New Quantum Inspired Meta-heuristic Methods for Multi-level Thresholding

Sandip Dey  
Department of Information Technology  
Camellia Institute of Technology  
Madhyamgram,Kolkata-700129, India  
Email: dsandip_vc@yahoo.com

Indrajit Saha and Ujjwal Maulik  
Department of Computer Science & Engineering, Jadavpur University  
Kolkata-700032, India  
Email: indra@icm.edu.pl and umaulik@cse.jdvu.ac.in

Siddhartha Bhattacharyya  
Department of Information Technology  
RCC Institute of Information Technology  
Beliaghat, Kolkata-700015, India  
Email: dr.siddhartha.bhattacharyya@gmail.com

Abstract—Thresholding is a simple, effective and popular method for image segmentation. It can be bi-level or multi-level depending on number of segments in an image. Multi-level thresholding computationally takes more time than the bi-level thresholding. To reduce the computational complexity, here we propose two quantum inspired meta-heuristic methods, namely Quantum Inspired Ant Colony Optimization and Quantum Inspired Simulated Annealing for multi-level thresholding. The basic quantum principles are coalesced with meta-heuristic approaches to design the proposed methods. The performance of the proposed methods is demonstrated in comparison with its conventional versions for two test images in terms of optimal threshold values at different levels with the fitness measure, standard deviation of the fitness measure and the computational time. It has been noticed that the Quantum Inspired meta-heuristic methods are superior in terms of computational time compare to the other methods. Finally, statistical significance test, called t-test, has performed to establish the superiority of the results.

Keywords—Image segmentation, multilevel thresholding, otsu’s function, statistical test.

I. INTRODUCTION

As far as the segregation of foreground and background information of an image is concerned, thresholding can be used as a most effective tool in this context. The pixel values of an image can be grouped into different classes for bi-level and multi-level image thresholding [1]. In soft computing paradigm, two important approaches are available to determine image thresholding. These include a deterministic analysis of image intensity distribution and heuristic search using different optimization techniques. The details of image thresholding and its significant applications can be found in [2].

Of late, the classical algorithms are combined with the features of quantum computing to speed-up its execution time [3]. The features of quantum computing are some physical phenomena like interference, entanglement, superposition and so on, which give some advantage to make the classical computing much faster [4]. A typical application of such quantum inspired algorithm for searching databases is mentioned in [5].

Meta-heuristic techniques are stochastic in nature, specifically designed to solve the optimization problems by covering large search space to find optimum solutions. It has been used as an alternative approach for underlying optimization problem of multilevel thresholding. Moreover, meta-heuristic algorithms have also been used in wide range of various applications. Some of its typical applications are depicted in [6]–[11]. Meta-heuristic algorithms are distinguishable based on execution time that needs to find the global optimum. These methods can be accelerated by adding some additional components. Hence, we apply the basic quantum principles in meta-heuristic techniques to propose quantum inspired meta-heuristic algorithms, namely Quantum Inspired Ant Colony Optimization (QIACO) and Quantum Inspired Simulated Annealing (QISA) for solving the underlying optimization problem of multi-level thresholding. The proposed algorithms are used the quantum computing features as qubits encoded form with the use of objective called Otsu’s function [12]. The comparative study of the proposed algorithms has been conducted in comparison with conventional Ant Colony Optimization technique [13] and Simulated Annealing [14]. Finally, the superiority of the proposed methods has been established by statistical significance test, called t-test [15].

II. FUNDAMENTALS OF QUANTUM COMPUTING

Quantum computing (QC) can be described by exploiting the doctrine of the quantum physics theory [4]. A wave function, \(|\psi>\), that is present in Hilbert space, is indispensable to describe quantum systems. A quantum system encompasses a set of states \(\phi_j\) as given by

\[
|\psi> = \sum_j c_j |\phi_j>
\] (1)

where, \(|\psi>\) is recognised as the linear superposition of the basis states, \(\phi_j\) and \(c_j\) are complex numbers that follow the unitary condition given by

\[
\sum_j c_j = 1
\] (2)

Coherence of QC is considered while the participating states are in superposed form and preserves a constant phase relationship. Alike to wave theory, decoherence will be in effect while the phase relationship comes to an end or the superposed form of the basis states is demolished. \(c_j\) is the required probability of collapsing into state \(|\phi_j>\) if it decoheres [16].

