An Integrated Outbound Logistics Model for Frito-Lay: Coordinating Aggregate-Level Production and Distribution Decisions

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In this paper, we describe research to improve Frito-Lay’s outbound supply chain activities by simultaneously optimizing its inventory and transportation decisions. Motivated by Frito-Lay’s practice, we first develop a mixed-integer programming formulation from which we develop a large-scale, integrated multiproduct inventory lot-sizing and vehicle-routing model with explicit (1) inventory holding costs, truck loading and dispatch costs, and mileage costs; (2) production, storage, and truck capacity limitations; and (3) direct (plant-to-store) and interplant (plant-to-plant) delivery considerations. Second, we present an iterative solution approach in which we decompose the problem into inventory and routing components. The results demonstrate the impact of direct deliveries on distribution costs and show that direct deliveries and efficient inventory and routing decisions can provide significant savings opportunities over two benchmark models, one of which represents the existing Frito-Lay system. We implemented our models using an application that allows strategy evaluation, analysis of output files, and technology transfer. This application was particularly useful in evaluating potential direct-delivery locations and inventory reductions throughout the supply chain.

Key words: coordinated logistics; integrated inventory; transportation decisions; vendor-managed inventory and delivery; inventory lot sizing; vehicle routing.

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Frito-Lay North America (FLNA) operates a large and complex supply chain. The company, a division of PepsiCo, employs more than 40,000 people and produces a wide variety of snack foods. It owns a private fleet, which consists of more than 1,400 over-the-road (OTR) trucks and 20,000 route-delivery (RD) trucks of various capacities, for the outbound transport of these products to many delivery locations; these include distribution centers (DCs), bins, and customers (stores). Its overall distribution operation also includes interplant (plant-to-plant) shipments because each plant does not produce each product group. FLNA’s offerings comprise 16 different groups of produced and nonproduced items, with thousands of stock-keeping units (SKUs). It operates 33 production facilities, 192 DCs, and 1,535 bins in the United States as well as additional plants, DCs, and bins in Canada. Consequently, FLNA faces complex outbound logistics planning issues.

FLNA participates in a vendor-managed inventory and delivery (VMI/D) program. Under a typical VMI/D program, a vendor is empowered to control its resupply timing and quantity at downstream locations (Çetinkaya and Lee 2000). Because effectively utilizing transportation resources is imperative, a vendor is more likely to dispatch full-vehicle outbound loads to achieve economies of scale. Using this program also makes coordinating inventory and outbound transportation decisions easier. We developed a model to allow simultaneous optimization of
these decisions to improve outbound supply chain efficiency and, thus, enhance the success of FLNA’s VMI/D program.

We focused on the outbound supply chain of a major FLNA plant in Irving, Texas, and developed an integrated cost-minimization model that links production output and shipment quantities destined to delivery locations; thus, it allows demand to be satisfied in a cost-effective and timely manner. The model is a mixed-integer programming formulation that seeks to achieve the following:

(1) Inventory optimization and reduction at the Irving plant’s warehouse (called the factory warehouse henceforth);

(2) Load optimization for transportation from the factory warehouse to the delivery locations (i.e., DCs, bins, plants, and direct-delivery (DD) customers), while allowing direct (plant-to-store) and interplant shipments; and

(3) Flow optimization throughout the entire outbound supply chain.

Our model’s uniqueness is its ability to coordinate inventory and shipment quantities with truck loading and routing decisions. The model also considers the option that DD customers can receive supplies directly from the factory warehouse, DCs, and bins. Hence, we also seek to identify the DD customers in a customer set and measure the value of direct deliveries to them.

The problem we address is a large-scale, integrated multiproduct inventory lot-sizing and vehicle-routing problem with explicit inventory holding costs, truck loading and dispatch costs, and mileage costs; production, storage, and truck capacity limitations; and direct and interplant delivery considerations. It is a challenging generalization of the vehicle-routing problem, which is NP-hard. Therefore, an exact optimal solution is impractical. We also cannot obtain a solution using existing software and FLNA’s real data. We overcome these difficulties by developing a new heuristic that decomposes the overall problem into inventory and routing subproblems.

In the inventory subproblem, we determine the final set of DD customers and the weekly replenishment and shipment quantities to the delivery locations while minimizing relevant costs. In the routing subproblem, we find the subsets of delivery locations that trucks dispatched from the factory warehouse must visit. In this subproblem, we utilize the weekly replenishment and shipment quantities, which we obtain from solving the inventory subproblem, as distribution requirements at the delivery locations. The objective of the routing subproblem is to minimize loading and routing costs and maximize truck utilization. Clearly, convergence might occur to a local optimum, and it is impossible to benchmark solution quality relative to the global optimum. To demonstrate the effectiveness of our optimization model and proposed solution approach, we compare our results to two practical benchmarks, the chase policy and the cube-out policy (see descriptions in Benchmark Models and Benefits Realized). The former is of interest to the company for policy comparison purposes; the latter represents its usual practice. The solution approach for the benchmark models relies on the solution that we developed for our model. Using numbers, we show that run times in our solution approach are reasonable, and our model provides significant cost savings.

We organize the remainder of the paper as follows. We present a brief review of the relevant literature in Related Literature. In Problem Setting and Formulation, we develop a mathematical formulation of the problem. In Solution Methodology, we discuss problem properties to show the difficulties of developing solutions for our problem; we also present our proposed approach. We describe the software application that combines the database and the solution approach in Application Modules. In Benchmark Models and Benefits Realized, we discuss our proposed model’s performance and demonstrate that our approach produces high-quality solutions when compared to the benchmark models. Finally, we summarize our findings in Conclusions.

Related Literature

Our problem deals with integrated logistical planning with a specific emphasis on coordinating inventory and transportation decisions. A large body of quantitative research exists to address integrated logistical planning for the simultaneous optimization of various types of decisions, including facility location and capacity, production, inventory, and distribution.
However, despite the growing interest in this topic, our specific problem has not been previously analyzed in the literature.

