t_1$ and $|V(x) + z'Qz| \leq t_2$ where $t_1, t_2$ are small positive user-defined thresholds then perform MMACM i.e., update $L_i$ using (18) and calculate the control signal according to (17). Add a small sufficiently rich signal to the control signal to ensure PE. |
| 1.c  | Otherwise, issue a command for ConvCD re-design. Let $T_{\text{ConvCD}}$ denote the time it takes to perform ConvCV re-design. Keep applying MMACM from the time the command for ConvCD re-design is issued and until ConvCD re-design is completed (i.e., keep MMACM active for $T_{\text{ConvCD}}$ time-units). Then, calculate the new $L_i$ matrices by performing the approximation (16) and apply the AdaptST Steps (AD2), (AD3). |

where $\delta_i, i = 1, 2$ are bounded nonnegative terms that satisfy $\delta_i = O(\text{identifier modeling error}) + \delta_{i0}(t)$ with $\delta_{i0}(t)$ being a term that exponentially decays to zero.

In essence the above Theorem states that – provided $T_{\text{ConvCD}} \leq T$ – the integrated AGILE scheme’s performance converges exponentially fast to the same performance that would have been attained by a ConvCD controller – that assumes perfect system knowledge – disturbed by a term that is proportional to the accuracy (efficiency) of the identifier employed. It has to be emphasized that the time $T_{\text{ConvCD}}$, which is actually the time needed to solve the optimization problem (11) whenever a command for ConvCD re-design is issued, can be drastically reduced if we allow ConvCD re-design – i.e., Step (AD4) of AdaptST – to run off-line: in this case ConvCD re-design is updated continuously using the on-line estimates of Step 1.(a) even when MMACM is active [Step 1.(b)]. In such a way it takes significantly less time to perform the ConvCD re-design whenever AdaptST is activated.

**A. Example**

The validity of the overall AGILE scheme has been tested using a motorway coordinated ramp control application. As the details of the simulations require quite a lengthy description, a brief description is presented here: A more detailed description will be reported in another paper. Figure 1 displays a schematic diagram of the motorway stretch assumed in the simulation. This stretch is part of the Monash-CityLink-West Gate Corridor in Melbourne, Australia, operated by VicRoads. The numbered circles in Figure 2 represent the network links (the links start and end at locations where there is either an on-ramp or an off-ramp or a lane drop or lane increase as it happens in node N13, where there is a lane increase from 3 to 4 lanes). The small dots numbered as 7838, ..., 7972, 14304, ..., 7973 indicate the sensor locations at the mainstream and the ramps which provide with occupancy, flow and speed measurements. The motorway stretch of Figure 1 has a total length of 17 km and contains a total of 8 on-ramps and 7 off-ramps; Congestion usually appears right upstream of ramp ONHIGHST – due to waving at the end of link L13a – and spills back, creating severe shockwaves. The macroscopic traffic simulation tool METANET [19] was used for the simulation experiments. The traffic model parameters used in METANET were obtained by using extensive validation experiments for minimizing the mismatch between METANET and actual traffic data. The aim of a control design is to design a ramp flow control logic which, based on the sensor measurements, controls the on-ramp flow (using traffic lights at the on-ramps) so that the daily mean speed of the overall stretch is maximized, while the ramp queues do not exceed certain physically-imposed limits.

Among the main challenges that AGILE faces in such an application is that the ramp queues, the system states and control inputs (controlled ramp flows) have to satisfy certain physical constraints (i.e. they should be non-negative and should not exceed certain physically-imposed limits).
To circumvent this problem, we first augmented the system by introducing a low-pass pre-compensator (rendering thus the actual control inputs as states of the augmented system) and then designed the matrix $Q$ of (2) so that $z^*(x)Qz(x)$ takes very large values when one or more of the constraints is violated. The resulting system (with the low-pass pre-compensator augmentation) comprises 41 states and 7 control input vector. A second-order approximation was imposed for the monomial vector $z(x)$ in the ConvCD design and a switching MMCM comprising 6 different linear elements was employed. The mixing signals of MMCM were dependent on the average congestion level on the stretch, i.e. on the average density along the stretch. Finally, as the METANET model is nonlinear with respect to its parameters, a linear-in-the-parameters polynomial approximator was used for the identification scheme employed within AGILE.