III. IMAGE THRESHOLDING: OPTIMIZATION PROBLEM

Image thresholding appears to be an optimization problem where the objectives like Otsu’s function [12] and Kapur’s
function [17], can be optimized to get the proper number of threshold. Here the proposed quantum inspired meta-heuristic algorithms are used otsu’s function [12] as an objective function for underlying optimization problem of multi-level image thresholding. Otsu’s function is a cluster based approach. It finds a set of optimal thresholds \( \{ \theta_1, \theta_2, \cdots, \theta_{K-1} \} \) that maximize the variance of classes. Formally, it can be defined as follows [12].

\[
F = f(\{\theta_1, \theta_2, \cdots, \theta_{K-1}\}) = \sum_{j=1}^{K} \omega_j (\mu_j - \mu)
\]

where \( K \) is the number of classes in \( C = \{ C_1, C_2, \cdots, C_K \} \) and

\[
\omega_j = \sum_{i \in C_j} p_i, \quad p_i = n_i / N, \quad \mu_j = \sum_{i \in C_j} ip_i / \omega_j
\]

where, \( n_i \) is the frequency of a pixel at the \( i \)th intensity level and \( p_i \) is the probability of \( i \)th pixel, whereas \( N \) represents total number of pixels in an image. \( \omega_j \) and \( \mu_j \) represent the probability and the mean of class \( C_j \), respectively, while \( \mu \) is the mean of class \( C \). Note that the maximization of \( F \) will provide the number of thresholds for an image.

IV. QUANTUM INSPIRED META-HEURISTIC METHODS FOR MULTI-LEVEL THRESHOLDING

The proposed quantum inspired meta-heuristic algorithms for multi-level thresholding are described in this section. The basic quantum principles are coalesced with two meta-heuristic algorithms to design the proposed algorithms.

A. Quantum Inspired Ant Colony Optimization for Multi-level Thresholding

The motivation of real ants struggling to get food helped to acquire the concept of Ant Colony Optimization (ACO). In 1996, Dorigo et al. invented ACO [13]. The aim of this meta-heuristic optimization technique was to solve the optimization problem. The real ants look for foods in different sites in order to find the shortest path, they spurt pheromone in their way to the exchange information with one another about the shortest path to be followed. The act of real ants motivated many researchers to develop algorithms to solve the combinatorial optimization problems in real life [18].

In Quantum Inspired Ant Colony Optimization (QIACO) for multi-level thresholding, a population \( \text{POP} \), having \( Z \) number of initial strings, is generated by selecting pixel intensities randomly. The length of each string in \( \text{POP} \) is \( \mathcal{L} = \sqrt{L_1 \times L_2} \), where \( L_1 \) and \( L_2 \) represents the width and height of an image. A real random number between \( (0,1) \) is assigned to each pixel encoded in \( \text{POP} \) by exploring the concept of qubits and then quantum interference [3] is used to produce \( \text{POP}' \) and \( \text{POP}'' \), respectively. In this algorithm, user defined number of thresholds as pixel intensities are selected from \( \text{POP} \) based on a condition. It instigates the participating ants to get the best search path in each generation. A pheromone matrix, \( \tau_{ij} \) is produced for each ant \( j \) by assigning random real value between \( [0,1] \) at the initial stage. For each individual \( j \) in \( \text{POP}' \), if \( \text{POP}''(ij) > q_0 \), where \( q_0 \) is the priority defined number between \( [0,1] \), the maximum pheromone assimilation is presumed as threshold value for the gray value, i.e., \( \text{POP}''(ij) = \arg \max \{ \tau_{ij} \} \); Otherwise, \( \text{POP}''(ij) = \text{rand}(0,1) \). At the end of each generation, the pheromone matrix is updated using \( \tau_{ij} = \rho \tau_{ij} + (1-\rho)\alpha \), where \( i \) and \( j \) represent the particular string and its corresponding position. \( \alpha \) is the best string of the generations and \( \rho \) is the persistence of trials, \( \rho \in [0,1] \). QIACO is executed for \( \text{MaxGen} \) number of generations. \( K \) is the number of thresholds used as user input. \( Z \) and \( \theta \) represent the size of the population and output as thresholds, respectively. The steps of the proposed QIACO are described in Algorithm 1.

