SPATIAL PROCESSING TECHNIQUE ADAPTIVE BEAMFORMING (SPTABF) VIA COMPACTLY SUPPORTED ORTHONORMAL WAVELETS

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ABSTRACT

In this paper, we propose a new adaptive beamformer which uses the Generalized Sidolobe Canceller (GSC) as the underlying structure. However, unlike the traditional adaptive beamformer, which uses the time-domain least mean squares (LMS) algorithm, the new one employs the wavelet-based LMS as the adaptive scheme for adjusting the system weights. The latter, as demonstrated in some recent literatures, can indeed yield faster convergence rate as opposed to the time-domain counterpart. In addition, the new beamformer has also incorporated the spatial smoothing technique so that it can handle the case of coherent interference. The resulting adaptive beamformer, thus, admits efficient hardware implementations as the GSC. It as well exhibits fast tracking capability, and, at the same time, works well for both the noncoherent and coherent interference. Simulation results are provided to verify this new structure.

1. INTRODUCTION

The design of beamformers is of importance in many signal processing applications such as radar, sonar and geophysical explorations [1]. A beamformer is a processor which can form the desired beampattern via spatial filtering the data collected by the sensors. In consideration of the change of the outside environment, the system weights must be updated in an appropriate manner in order to effectively suppress the interference. Thus, the crux of the whole adaptive beamformers lies in the ways of processing the received spatial data, and, most importantly, the ways of adjusting the system weights which are preferably more amenable to practical implementations and provide fast tracking capability under varying environment.

In this paper, we consider a new adaptive beamformer which possesses several desired features. First, the proposed beamformer uses the GSC [2] as the underlying structure so that it, as the GSC, admits efficient hardware implementations. Second, in order to account for the coherent interference, the spatial smoothing technique [3, 4] has also been incorporated into the new structure. Third, the employed adaptive scheme to update the weights of the system is the wavelet-based LMS algorithm. This is in contrast to the traditional adaptive beamformers, which perform the LMS algorithm based on the time-domain input (spatial) data. It is well-known that the time-domain LMS algorithm, although computationally efficient, is notorious for its slow convergence rate, especially for colored signals with a large eigenvalue spread [5]. An approach to overcome this difficulty is to pre-whiten the input signals by transforming them into another domain using transforms such as the DFT and DCT. These transforms, however, are not optimal from the statistical point of view.

The adopted wavelet transform has recently received much attention [6]. This new transform works effectively in analyzing the high and low frequency components of signals, as it forms a frequency-adaptive window on the time-scale plane. In fact, it has been shown that the wavelet transform behaves similarly to that of the statistically optimal Karhunen-Loeve transformation for a large class of stochastic processes [5]. As a result, the eigenvalues of the transformed correlation matrix in general will not spread overly wide, leading to a faster convergence speed of the LMS algorithm. Some simulations reported in recent literatures [7, 8, 9, 10] have demonstrated the effectiveness of this method.

Hence, the resulting WAvelet-based Spatial Processing Adaptive Beamformer (WASPAB) not only renders efficient hardware implementations, but it also exhibits the characteristic of fast convergence. Furthermore, it can also handle the case of coherent interference. The provided simulation results justify this new approach.

2. WAVELET-BASED LMS ALGORITHM

Since the proposed adaptive beamformer is to first transform the time-domain input signal using the wavelet transform, and then utilize the LMS algorithm. In this section, we briefly review the wavelet-based LMS algorithm.

The wavelet transform is a new type of transform, the basis function of which can be generated by the dilation and translation of a prototype (“mother”) wavelet function ψ(t). Using the discrete wavelet transform, any square integrable function f(t) can be expressed as the superposition of the basis functions as [5, 6]

$$f(t) = \sum_{j,k} 2^{j/2} f_{j,k} \psi(2^j t - k),$$  \hspace{1cm} (1)

where j and k denote the scaling and translation parameters.
eters, respectively. Here, we consider the compactly supported wavelets proposed by Daubechies [6], which also form an orthonormal basis.

Let \( x(n) = [x(n), \ldots , x(n - (N - 1))]^T \) and \( v(n) = [v(n), \ldots , v(n - (N - 1))]^T \) be the input and transformed data vectors, respectively, where the superscript \(^T\) denotes the matrix transposition and \( N \) is the transformed data length, then the wavelet transform can be expressed in the matrix notation as

\[
v(n) = Qx(n),
\]

(2)

where \( Q \) is a discrete wavelet transform matrix, constituted by the Daubechies wavelet coefficients [5]. Note that in practice the transform of (2) can be efficiently implemented via the filter bank structure [6].