Academic research has successfully documented the cost savings associated with integrated decision making for logistical planning. A detailed review of this general topic is beyond our scope. For related literature, we refer the reader to Baita et al. (1998), Çetinkaya (2004), Keskin and Üster (2007), Sarmiento and Nagi (1999), and Thomas and Griffin (1996); we restrict our attention to previous research that specifically investigates the interaction between inventory and transportation decisions. We note that the traditional inventory models largely ignore this interaction because the early theory assumes that demands should be satisfied as soon as they arrive and the customer should cover delivery costs. Consequently, the outbound transportation costs are sunk costs (Hadley and Whitin 1963, pp. 17–18). However, this is no longer true for private-fleet outbound distribution in general and for VMI/D in particular, both of which have received significant academic attention during the past decade (Bertazzi et al. 2005, Çetinkaya and Lee 2000, Kleywegt et al. 2002, Toptal and Çetinkaya 2006).

Specifically, two types of research emphasize the interaction between inventory and transportation decisions. The first type focuses on including transportation costs and (or) capacities in traditional inventory problems without explicitly modeling transportation-related (e.g., routing, consolidation, etc.) decision variables such as classical economic order quantity, dynamic lot sizing, stochastic dynamic demand, one-warehouse multiretailer, joint replenishment, and buyer-vendor coordination problems. It is worthwhile to note that the previous work on this topic (Alp et al. 2003, Chan et al. 2002, Diaby and Martel 1993, Lee et al. 2003, Lipmann 1969, Toptal and Çetinkaya 2006, Üster et al. 2008, Van Eijs 1994) focuses on stylistic supply chain settings (e.g., single vendor and single buyer, single vendor and multi-buyer, etc.) and neglects the vehicle-routing considerations that we address.

The second set addresses the integration of inventory and transportation decisions by explicitly considering both inventory- and transportation-related decision variables. The literature on this latter type of research is also abundant (Anily and Federgruen 1993, Baita et al. 1998, Çetinkaya 2004, Federgruen and Simchi-Levi 1995, Viswanathan and Mathur 1997). We note that our problem has several characteristics of the inventory-routing problem and its extensions (Archetti et al. 2007, Bertazzi 2008, Bertazzi et al. 2005, Campbell and Savelsbergh 2004, Fumero and Vercellis 1999, Kleywegt et al. 2002, Lei et al. 2006). The inventory-routing problem, studied by Campbell and Savelsbergh (2004), is a variation of the vehicle-routing problem in which the vendor can make decisions about the timing, sizing, and routing of deliveries without allowing any shortages. However, most existing literature on this problem is method-oriented (e.g., large-scale mixed-integer programs); a few exceptions (e.g., Lei et al. 2006) focus on real-life applications.

Although the specific application setting in this paper is similar to the inventory-routing problem, we consider a more general supply chain setting than the previous literature does. Our setting includes multiple product groups and intermediate tiers; these represent the DCs and bins in which customers do not carry more than one period’s inventory and can replenish from the DCs, bins, and directly from the factory warehouse.

**Problem Setting and Formulation**

We consider a finite planning horizon in which each period represents one week. Multiple DCs, bins, plants (that receive interplant shipments), and customers constitute the delivery locations on the outbound supply chain. We model the tiers and flows of the outbound supply chain (Figure 1). The flows include (1) production output at the Irving plant; (2) finished-product inventories at the factory warehouse; (3) replenishment quantities from the factory warehouse to the DCs and bins, plant-to-store shipment quantities to DD customers, and interplant shipment quantities to the other plants; (4) shipment quantities from the DCs and bins to customers; and (5) customer demand that can be satisfied by shipments from DCs and bins and (or) direct supply from the factory warehouse. Our model’s objective is to minimize the handling, inventory, loading, routing, and delivery costs associated with the flows. Because
our focus is outbound logistics, we do not model the production process in detail, and we omit production costs. However, we explicitly model the production capacities. One output of our model is a target production output plan that is useful in developing detailed production schedules.

We introduce our notation and present the complete mathematical formulation in the appendix. To explain our formulation, we next examine the tiers of the underlying supply chain from customers to the Irving plant and introduce the problem constraints.

**Constraints Related to Inventory Decisions for Tiers 1–4**

Analyzing the chain in Figure 1 backwards, the first tier consists of the customers who generate the demand flows that can be satisfied by shipping from DCs and bins. For some customers, a DD can be sent from the factory warehouse; they can be served on the same route with DCs and bins. To determine the source(s) of supply for each customer, we use the customer’s DC or bin assignment and DD information.

Then, the demand constraints (1) and (2) dictate that customer demand is satisfied by timely shipments to customers from DCs, bins, and the factory warehouse. These constraints also dictate that weekly total shipments to a customer location do not exceed the weekly demand, because FLNA’s VMI/D program ensures that customer locations never receive more inventory than their weekly demand specifies.

We note that we model the demand for each product group at a customer location in a given week using an aggregate number of cartons. The actual cartons used for each product group are characterized by varying sizes, i.e., storage volumes. To represent demand in an aggregate fashion, we define a cubic carton, i.e., a unit measure for our volume calculations, as the following example shows. Suppose that, for a product group, we use two types of cartons with storage capacities of 0.5 cubic feet and 1.5 cubic feet, respectively. Furthermore, let the usage proportions (historical data provided) for the two carton types be 0.75 and 0.25, respectively. We define a cubic carton for this product group with a
storage volume given by the weighted average of the volumes of the carton types available for the product group: \((0.75)(0.5) + (0.25)(1.5) = 0.75\) cubic feet. Then, we compute the total demand for a product group at a customer location in a given week (in required cubic feet) by multiplying the storage volume of the cubic carton for the product group by the total number of actual cartons (data provided).

Next, considering the second tier of the chain, it is straightforward to develop the inventory-balance constraints at the DCs (3) and bins (4). The variables representing the replenishment quantities of the products sent from the factory warehouse to the DCs and bins, in successive periods, link the second (DCs and bins) and the third (staging and loading area of the factory warehouse) tiers of the chain in Figure 1. These variables also represent the truck loads that are en route from the factory warehouse to the DCs and bins in successive periods. Hence, we also use them to develop additional constraints that represent the loading and routing operations. We note that inventory levels at the DCs and bins should not violate the capacity limits; hence, we have the corresponding storage capacity constraints (5) and (6).

Considering the flow conservation at the staging and loading area of the factory warehouse, we have the load balance constraints (7), which consider the total load from the factory warehouse to the DCs, bins, DD customers, and other plants. It is worth noting that, in these constraints, the interplant replenishment quantities are model parameters that represent the nonproduced item requirements at the other plants. The variables representing the total loads of the products shipped from the factory warehouse, in successive periods, link the staging and loading area with the fourth tier, the factory warehouse. Considering the replenishment (production) quantities of the products in successive periods, we also have the corresponding inventory balance constraints at the factory warehouse (8). The total inventory at the factory warehouse should not violate the capacity therein; therefore, we also have the storage capacity constraints at the factory warehouse (9).