1) Complexity analysis: The worst case time complexity of the proposed QIACO is discussed below.

- For generation of initial strings in QIACO, the complexity is \( O(Z \times \mathcal{L}) \), where \( Z \) is the population size in QIACO. It should be noted that length of each of chromosome is \( \mathcal{L} = \sqrt{L_1 \times L_2} \) where, \( L_1 \) and \( L_2 \) are the width and height of an image, respectively.
- For the assignment of real value to each pixel encoded as population of strings, the complexity becomes \( O(Z \times \mathcal{L}) \).
- For performing the quantum interference in QIACO, the complexity turn into \( O(Z \times \mathcal{L}) \).
- The time complexity to generate \( \text{POP}'' \) is \( O(Z \times \mathcal{L}) \).
- The time complexity for the fitness computation in QIACO is \( O(Z \times K) \).
- To determine \( \text{POP}''' \) from \( \tau \), the time complexity at each generation is \( O(Z \times \mathcal{L}) \).
- To build the pheromone matrix, \( \tau \), the complexity is \( O(Z \times \mathcal{L}) \).
- The pheromone matrix is updated at each generation with the time complexity \( O(Z) \).
- The time complexity to execute for a predefined number of generations in QIACO is \( O(Z \times \mathcal{L} \times \text{MaxGen}) \), where \( \text{MaxGen} \) represents the number of generations.

Therefore, it can be concluded that the overall worst case time complexity for the proposed QIACO for multi-level thresholding is \( O(Z \times \mathcal{L} \times \text{MaxGen}) \).

B. Quantum Inspired Simulated Annealing for Multi-level Thresholding

Simulated annealing (SA) is known as one of most important meta-heuristic optimization technique. The concept of SA is to heat a physical substance and then cool it slowly. It can help to avoid stuck at local optimum problem. A new quantum inspired simulated annealing (QISA) for multi-level thresholding is built by coupling the basic principles of QC with SA.

At the first stage, the pixel intensities randomly selected to produce the initial configuration, \( P \). The length of the generated configuration is \( \mathcal{L} = \sqrt{L_1 \times L_2} \), where, \( L_1 \) and \( L_2 \) are the width and height of an image. The random real value between \( [0,1] \) is assigned to each pixel encoded in \( P \) with the notion of qubits which later used for quantum interference to produce \( P' \) and \( P'' \), respectively. Using the condition, user defined number of thresholds as pixel intensities are selected from the configuration \( P \) to generate \( P'' \). Subsequently, a temperature parameter, \( T \), is initialised with a very high temperature, \( T_{max} \). At each temperature, QISA runs for \( I \)
number of times and it stops executing until and unless $T$ reaches to the final temperature, $T_{\text{fin}}$. During the each run of $T$, it aims to find a better configuration ($S$) by perturbing the multiple points in the configuration ($P$) randomly. The new configuration $S$ undergoes the similar process, as used for $P^*$ from $P$, to generated the configuration $S^*$. The $T$ decreases by a reduction factor $\rho$ at the end of each iteration. The new configurations $S^*$ and $S$ are accepted, if $F(S^*) > F(P^*)$, by updating the old configurations $P^*$ and $P$, respectively; otherwise, it may accept the newly generated configurations with a probability $\exp(-F(P^*) - F(S^*)) / T$. The value of $\rho$ is selected within the range $[0.5, 0.99]$. $K$ represents user defined number of thresholds and $\theta$ signifies output as thresholds. The steps of the proposed QISA for multi-level thresholding are shown in Algorithm 2.