The resulting \( w_{opt} \) of the wavelet-based LMS algorithm after convergence is equal to that derived by the Wiener-Hopf equation [9], i.e.

\[
w_{opt} = R_v^{-1}p_v;
\]

(3)

where

\[
R_v = E[v(n)v^*(n)]
\]

indicates the auto-correlation matrix of the input signal in the wavelet domain, the superscript \(^*\) denotes the complex conjugation;

\[
p_v = E[d(n)v(n)]
\]

indicates the cross-correlation matrix of ideal output \( d(n) \) and input signals in the wavelet domain.

Using (2), it is easy to verify that

\[
R_v = QE[x(n)x^*(n)]Q^* = QR_vQ^*;
\]

(4)

\[
p_v = QE[d(n)x(n)] = Qp_d.
\]

(5)

Also, the minimum mean square error is

\[
\epsilon_{min} \triangleq E[\tilde{d}(n)\tilde{d}(n) - p_v^*R_v^{-1}p_v] = E[\tilde{d}(n)\tilde{d}(n)] - p_v^*R_v^{-1}p_v = \epsilon_{min},
\]

(6)

where we have used (4), (5), and the fact that \( Q \) is a unitary matrix. Since the minimum mean square error remains the same after the wavelet transformation, this approach is justified.

The adjustment of the weight vector is according to

\[
w(n + 1) = w(n) + \mu v(n)\tilde{e}(n);
\]

(7)

where \( \tilde{e}(n) = d(n) - d(n) \) and the range of the convergence factor \( \mu \) is

\[
0 < \mu < \frac{2}{\text{trace} R_v}.
\]

(8)

3. THE PROPOSED WASPAB

The proposed adaptive beamformer uses the GSC proposed by Griffiths and Jim as the underlying structure. The GSC reformulates the linear constraint minimum variance (LCMV) schemes considered by Frost [2, 11], resulting in a more efficient hardware implementation. In particular, the GSC is more amenable to the inclusion of adaptive schemes. However, as mentioned above, the time-domain based LMS algorithm in general can not yield satisfactory convergence speed. Hence, the proposed beamformer employs the wavelet-based LMS algorithm addressed in Section 2. Now, suppose that there are \( M \) omnidirectional sensors in a linear array and each spatial filter has \( K \) taps. The input data vector can be expressed as the following \( MK \times 1 \) vector:

\[
\hat{u}(n) = \begin{bmatrix}
    u(n) \\
    u(n - 1) \\
    \vdots \\
    u(n - (K - 1))
\end{bmatrix};
\]

(9)

where \( u(n) = [u_1(n), u_2(n), \ldots , u_M(n)]^T \) denotes the received data by these \( M \) sensors at time \( n \).

The basic principle of the GSC is to decompose the weight vector into two components. The first component \( w_0 \), which stands for the fixed target signal filter of GSC. The second component is composed of a blocking matrix and the unconstrained adaptive weight part. Note that the noise and interference subspace can be represented as \( B^H\hat{u}(n) \), with the superscript \(^H\) denoting Hermitian transposition. The matrix \( B \) is constituted by a set of signal block matrix \( b \), which is of dimension \( M \times (M - 1) \). In general,

\[
b = \begin{bmatrix}
    1 & 0 & \cdots & 0 & 0 \\
    -1 & 1 & \cdots & 0 & 0 \\
    0 & -1 & \cdots & 0 & 0 \\
    \vdots & \vdots & \ddots & \vdots & \vdots \\
    0 & 0 & \cdots & -1 & 1 \\
    0 & 0 & \cdots & 0 & -1
\end{bmatrix};
\]

(10)

the block signal blocking matrix \( B \) can be shown to be

\[
B = \begin{bmatrix}
    b & 0 & \cdots & 0 \\
    0 & b & \ddots & \vdots \\
    \vdots & \ddots & \ddots & 0 \\
    0 & \cdots & 0 & b
\end{bmatrix},
\]

(11)

which \( B \) has size of \( MK \times (M - 1)K \). Such a choice of \( B \) results in a substraction of two temporal successive data samples in each sensor. Since the target signal is of the same phase, it is “blocked” by \( B \) and \( B^H\hat{u}(n) \) will contain only the noise and interference. After is further manipulation on \( B^H\hat{u}(n) \), we can get rid of the target signal of the original received signals.