Because we model the target production output levels of different products in different periods as decision variables, we say that our formulation translates customer demand into a target production output plan. These target production output levels account for the loading and delivery schedules explicitly (we discuss the loading and delivery considerations in the next paragraph). When we have computed a numerical solution for the proposed model, we can use the resulting target production output levels (of individual products in successive periods) as “translated demand” values in the existing production planning and scheduling software. The target production output levels should not violate the aggregate capacity and product-based constraints (10) and (11).

**Constraints Related to Truck Loading and Routing at Tier 3**

Figure 2 illustrates the loading and routing operations between the factory warehouse and the DCs, bins, DD customers, and other plants. We model these operations explicitly using constraints (14)–(23).

First, let us consider the loading and routing component of the problem associated with the outbound shipments from the factory warehouse (Figure 2). Our objective is to minimize the outbound transportation costs from the factory warehouse and to maximize truck utilization. The OTR trucks based at the factory warehouse are identical.

The link between the routing operations and the flow constraints is as follows. The load balance constraints consider the total loads of the products to be shipped in successive periods from the factory warehouse to the delivery locations. The components of the total load shown on the left side of the load balance constraints represent the replenishment and shipment quantities that should be loaded on trucks that are en route from the factory warehouse to the delivery locations. Then, it is straightforward to verify the total load and truck volume constraints (14) and drop-off constraints (15)–(18). Note that the drop-off constraints are associated with the set of delivery locations of the routing component of interest. The consequent truck arrival and departure constraints (constraints 19–22) assure that (1) the number of trucks arriving at a delivery location is equal to the number of trucks leaving that location, and (2) all trucks leaving the factory warehouse eventually return to this location. Any subset of the delivery locations can be on the route of any truck. Hence, we must assure that the total load on a truck destined to visit any such subset does not exceed
the truck volume; the ensuing route-based truck volume constraints (23) follow.

**Constraints Related to Market Area Deliveries at Tier 2**

The final set of constraints is associated with smaller trucks (RD trucks) delivering indirect shipments to customers from the DCs and bins. At this level, we differentiate between the RD trucks based at the DCs and bins based on their distinct capacities. The sets of customers that each DC and bin serve are fixed. Each set represents a market area, which consists of customers in close proximity and assigned to a specific DC or bin. Under the VMI/D program, FLNA serves all customers in each market area using preset delivery routes. Hence, our main interest at the customer level is not load or route optimization; it is to assure that the truck volume constraints associated with deliveries from the DCs and bins are satisfied. The two sets of dispatch capacity constraints (12) and (13) ensure that these requirements are satisfied.

**Cost Function**

The cost function has five specific components that represent (1) touch costs (i.e., product-based per-unit handling costs at DCs and bins) associated with indirect customer deliveries via DCs and bins, (2) fixed delivery costs to market areas from the DCs and bins, (3) holding costs at inventory-keeping locations, (4) fixed costs of loading and dispatching OTR trucks at the factory warehouse, and (5) distance-based routing costs of OTR trucks leaving the factory warehouse. The first two terms of the objective function take into account the touch costs. A fixed delivery cost, which is associated with each dispatch from each DC (bin) to its market area, is included in the third and fourth terms of the objective function. The fifth, sixth, and seventh terms correspond to the holding costs.
costs at the inventory-keeping locations. Next, associated with each OTR truck released from the factory warehouse, we consider a fixed loading cost. Finally, we keep track of the number of drop-offs at each delivery location of the truck routes via the route-selection variables in successive periods; thus, we can incorporate the total mileage cost in the last term of the objective function.

Solution Methodology

As we noted earlier, our solution methodology decomposes the overall model into two subproblems that involve complementary inventory and routing components. We illustrate the relationship between the subproblems in the flowchart of our algorithm in the appendix (Figure 8), and we also provide the specific formulations of the subproblems. We solve the subproblems iteratively until we cannot find a cost-based improvement for the overall solution or reach the limit on maximum number of iterations.

The inventory subproblem seeks to determine the weekly replenishment and shipment quantities at the DCs, bins, and DD customers, and also considers the requirements at the other plants, the touch costs, inventory holding costs, fixed delivery costs to market areas from the DCs and bins, and additional fixed costs associated with loading and routing (i.e., the route-based setup costs), as we describe in detail below. Given the weekly replenishment and shipment quantities, the routing subproblem specifies the truck routes and minimizes the actual loading and routing costs. Our algorithm begins with an initialization module. Its main role is to set the replenishment and shipment quantities as input into the routing subproblem in the first iteration, as we describe next.

Initialization

In the initialization module, in which we address each product in each period, we set the shipment quantities for each potential DD customer based on that customer’s corresponding demand, to satisfy the demand constraints (1). Hence, at this iteration, the DD customers do not receive any shipments from their respective DCs or bins. We set the replenishment quantities from the factory warehouse to the DCs, bins, and other plants based on the remaining overall requirements of the corresponding locations and using the inventory balance constraints (3) and (4). We specify the remaining overall requirements by the shipment quantities of the other customers, as dictated by the demand constraints (2) and the product requirements of the other plants. We also set the total load in a similar fashion using the load balance constraints (7).

Utilizing the weekly DD, replenishment, and shipment quantities—based either on the outcome of the initialization module discussed above or the solution of the inventory subproblem—in the routing subproblem, we determine the corresponding truck routes while minimizing the loading and routing costs of the OTR trucks based at the factory warehouse. Using the solution to the routing subproblem, we calculate the route-based setup costs associated with each delivery location of the routing subproblem in each period. In the appendix, we describe how we calculate these costs and consider them as an input while solving the inventory subproblem. We now proceed with a detailed discussion of the routing and inventory subproblems.