1) Complexity analysis: The worst case time complexity for the proposed QISA is described below.

- The time complexity for the initial configuration in QISA is $O(L)$, where length of configuration is $L = L_1 \times L_2$ where, $L_1$ and $L_2$ represent the width and height of an image respectively.
- To assign real value to each pixel encoded in the population of configuration, the time complexity is $O(L)$.
- The time complexity for quantum interference is $O(L)$.

### Algorithm 2: Steps of QISA for multi-level thresholding

**Input:** Initial temperature: $T_{\text{max}}$

**Final temperature: $T_{\text{min}}$**

**Number of Iterations: $I$**

**Reduction fraction: $\rho$**

**Output:** Optimal threshold values: $\theta$

1. An initial configuration, $P$ is produced by selecting pixel intensity values randomly, where length of the configuration is $L = L_1 \times L_2$, where, $L_1$ and $L_2$ represent the width and height of an image.
2. The concept of qubits is applied to assign real value between (0,1) to each pixel encoded in $P$. Let it produces $P'$.
3. The $P'$ endures an quantum interference to create $P^{**}$
4. Finding $K$ number of thresholds as pixel intensities from the configuration $P$ after satisfying corresponding value in $P^{**} > rand(0, 1)$. Let it produces $P^*$.
5. Compute fitness of the configuration $P^*$ using equation (3). Let it denotes as $F(P^*)$.
6. $T = T_{\text{max}}$
7. while $T > T_{\text{min}}$, do
   8. for $i = 1$ to $I$ do
      10. Repeat step (2), (3) and (4) to produce $S^*$.
      11. Compute fitness $E(S^*, T)$ of the configuration $S^*$ using equation (3).
      12. if $(F(S^*) - F(P^*)) > 0$ then
         13. Set $P^* = S^*$, $P = S$ and $F(P^*) = F(S^*)$.
         14. else
            15. Set $P^* = S^*$, $P = S$ and $F(P^*) = F(S^*)$ with probability $\exp(-F(P^*) - F(S^*)) / T$.
      16. end if
   17. end for
   18. $T = T \times r$.
   19. end while
20. Threshold values $\theta = P^*$ are reported.

- The time complexity to generate $P^*$ in QISA is $O(L)$.
- For fitness computation in QISA, the time complexity becomes $O(K)$.
- Similarly, for evaluating fitness of the configuration after perturbing in QISA, the time complexity turns into $O(K)$.
- Let the outer loop executes $MaxIntr$ number of times, while the inner loop executes $I$ times. Hence, the time complexity for this step in QISA is $(I \times MaxIntr)$

Therefore, aggregating the above steps, the overall worst case time complexity for the proposed QISA for multi-level thresholding is $O(L \times I \times MaxIntr)$

<table>
<thead>
<tr>
<th>QIACO</th>
<th>QISA</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Generation: MaxGen = 100</td>
<td>Initial temperature: $T_{\text{max}}$ = 100</td>
</tr>
<tr>
<td>Population size: $N = 50$</td>
<td>Final temperature: $T_{\text{min}}$ = 0.5</td>
</tr>
<tr>
<td>Priori constant: $q_0 = 0.5$</td>
<td>Number of Iterations: $I = 50$</td>
</tr>
<tr>
<td>Persistence of trials: $\rho = 0.2$</td>
<td>Reduction factors: $r = 0.9$</td>
</tr>
<tr>
<td>No. of thresholds: $K = 2, 3, 4, 5$</td>
<td>No. of thresholds: $K = 2, 3, 4, 5$</td>
</tr>
</tbody>
</table>