Table IV summarizes the results obtained for three different daily traffic demand scenarios (these scenarios correspond to actual traffic data collected at three different days) and for the following cases:

- The “No Control” case, i.e., the case where no ramp metering is imposed on the on-ramps.
- The ConvCD case, i.e. the case where the design methodology of Table I is imposed (using all the design parameters and selections described above) by assuming perfect knowledge of the system dynamics (i.e. the ConvCD approach incorporates the full model of METANET).
- The AdaptST and integrated AGILE approach where no knowledge of the system dynamics is assumed (in particular, at the beginning of each day the polynomial identifier is initialized with all of its parameters equal to zero).

The results in Table IV exhibit the efficiency of AGILE as well as all of its ingredients: as compared to the “No Control” case – present in the vast majority of motorway systems – the optimal control-based ConvCD design can achieve very significant improvements that can be as high as 40.38% in case of high traffic demand scenarios (e.g., scenario 2). On the other hand, the adaptive versions of ConvCD (AdaptST and AGILE) produce a performance that is about the same as the non-adaptive ConvCD one. A closer look to the decisions of the non-adaptive (ConvCD) and adaptive (AdaptST and AGILE) cases indicate that they achieve the improvements shown in Table IV by “intelligent” handling of the ramp queues so that the mainstream traffic does not reach capacity. The adaptive versions behave pretty much the same as the non-adaptive: the only difference is that in the adaptive cases the controller needs a transient period to converge to the decisions that are similar as the ConvCD ones. However, as such a transient period takes place while traffic demand is increasing and way before its peak, its effect on the overall daily system performance is negligible.

Table IV also shows that AdaptST and the integrated AGILE system behave similarly, with integrated AGILE being slightly worse. This is due to the time it takes (equal to $T_{convCD}$ in Table III) for completing Step 1.(c) of Table III. In the particular application, $T_{convCD}$ was found to be less than 90secs which, practically means that it takes 3 time-steps – the controller time-step is 30 secs – from the time a command for activating AdaptST is issued to the time the AdaptST-generated ConvCD re-design is implemented. In other words, the Integrated AGILE scheme achieves to produce a similar performance as AdaptST which is infeasible to be implemented in the particular implementation due to its computational complexity.

IV. CONCLUSIONS & FUTURE RESEARCH

Theoretical analysis and simulation experiments have demonstrated that the AGILE methodology meets the requirements of arbitrarily-close-to-optimal, self-tunable, scalable and modular LSCS performance. However, real-life large-scale systems include a number of partly unpredictable phenomena, such as sensor/communication/controller failures, peculiar demand and environmental changes, atypical operation phases and, most importantly, end-user stochastic behavior that are likely to challenge AGILE as to whether it can ensure a reliable and efficient operation under all practical conditions. Furthermore, the AGILE design methodology can only enjoy broad employment and utilization if it is
Table IV: Coordinated Motorway Ramp Metering Results

<table>
<thead>
<tr>
<th>Traffic Demand Scenario</th>
<th>Daily Mean Speed (km/h)</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>No Control</td>
<td>57.86</td>
<td>23.77</td>
<td>69.44</td>
<td></td>
</tr>
<tr>
<td>ConvCD (perfect system knowledge)</td>
<td>83.24</td>
<td>32.64</td>
<td>77.14</td>
<td></td>
</tr>
<tr>
<td>AdaptST (zero system knowledge)</td>
<td>80.32</td>
<td>32.78</td>
<td>75.18</td>
<td></td>
</tr>
<tr>
<td>AGILE (zero system knowledge)</td>
<td>81.67</td>
<td>30.67</td>
<td>71.12</td>
<td></td>
</tr>
</tbody>
</table>

successfully applied under real conditions. Under a research project funded by the EC [22], the AGILE system will be implemented, tested and evaluated into two real-life large-scale Test Cases possessing a rich variety of design and performance characteristics, extremely complex nonlinear dynamics, highly stochastic effects, uncertainties and modeling errors, as well as reconfiguration and modular design requirements: (a) the traffic control system of the City of Chania (Greece) urban network comprising 23 junctions, more than 100 control inputs (traffic light signal settings) and around 200 sensor measurements and (b) control of an Energy-Positive Building in Kassel, Germany that is equipped with a variety of renewable self-generation energy systems (solar arrays, wind turbines and geo-thermal soil collector) and a variety of different control elements (HVACs, automatically-controlled shading devices and natural-ventilation systems, floor heating and cooling systems) in each of the buildings.

REFERENCES