Routing Subproblem

The constraints and cost terms related to loading and routing operations in the overall model constitute the routing subproblem. Its main goal is to determine the OTR truck delivery routes from the factory warehouse to the delivery locations given the weekly replenishment and shipment quantities. A closer examination of the formulation in the appendix reveals that the routing subproblem is separable for each period. Hence, it suffices to solve a routing problem successively for each period (week) of the planning horizon. We use a savings algorithm, the Clarke and Wright algorithm (Clarke and Wright 1964, Chopra and Meindl 2001) with an additional step for improvement. When the savings algorithm terminates with a set of routes, we improve each generated route with more than four delivery locations using a cheapest insertion heuristic, a well-known travelling salesman problem heuristic.

Before we solve a routing problem for each period, we preprocess the given replenishment and shipment quantities to see if full-truck load (FTL) shipments are needed for a specific location; this might be the case for major customers, such as Sam’s Club and Costco, and metropolitan area DCs. That is, we first process
FTL shipments so that the quantity leftover for each location is less than the truck capacity. We note that each FTL shipment constitutes a route with a single destination. We then specify the additional truck routes and consider the leftover quantities, which are called less-than-truck load (LTL) shipments (each on a route with others).

**Inventory Subproblem**

Traditional inventory models evaluate the trade-offs between fixed ordering and inventory holding costs to determine replenishment quantities. In our inventory subproblem, because of the influence of the routing subproblem, the trade-off also includes touch costs at the bins and DCs, fixed delivery costs to the market areas, and route-based setup costs dictated by the loading and routing costs of the OTR trucks based at the factory warehouse. We discussed these cost components above, except for the route-based setup costs, which are an input from the routing subproblem.

In each period, depending on its replenishment and shipment quantity, a particular delivery location can be visited by a truck(s) on a route(s). By sending a truck on a particular route, we incur a route-based, fixed-loading total-mileage cost. Each delivery location on the route is responsible for the corresponding cost. We allocate this cost component above, except for the route-based setup costs, which are an input from the routing subproblem.

Our proposed solution for the routing subproblem is such that the set of routes of a given delivery location, $l$, includes at most one route in which $l$ shares a truck with other delivery locations, and possibly other routes carrying FTL shipments to $l$ (if the requirements at $l$ in a given period exceed the truck capacity). To ensure that the right delivery locations incur the route-based setup costs in a given period, we use binary variables. The impact of route-based setup costs are reflected by the last four terms of the objective function and the route-based setup constraints of the inventory subproblem. These costs and constraints establish the link between the inventory and routing subproblems.

Except for the addition of the route-based setup costs and constraints, the remaining formulation of the inventory problem is based on our overall model. That is, the inventory subproblem includes the constraints associated with the demand, inventory balance, storage capacity, load balance, production capacity, and dispatch capacity from the overall formulation, and the additional route-based setup constraints. The objective function includes the corresponding route-based setup costs and all cost terms of the overall model, except the loading- and routing-related cost parameters considered in the routing subproblem. We solve the inventory subproblem using CPLEX 9.0 (CPLEX is a trademark of ILOG, Inc.).

**Application Modules**

Our application includes the database application and solution algorithms, and provides broad functionality for optimization, strategy evaluation, and analysis of output files. The database application is useful for importing input data and processing the data to eliminate discrepancies in format and range. The application also includes (1) a graphical user interface, (2) application programming interfaces (APIs) for constructing a network to facilitate the generation of interdistances between the delivery locations based on the address information, and (3) modules for performing sensitivity analysis and report generation (Figure 3). We designed the user interface and the normalized database tables using Visual Basic for Applications and MS Access 2003 and implement our solution using the C++ programming language, Concert Technology, and CPLEX 9.0. We perform the runs on a computer with a 3 GHz Intel XEON processor and 6 GB RAM.

The application’s information flow begins with importing data using an interface that has a step-by-step procedure for performing the required data import and data-integrity verification. The graphical user interface (Figure 4) allows the user to easily import, effectively manage, and efficiently query data pertaining to the logistics network, capacity, demand, and cost. The interface is also useful for selecting a set of potential DD customers based on a variety of user-specified criteria.

The interdistance data are generated through the API for input data, such as network-construction data, and are passed to the solution algorithms. The API for solution algorithms provides formatted input, executes the algorithms, and reads the output solution into the database tables for later analysis and report generation.
Finally, the user can use the analysis module to change the network, capacity, demand, and cost data, and to perform scenario analyses to evaluate new strategies for incorporating DD customers for other plants. The application requires little user training; in addition, it provides effective solutions in reasonable time and facilitates technology transfer for easy implementation at FLNA plants. FLNA executives have used it for distribution policy, strategy evaluation, and scenario analyses. Their main objectives have been to assess potential cost savings that would result from (1) direct deliveries for varying demand forecasts and (2) inventory optimization and reduction at the factory warehouse. A plant in Perry, Georgia has implemented the application for these purposes.

Benchmark Models and Benefits Realized

We present our numerical results to demonstrate our approach’s effectiveness and benefits. First, we compare our results to two benchmarks, the chase and cube-out policies. Neither considers DD customers.

The chase policy matches the demand by sending the exact weekly requirements (including the demand and safety stock) to the DCs and bins. Using this policy, we obtain a feasible solution to the inventory subproblem by satisfying the replenishment and shipment requirements, and then solve the routing subproblem. Hence, it provides a basis for measuring the relative gains associated with inventory optimization and for considering DD customers. Using the cube-out policy, FTL shipments, which include the demand and some excess supply for the most popular product, potato chips, are sent to the DCs and bins to achieve FTL shipments. This benchmark, which aims to increase truck utilization and routing efficiency without explicitly addressing the role of inventory optimization, represents the usual company practice.

We consider 5 product groups that are produced at Irving, 8 FLNA plants located elsewhere to which interplant shipments are sent, 8 DCs, 74 bins, 8,460 customers with 85 potential DD customers, and 12 weeks of demand data. We observe that our solution for the optimization and benchmark models runs in a very reasonable time using real data. We obtain
the chase and cube-out policy solutions in a few minutes and obtain a solution for our model within 10 minutes. We also obtained similar run-time results for a relatively larger problem at another FLNA plant in Perry, Georgia.

In Table 1, we present the overall cost and its main components for each model. In the last row of this table, we report the percentage difference between the cost using our model and the benchmark models. In the following discussion, we elaborate on the detailed benefits of our model relative to the two benchmark models and the individual strengths of each model.