Note that such separation of the constrained and unconstrained parts of the LCMV allows for an easy incorporation of an adaptive scheme. Since \( w_0 \) is data independent, we only need to consider the adaption for \( w(n) \). Let \( v(n) \) denote the wavelet transformation of the input data after passing through the unconstrained part, then \( v(n) \) can be expressed as

\[
v(n) = QB^H\hat{u}(n).
\]

(12)

Thus, the output of the proposed beamformer is

\[
y(n) = w_0^H\hat{u}(n) - w^H(n)QB^H\hat{u}(n)
\]

(13)
Let $w(n)$ be a $K(M - 1) \times 1$ adaptive weight vector, then $w(n)$ can be updated using the wavelet-based LMS as

$$w(n + 1) = w(n) + \mu v(n)y(n),$$

where $v(n)$ and $y(n)$ are as those of (12) and (13), respectively. When the condition of (8) is satisfied, $w(n)$ will converge to that of (3).

When the input interference is coherent with the target signal, the traditional beamformers not only cancel the undesired interference, but they also suppress the target signal. So we advance the spatial processing technique to effectively remove the coherent interference [3, 4]. To employ the spatial smoothing technique, signals collected by $N$ sensors are divided into $N$ groups, where $N$ is the number of subbeamformers in the GSC. Every subbeamformer employs the GSC along with the wavelet-based LMS algorithm discussed above. After the system weights of the first subbeamformer are being adjusted according to the adaptive scheme of (14), the new weights are copied to the second one, and then go on. Hence, the output signals can be obtained as the average of the outputs of these $N$ subbeamformers, thereby reducing the coherent interference effect. The overall block diagram of the proposed adaptive beamformer is shown in Fig. 1.

### 4. EXPERIMENTAL RESULTS

In this section, some simulations are provided to demonstrate the performance of the proposed algorithm.

**Example 1:**

In this example, we compare the convergence rate of the time-domain and two transform-domain (FFT-based and wavelet-based) LMS algorithms using an adaptive line enhancer. The simulations results are shown in Fig. 2 and from which we can observe that the wavelet-based LMS has the fastest convergence rate, while the time-domain LMS is the slowest one.

**Example 2:**

Consider a linear array with $N = 9$ omnidirectional sensors with an equal distance of half wavelength. Each spatial filter has 2 taps ($K = 2$). The incident angle of the far field complex input signal is 0 degree with a frequency of 200 Hz, while that of the (noncoherent) interference signal is 20 degrees with a frequency of 100 Hz. The spatial smoothing technique is not used here, i.e., $N = 1$. The block matrix $B$ used is that of (11), and the employed LMS algorithm has a constant step-size of $2 \times 10^{-8}$. The interference to noise ratio (INR) is 30 dB and the contaminated noise is Gaussian with SNR = 10 dB. The underwater wave speed is 1500 m/sec. The resulting array output beam patterns using the WASPAB and the traditional one are shown in Figs. 3 and 4, respectively. From the figures, we can find that the proposed WASPAB has better interference rejection capability than that of the traditional one. Also, it is observed that the former has faster convergence speed.

**Example 3:**

Consider the same array as that of Example 2 but with $M = 11$. The array is now divided into 3 groups ($N = 3$) and each of which contains 9 subarray sensors. The spatial filter in every subbeamformer has 16 taps ($K = 16$). The block matrix $B$ used is that of (11) and the employed LMS algorithm uses a constant step-size of $9 \times 10^{-11}$. The incident angle of far field complex input signal is 0 degree with a frequency of 100 Hz. Now there are two coherent interference signals, which have incident angles −45 degrees and 35 degrees, respectively. The INRs of both interference are equal to 30 dB. The contaminated noise is Gaussian with SNR = 10 dB. The underwater wave speed is 1500 m/sec.

The resulting beam patterns using the WASPAB and the traditional one (spatial smoothing technique has been employed) are shown in Figs. 5 and 6, respectively. We can observe that the WASPAB can effectively suppress the two coherent interferences, when compared with the other approach.

### 5. CONCLUSION

In this paper, we describe a new adaptive beamformer, WASPAB, which utilizes the GSC as the underlying architecture with the wavelet-based instead of the time-domain based LMS algorithm as the computational engine, leading to a faster adaption capability. Furthermore, the spatial processing technique has also been appropriately included so that even the coherent jamming can be effectively suppressed. As shown by simulations, this new structure performs well under various situations.

### REFERENCES


Figure 1. The block diagram of the proposed WASPAB

Figure 2. The learning curve of the Time, FFT & Wavelet-based LMS algorithms.

Figure 3. The beam pattern of example 2, using the proposed WASPAB.

Figure 4. The beam pattern of example 2, using the traditional GSC.

Figure 5. The beam pattern of example 3, using the proposed WASPAB.

Figure 6. The beam pattern of example 3, using the traditional GSC.