**V. EXPERIMENTAL RESULTS**

The proposed Quantum Inspired Ant Colony Optimization and Quantum Inspired Simulated Annealing (QISA) for multi-level thresholding has been tested on two gray scale images by maximizing an objective function $F$. The best fitness along
TABLE II. BEST RESULTS OF QIACO, ACO, QISA AND SA FOR MULTI-LEVEL THRESHOLDING

<table>
<thead>
<tr>
<th>K</th>
<th>B2</th>
<th>QIACO</th>
<th>ACO</th>
<th>QISA</th>
<th>SA</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>θ</td>
<td>F&lt;sub&gt;best&lt;/sub&gt;</td>
<td>t</td>
<td>θ</td>
<td>F&lt;sub&gt;best&lt;/sub&gt;</td>
</tr>
<tr>
<td>2</td>
<td>128</td>
<td>3108.569</td>
<td>17.17</td>
<td>128</td>
<td>3108.569</td>
</tr>
<tr>
<td>3</td>
<td>93,175</td>
<td>3612.926</td>
<td>17.23</td>
<td>93,175</td>
<td>3612.926</td>
</tr>
<tr>
<td>5</td>
<td>73,138,181,218</td>
<td>3838.779</td>
<td>18.73</td>
<td>70,142,180,215</td>
<td>3836.817</td>
</tr>
</tbody>
</table>

Table 2.1: Best results of QIACO, ACO, QISA and SA for multi-level thresholding

Fig. 1. Original test images (a) B2 and (b) Barbara.

Fig. 2. Images for $K = 2, 3, 4, 5$, in (a), (c), (e) and (g), for B2 and, (b), (d), (f), and (h), for Barbara, respectively, after using QIACO for multi-level thresholding.

Each method has been executed 20 times for four different levels of thresholds. The best results of each method are reported in Table II, which contains the number of classes ($K$), optimal threshold values ($θ$), the fitness value ($F_{best}$), and the corresponding execution time ($t$) (in seconds). Moreover, the average fitness ($F_{avg}$) and standard deviation ($σ$) over 20 runs for each method are reported in Table III. A two-tailed $t$-test were conducted at 5% confidence level to see whether the $p$-value less than 0.05 or not for alternative hypothesis. The results of $t$-test among two methods are reported in Table IV.

For $K = 2$, QIACO, ACO, QISA and SA are found optimal threshold values 128 and 115 for B2 and Barbara images, respectively. The average fitness ($F_{avg}$) and standard deviation values for all the methods are same. Hence, the $t$-test is unable to reveal a method that performs better among themselves. Similar result is also obtained for $K = 3$. However, for both the images, little change is observed in standard deviation values for QIACO and ACO. For $K = 4$ and 5, optimal threshold values, average fitness and standard deviation values are different for all the methods. Among them, results of QIACO are superior to others for both the images. This fact also established by statistical test shown in Table IV. The tests have been performed on Toshiba Intel(R) Core (TM) i3, 2.53GHz PC with 2GB RAM. The execution time for each method is reported in Table II. It can be noted that QIACO takes least time among all the methods. The images of QIACO after thresholding are depicted in Fig. 2 for each level. As the thresholds of QIACO and QISA are same or close to each other, one set of images are shown in Fig. 2.
VI. CONCLUSION

In this paper, Quantum Inspired Ant Colony Optimization (QIACO) and Quantum Inspired Simulated Annealing (QISA) for multi-level thresholding are developed. These methods are able to determine optimal threshold values from gray level images. Here for this purpose, otsu’s function is employed in the developed methods as an objective function. Experiments have been demonstrated by computing optimal threshold values at different level, fitness values, standard deviation and the computational time for each method. Statistical significance test, known as t-test, has been conducted to establish the superiority of the results. It has been noticed that for higher level of thresholding, the efficiency differs and the proposed quantum algorithms are shown more effective than the conventional algorithms. Moreover, QIACO is better than other when execution time is concerned.

As a scope of further research, multiobjective version of differential evolution [19] can be studied for multi-level thresholding using conventional and quantum forms. Moreover, edge detection using thresholding may also be investigated [20][21]. Authors are currently working in this direction.

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