**Detailed Analysis of Cost Components**

In Table 2, we show the cost-component results for our model and for each policy. Relative to the chase policy and cube-out policy, the inventory levels in our model at all stock-keeping locations are significantly lower because of inventory optimization; the factory warehouse achieves the greatest inventory reduction. Compared to the two benchmark policies, the inventory holding costs at the factory warehouse in our model are lower by as much as 76.92 percent (chase policy) and 73.62 percent (cube-out policy). The inventory holding costs at the DCs are lower by 24.52 percent and 25.59 percent compared to the chase policy.

<table>
<thead>
<tr>
<th></th>
<th>Our model ($)</th>
<th>Chase policy ($)</th>
<th>Cube-out policy ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Touch cost</td>
<td>6.75</td>
<td>8.64</td>
<td>8.64</td>
</tr>
<tr>
<td>Delivery cost</td>
<td>33.77</td>
<td>45.17</td>
<td>45.17</td>
</tr>
<tr>
<td>Inventory holding cost</td>
<td>2.83</td>
<td>4.08</td>
<td>4.34</td>
</tr>
<tr>
<td>Loading cost</td>
<td>28.63</td>
<td>28.30</td>
<td>25.72</td>
</tr>
<tr>
<td>Mileage cost</td>
<td>28.02</td>
<td>25.01</td>
<td>22.51</td>
</tr>
<tr>
<td>Total cost</td>
<td>100</td>
<td>111.20</td>
<td>106.38</td>
</tr>
</tbody>
</table>

Table 1: Examining the costs using our model and each policy indicates that our model outperforms both policies, and the cube-out policy outperforms the chase policy.
One of the most striking results from the data in Table 1 is that, in each model, transportation-related expenses (i.e., delivery costs associated with shipments from DCs and bins and loading and total mileage costs associated with shipments from the factory warehouse) represent approximately 90 percent of the cost. In our model, the loading costs at the factory warehouse are higher by 1.16 percent and 11.31 percent compared to the chase and cube-out policies, respectively. This is a consequence of the increased number of delivery locations served from the factory warehouse (because of including DD customers). For the same reason, the total mileage cost from the factory warehouse is higher by 12.03 percent and 24.48 percent compared to the chase and cube-out policies, respectively. However, in terms of overall cost, our model is better than the chase and cube-out policies by $11.20 and $6.38, respectively, because our explicit consideration of potential DD customers and inventory optimization helps to significantly reduce the touch costs, inventory holding costs, and delivery costs from DCs and bins.

As we note above, the potential DD customers include 85 locations identified based on demand volume. However, each potential DD customer does not receive its supplies directly from the factory warehouse in each period. The selected number of DD customers varies between 66 and 74, as Figure 5 shows. For the specific planning horizon considered, in each

**Measuring the Value of Direct Deliveries**

A closer examination of the results in Tables 1 and 2 is useful for illustrating both the savings resulting from optimization and the explicit consideration of DD customers.

<table>
<thead>
<tr>
<th></th>
<th>Our model ($)</th>
<th>Chase policy ($)</th>
<th>Cube-out policy ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Inventory holding</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Overall</td>
<td>2.83</td>
<td>4.08</td>
<td>4.34</td>
</tr>
<tr>
<td>Factory</td>
<td>0.24</td>
<td>1.04</td>
<td>0.91</td>
</tr>
<tr>
<td>DCs</td>
<td>1.57</td>
<td>2.08</td>
<td>2.11</td>
</tr>
<tr>
<td>Bins</td>
<td>1.02</td>
<td>0.97</td>
<td>1.31</td>
</tr>
<tr>
<td><strong>Touch</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Overall</td>
<td>6.75</td>
<td>8.64</td>
<td>8.64</td>
</tr>
<tr>
<td>DCs</td>
<td>4.50</td>
<td>6.19</td>
<td>6.19</td>
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<tr>
<td>Bins</td>
<td>2.25</td>
<td>2.46</td>
<td>2.46</td>
</tr>
<tr>
<td><strong>Deliveries from DCs and bins</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Overall</td>
<td>33.77</td>
<td>45.17</td>
<td>45.17</td>
</tr>
<tr>
<td>DC</td>
<td>29.76</td>
<td>40.79</td>
<td>40.79</td>
</tr>
<tr>
<td>Bin</td>
<td>4.01</td>
<td>4.37</td>
<td>4.37</td>
</tr>
</tbody>
</table>

Table 2: A detailed cost breakdown of the inventory subproblem for our model and each policy is helpful in evaluating the impact of positioning inventory at the DCs, bins, or factory warehouse.
week, most of the selected DD customers (varying between 66.2 percent in week 2 and 98.6 percent in week 7) receive FTL shipments from the factory warehouse. Shipping these large quantities directly, rather than via DCs and (or) bins, helps to save on touch costs and inventory holding costs and also on delivery costs from DCs and bins. For example, in week 7, 73 of 74 selected DD customers receive FTL shipments. The touch, inventory holding, and delivery costs from DCs and bins decrease by sending smaller quantities to the DCs and bins.

We note that when a customer is identified as a DD customer, that customer has the opportunity to receive shipments directly from the factory warehouse; however, this opportunity does not forbid shipments from the DCs and bins. In Figure 6, we illustrate the shipments to DD customers from the factory warehouse. Adding the FTL and LTL percentages sent from the factory warehouse, we observe that 79.8 to 89.1 percent of the DD customer demand is shipped directly from the factory warehouse. In Figure 6, the percentage of FTL shipments varies between 48.4 and 61.3 percent in different periods, and the percentage of LTL shipments varies between 27.8 and 35.5 percent.

Trade-Off Between Truck Utilization and Inventory Optimization

We next discuss the results of the trade-off between truck utilization, which FLNA considers a success criterion for efficient routing, and inventory optimization. As we mentioned above, the company uses this principle for designing the routes by following the cube-out policy. One of its main concerns about our model was whether it could achieve high truck utilization. In Figure 7, we address this issue by showing the truck utilization percentage for each model.

As expected, the cube-out policy provides the highest truck utilization; it ranges between 81.8 and 88.3 percent during the planning horizon; however, our model is comparable to the chase policy and performs well relative to company expectations. Based on overall benefits, FLNA evaluates the model as providing significant value to the company. In addition to its cost advantage (Table 1), the model allows FLNA to directly serve DD customers from the factory warehouse with either FTL or LTL deliveries. Neither policy can provide this benefit because neither can accommodate a route that includes DD customer locations.

Conclusions

Our research resulted in quantifiable benefits for FLNA. Both the proposed model and the solution approach are innovative because, despite the recent interest of academia and industry in integrating logistical decisions, no published results exist that address the specific problem we discuss in this paper.

A major goal of our research was inventory elimination at the Irving plant. To this end, FLNA had been investigating the use of a new packing and shipping technology that would make direct supply from the factory warehouse to DD customers a viable option. Our model provided input into FLNA’s decision making by evaluating the impact of DD customers.

A careful investigation of the model’s results in comparison to the chase and cube-out policies showed that DD options, in addition to making efficient
routing and inventory decisions, eliminated inventory at the factory warehouse. FLNA could also realize significant cost savings in comparison to the benchmark models, which did not allow direct deliveries. Furthermore, we were able to evaluate the criteria for specifying potential DD customers, the impact of positioning inventory at the DCs and bins versus the factory warehouse, and the trade-offs between truck utilization and inventory optimization. An additional goal was to evaluate the cube-out policy as implemented for outbound shipments from plants to DCs. Our results revealed that DDs from the Irving plant, when supported by efficient routing and inventory replenishment decisions, presented a significant opportunity for cost savings compared to an estimate for the existing system. When we implemented our model and the two benchmark models for the FLNA plant in Perry, Georgia, we saw similar results. The study and results received enthusiastic support from FLNA, and our collaboration continued with a closer examination of the implications of operational (i.e., daily delivery) requirements of the selected DD customers. Additional research, which we will document in a future publication, is in progress.

Appendix

Notation

We use the following notation in our model formulation.

Sets and Indices

- \( t \): time index, \( t = 1, 2, \ldots, T \).
- \( J \): set of product groups, \( j \in J \).
- \( D \): set of DCs, \( d \in D \).
- \( B \): set of bins, \( b \in B \).
- \( P \): set of other plants receiving interplant shipments, \( p \in P \).
- \( I \): set of customers, \( i \in I \).
- \( D_i \): set of DCs that serve customer \( i \); i.e., \( D_i \subseteq D \).
- \( B_i \): set of bins that serve customer \( i \); i.e., \( B_i \subseteq B \).
- \( o \): factory warehouse.
- \( I_o \): set of potential DD customers; i.e., \( I_o \subseteq I \).
- \( I_{Di} \): set of customers served by DC \( d \); i.e., \( I_{Di} \subseteq I \).
- \( I_{Bb} \): set of customers served by bin \( b \); i.e., \( I_{Bb} \subseteq I \).
- \( V \): \( \{o\} \cup D \cup B \cup P \cup I \).
- \( E \): links connecting the locations in \( V \).
- \( G(V,E) \): road network that consists of vertices (locations) \( V \) and edges (links) \( E \).

Demand Parameters

- \( D_{ijt} \): demand (in cubic cartons) at customer \( i \) for product \( j \) in period \( t \).
- \( R_{ijt} \): product requirement (shipment quantity) of plant \( p \) for product \( j \) in period \( t \) sent from \( o \).

Parameters Related to Production, Inventory, and Transportation Capacities

- \( cc_{cap} \): storage capacity at DC \( d \).
- \( bc_{cap} \): storage capacity at bin \( b \).
- \( oc_{cap} \): storage capacity at \( o \).
- \( pc_{cap} \): production capacity (in cases) for product \( j \) in period \( t \).
- \( p_j \): production time coefficient of each case of product \( j \).
- \( ac_{cap} \): aggregate production capacity in \( t \) (in hours or minutes).
- \( tcap \): volume capacity of trucks based at \( o \).
- \( tcap_d \): volume capacity of trucks based at DC \( d \).
- \( tcap_b \): volume capacity of trucks based at bin \( b \).

Cost Parameters

- \( s_j \): per-unit touch cost of product \( j \) at DC \( d \).
- \( s_{ob} \): per-unit touch cost of product \( j \) at bin \( b \).
- \( a_d \): fixed delivery/dispatch cost of each truck leaving DC \( d \).
- \( a_{ob} \): fixed delivery/dispatch cost of each truck leaving bin \( b \).
- \( h_j \): per-unit-per-period inventory holding cost at DC \( d \).
- \( h_{ob} \): per-unit-per-period inventory holding cost at bin \( b \).
- \( h_o \): per-unit-per-period inventory holding cost at \( o \).
- \( A \): fixed loading cost of each truck leaving \( o \).
- \( g_{mn} \): mileage cost between \( m \) and \( n \) for \( m, n \in V \).

Decision Variables

- \( S_{djt} \): shipment quantity from DC \( d \) to customer \( i \in I_{Di} \) of product \( j \) in period \( t \).
- \( S_{bij} \): shipment quantity from bin \( b \) to customer \( i \in I_{Bb} \) of product \( j \) in period \( t \).
- \( S_{oi} \): DD quantity (from \( o \) to customer \( i \in I_o \) of product \( j \) in period \( t \).
- \( I_{dj} \): beginning inventory of product \( j \) in period \( t \) at DC \( d \).
- \( I_{bj} \): beginning inventory of product \( j \) in period \( t \) at bin \( b \).
- \( R_{dj} \): replenishment quantity of product \( j \) in period \( t \) sent to DC \( d \) from \( o \).
- \( R_{bj} \): replenishment quantity of product \( j \) in period \( t \) sent to bin \( b \) from \( o \).
- \( R_{o} \): total load of product \( j \) destined to the DCs and bins from \( o \).
- \( I_{dj} \): beginning inventory of product \( j \) in period \( t \) at \( o \).
- \( F_{ij} \): target replenishment quantity of product \( j \) in period \( t \) sent to \( o \).
- \( C \): number of dispatches in period \( t \) from \( o \).
- \( y_{d} \): number of drop-offs at DC \( d \) in period \( t \).
- \( y_{ob} \): number of drop-offs at bin \( b \) in period \( t \).
- \( y_{ob} \): number of drop-offs at plant \( p \) in period \( t \).
- \( y_{oi} \): number of drop-offs at customer \( i \in I_o \) in period \( t \) by trucks originating at \( o \).
- \( z_{mn} \): number of vehicles traveling from location \( m \in V \) to location \( n \in V \).
- \( x_{d} \): number of trucks leaving DC \( d \) in period \( t \).
- \( x_{ob} \): number of trucks leaving bin \( b \) in period \( t \).
Additional Notation Related to Subproblems

\( K_{dt} \): route-based setup cost at DD customer \( i \in I_o \) in period \( t \).

\( K_{dt} \): route-based setup cost at DC \( d \) in period \( t \).

\( K_{bt} \): route-based setup cost at bin \( b \) in period \( t \).

\( K_{pt} \): route-based setup cost at plant \( p \) in period \( t \).

\( w_{pt} \): binary indicator of route-based setup at plant \( p \) in period \( t \).

\( w_{bt} \): binary indicator of route-based setup at bin \( b \) in period \( t \).

\( w_{dt} \): binary indicator of route-based setup at DC \( d \) in period \( t \).

\( M \): a large number.

Model

We formulate the overall problem as a mixed-integer program as follows.

\[
\begin{align*}
\min & \sum_{t=1}^{T} \sum_{i \in I_o} \sum_{d \in D} s_{di}^i + \sum_{t=1}^{T} \sum_{i \in I_o} \sum_{b \in B} s_{bi}^b \\
& + \sum_{t=1}^{T} \sum_{d \in D} a_d x_{dt} + \sum_{t=1}^{T} \sum_{b \in B} a_b x_{bt} \\
& + \sum_{t=1}^{T} \sum_{i \in I_o} \sum_{d \in D} h_d I_{dt} + \sum_{t=1}^{T} \sum_{b \in B} h_b I_{bt} + \sum_{t=1}^{T} \sum_{i \in I_o} h_i I_{it} \\
& + A \sum_{i \in I_o} C_i + \sum_{t=1}^{T} \sum_{n \in V \cup V'} s_{mn} z_{mnt} \\
& + A \sum_{t=1}^{T} x_{dt} + \sum_{t=1}^{T} \sum_{b \in B} x_{bt} \\
& + \sum_{t=1}^{T} \sum_{i \in I_o} s_{di}^i + \sum_{t=1}^{T} \sum_{i \in I_o} s_{bi}^b \\
& \; \text{subject to} \; D_{dt} = \sum_{d \in D} s_{di}^i + \sum_{b \in B} s_{bi}^b + s_{a_{ij}} \; \forall i \in I_o, \forall j \in J, \forall t \\
& \; \text{(Demand),} \; \text{(1)} \\
& D_{bt} = \sum_{d \in D} s_{di}^i + \sum_{b \in B} s_{bi}^b \; \forall i \in I_o, \forall j \in J, \forall t \\
& \; \text{(Demand),} \; \text{(2)} \\
& \sum_{i \in I_o} s_{di}^i = I_{dt} + R_{dt} - I_{dt(t+1)} \; \forall d \in D, \forall j \in J, \forall t \\
& \; \text{(Inv. bal. at DCs),} \; \text{(3)} \\
& \sum_{i \in I_o} s_{bi}^b = I_{bt} + R_{bt} - I_{bt(t+1)} \; \forall b \in B, \forall j \in J, \forall t \\
& \; \text{(Inv. bal. at bins),} \; \text{(4)} \\
& \sum_{j \in J} I_{dt} \leq d_{cap} \; \forall d \in D, \forall t \\
& \; \text{(Storage cap. at DCs),} \; \text{(5)} \\
& \sum_{j \in J} I_{bt} \leq b_{cap} \; \forall b \in B, \forall t \\
& \; \text{(Storage cap. at bins),} \; \text{(6)} \\
& \sum_{i \in I_o} s_{ajt}^i + \sum_{d \in D} R_{dtj} + \sum_{b \in B} R_{btj} + \sum_{p \in P} R_{ptj} = R_{ajt} \; \forall j \in J, \forall t \\
& \; \text{(Load balance),} \; \text{(7)} \\
& R_{ajt} = I_{ajt} + P_{jt} - I_{aj(t+1)} \; \forall j \in J, \forall t \\
& \; \text{(Inv. bal. at o),} \; \text{(8)} \\
& \sum_{j \in J} I_{ajt} \leq o_{cap} \; \forall t \\
& \; \text{(Storage cap. at o),} \; \text{(9)} \\
& \sum_{j \in J} P_{jt} \leq p_{cap} \; \forall t \\
& \; \text{(Aggregate prod. cap.),} \; \text{(10)} \\
& \sum_{j \in J} \sum_{i \in I_o} s_{ajt}^i \leq l_{cap} \; \forall d \in D, \forall t \\
& \; \text{(Dispatch cap. for DCs),} \; \text{(11)} \\
& \sum_{j \in J} \sum_{b \in B} s_{bi}^b \leq l_{cap} \; \forall b \in B, \forall t \\
& \; \text{(Dispatch cap. for bins),} \; \text{(12)} \\
& C_t \geq \sum_{j \in J} R_{ajt} \; \forall t \\
& \; \text{(Total load and truck volume),} \; \text{(14)} \\
& y_{dt} \geq \sum_{i \in I_o} R_{dit} \; \forall d \in D, \forall t \\
& \; \text{(Drop-offs at DCs),} \; \text{(15)} \\
& y_{bt} \geq \sum_{b \in B} R_{bit} \; \forall b \in B, \forall t \\
& \; \text{(Drop-offs at bins),} \; \text{(16)} \\
& y_{pt} \geq \sum_{p \in P} R_{ptj} \; \forall p \in P, \forall t \\
& \; \text{(Drop-offs at other plants),} \; \text{(17)} \\
& y_{it} \geq \sum_{i \in I_o} s_{ajt}^i \; \forall i \in I_o, \forall t \\
& \; \text{(Drop-offs at DD customers),} \; \text{(18)} \\
& \sum_{n \in V} z_{mnt} = y_{nt} \; \forall n \in V \setminus \{o\}, \forall t \\
& \; \text{(Arrivals to Tier 2),} \; \text{(19)} \\
& \sum_{m \in V} z_{mnt} = y_{nt} \; \forall m \in V \setminus \{o\}, \forall t \\
& \; \text{(Departures from Tier 2),} \; \text{(20)}
\]
\[ \sum_{m \in V} z_{mot} = C_i \quad \forall t \quad \text{(Arrivals to o),} \]  
(21)

\[ \sum_{m \in V} z_{out} = C_i \quad \forall t \quad \text{(Departures from o),} \]  
(22)

\[ \sum_{m \in S \in S} z_{mat} \geq \frac{\sum_{m \in S} \sum_{j \in \ell} R_{mj}}{tcap} \quad \forall S \subseteq V \setminus \{o\}, \forall t \]  
(Route-based truck volume). \( (23) \)

Nonnegative Integer Variables

\[ S_{dijt} \quad \forall d \in D, \, i \in I, \, j \in J, \, \forall t, \]  
(24)

\[ S_{bijt} \quad \forall b \in B, \, i \in I, \, j \in J, \, \forall t, \]  
(25)

\[ S_{iwt} \quad \forall i \in I, \, j \in J, \, \forall t, \]  
(26)

\[ R_{djt}, I_{djt}, I_{ijt} \quad \forall d \in D, \, b \in B, \, j \in J, \, \forall t, \]  
(27)

\[ R_{ojt}, P_{jt} \quad \forall j \in J, \, \forall t, \]  
(28)

\[ x_{dt} \quad \forall d \in D, \, \forall t, \]  
(29)

\[ x_{jt} \quad \forall b \in B, \, \forall t, \]  
(30)

\[ C_t \quad \forall t, \]  
(31)

\[ y_{dt} \quad \forall d \in D, \, \forall t, \]  
(32)

\[ y_{jt} \quad \forall b \in B, \, \forall t, \]  
(33)

\[ z_{mnt} \quad \forall m \in V, \, n \in V, \, \forall t. \]  
(34)

Obtaining an exact optimal solution is impractical for the overall model because of its size and complexity. We overcome this computational difficulty by developing a heuristic that builds on the decomposition of the above model into two subproblems involving inventory and routing components. We first describe these subproblems and then present a flowchart of our iterative heuristic.

Routing Subproblem

Input parameters are the inventory-related decision variables \( S_{dijt}, R_{djt}, R_{bijt}, \) and \( R_{ojt} \) and input parameters \( A, g_{unt}, R_{pjt}, \) and \( tcap \). The output includes (1) routing-related decision variables \( C_t, y_{dt}, y_{jt}, y_{pjt}, \) and \( z_{mnt} \), and (2) setup costs \( K_{djt}, K_{bijt}, \) and \( K_{pjt} \).

\[
\min \sum_{t=1}^{T} A + \sum_{t=1}^{T} \sum_{m \in V \setminus \{o\}} g_{mat} \quad \text{subject to constraints (14)–(23) with variables C_t, y_{dt}, y_{jt}, y_{pjt}, and z_{mnt} as defined above.}
\]

When we obtain a solution, we calculate the route-based setup costs of the delivery locations on the resulting routes. For a given delivery location \( t \in V \setminus \{o\} \), let \( RouteSet_t \) denote the set of routes to which delivery location \( t \) belongs in period \( t \). Also, let \( r_{sjt} \) denote the number of delivery locations on route \( t \in RouteSet_t \). Then, the route-based setup cost of delivery location \( t \) in period \( t \) is

\[
K_{jt} = \sum_{i \in RouteSet_t} A + \sum_{i \in RouteSet_t} \sum_{m \in V \setminus \{o\}} g_{mat} r_{sjt}.
\]

We explicitly consider it in the subsequent inventory subproblem.

Inventory Subproblem

Input parameters include the route-based setup costs denoted by \( K_{djt}, K_{bijt}, \) and \( K_{pjt} \) for \( i \in I, d \in D, b \in B, p \in P, \) and \( t = 1, \ldots, T \). We consider these costs and inventory-related cost parameters \( s_j, h_{dj}, h_{bij}, \) and \( h_{pjt} \) loading-related cost and capacity parameters \( a_d, a_b, tcap_d, \) and \( tcap_b \) customer and interplant demand parameters \( D_{djt}, R_{pjt}, \) and \( R_{pjt} \) and initial and end-of-horizon inventory levels at the inventory-keeping locations \( I_{djt}, I_{bjt}, I_{djt}, I_{djt(T_1)}, I_{djt(T_2)}, \) and \( I_{djt(T_3)} \). The output includes (1) all inventory-related decision variables \( S_{dijt}, S_{bijt}, S_{dijt}, R_{djt}, R_{bijt}, R_{pjt}, \) and \( R_{pjt} \); (2) the number of trucks required at DCs, \( x_{dt}, \) and bins, \( x_{jt}, \) for customer deliveries; and (3) binary route-based setup indicators \( w_{dt}, w_{jt}, \) and \( w_{pjt} \).

\[
\min \sum_{t=1}^{T} \sum_{j \in J} \sum_{d \in D} s_j S_{dijt} + \sum_{t=1}^{T} \sum_{j \in J} \sum_{b \in B} s_j S_{bijt} + \sum_{t=1}^{T} \sum_{d \in D} a_d x_{dt} + \sum_{t=1}^{T} \sum_{b \in B} a_b y_{jt} + \sum_{t=1}^{T} \sum_{j \in J} h_{dj} I_{djt} + \sum_{t=1}^{T} \sum_{j \in J} h_{bj} I_{bjt} + \sum_{t=1}^{T} \sum_{j \in J} K_{djt} w_{dt} + \sum_{t=1}^{T} \sum_{j \in J} K_{bijt} w_{jt} + \sum_{t=1}^{T} \sum_{j \in J} K_{pjt} w_{pjt}
\]

subject to constraints (14)–(34), and

\[
\frac{\sum_{j \in J} S_{dijt}}{M} \leq w_{dt} \quad \forall i \in I, \forall t
\]
(Route-based setup at DD customers), \( (24) \)

\[
\frac{\sum_{j \in J} R_{djt}}{M} \leq w_{dt} \quad \forall d \in D, \forall t
\]
(Route-based setup at DCs), \( (25) \)

\[
\frac{\sum_{j \in J} R_{pjt}}{M} \leq w_{pjt} \quad \forall b \in B, \forall t
\]
(Route-based setup at bins), \( (26) \)

\[
\frac{\sum_{j \in J} R_{pjt}}{M} \leq w_{pjt} \quad \forall p \in P, \forall t
\]
(Route-based setup at plants), \( (27) \)

with variables \( S_{dijt}, S_{bijt}, S_{dijt}, R_{djt}, R_{bijt}, x_{dt}, \) and \( x_{jt} \) as defined above and binary integer variables \( w_{dt}, w_{jt}, \) and \( w_{pjt} \).
Solution Algorithm

The algorithm involves iteratively solving the routing and inventory subproblems described above, which we outline in Figure 8.

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