Modelling and performance analysis of networked control systems under different driven modes

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Abstract: In remote Networked Control Systems (NCSs) the system setup depends on the driven mode for each network node, such as time-driven or event-driven. In this paper, based on the NCS with a special kind of network delay, the general formulae of system models are deduced for different cases, in which the sensor adopts the time-driven mode, with the controller and the actuator using either the time-driven or the event-driven mode. The effects of different driven modes for network nodes on the system performance are studied and compared. The issue about how to select suitable driven modes for nodes is discussed with regard to different system requirements and delay ranges. The theoretical results are further illustrated and confirmed by a numerical example.

Keywords: NCS; networked control system; time-driven; event-driven.


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1 Introduction

Motivated by the increasing technological integration of computing, communication and control in modern industrial systems, more and more researchers have become interested in Networked Control Systems (NCSs) (Zhivoglyadov and Middleton, 2003; Vozdolsky and Djaferis, 2005; Burger and Frieder, 2006; Chang et al., 2006), which means feedback control systems with control loops via a serial communication channel. Many applications in this field are available, including telerobotics and telesurgery. The defining characteristic of NCSs is that the system components such as sensors, controllers, actuators, are typically spatially isolated from one another, exchanging information with each other over a network. Inserting computer networks into control systems to replace the traditional point-to-point wiring has enormous advantages (Raji, 1994), including lower cost, reduced weight and power, simpler installation, and higher reliability. As a consequence, the study of NCSs has recently received much attention.

The shared communication network in the closed-loop control systems, however, will inevitably bring about some new problems, a notable one among which is time delay. The network-induced time delay, constant, bounded, or random, can degrade the performance of control systems and even destabilise the systems (Walsh et al., 2002; Yepez et al., 2002; Montestruque and Antsaklis, 2004). Many control schemes with slightly different system setups have been suggested to overcome this issue. The difference in the setups comes from the driven mode for system nodes, whether time-driven or event-driven. The mode that the node is driven to sample, signals regularly to works by the action of system clock is called time-driven mode, for which the synchronisation among all units is required. The event-driven mode does not need a synchronised clock and it means that the transmitted signal is used as soon as
it arrives at the system node. In the work done by Luck and Ray (1990), the deterministic control methodology was developed with the assumption that all network nodes used the time-driven mode. The method reshapes random time delays from networks to deterministic delays so that the control system becomes time-invariant. The optimal stochastic control methodology was found by Nilsson and Bernhardsson (1996, 1997) to control a NCS with a time-driven sensor, an event-driven controller, and an event-driven actuator. The effect of random network delays on the system is treated as a Linear-Quadratic-Gaussian (LQG) problem. Yu et al. (2000) proposed a novel control model of the NCS based on the setup with an event-driven controller and a time-driven actuator. They discussed the network with Markov delay characteristics. In fact, the system performance and the complexity of analysis and design depend greatly on the configuration of driven modes for system nodes, so the issue about how to select the appropriate driven modes becomes very important. To our best knowledge, little systematic work has been done on NCSs with different kinds of driven modes.

In this paper, we try to build the models for the NCS within a special range of time delays under different driven modes and to discuss the effects of different kinds of driven modes on the system performance. The research results are expected to be useful for other researchers to select the suitable driven modes for system nodes according to network conditions so that a relatively good performance, as well as relatively low complexity, can be obtained for the control system designed without considering the delays.

This paper is arranged as follows. In Section 2, the general formulas of the system model are deduced under the four different combinations of driven modes. The effects of the driven modes on the system performance are analysed in Section 3. The features and the appropriate application ranges for the four different cases are proposed in Section 4. In Section 5, the simulation results are given to support the theoretical claims. The conclusion of this paper is presented in Section 6.

2 System models with different driven modes

In general, the sensor of the system samples signals in a regular way, that is, it uses the time-driven mode, while the controller and the actuator both usually use either time-driven or event-driven mode, so there are four different combinations of driven modes to be considered. The form of system driven mode can be described as Node name (Driven mode) + Node name (Driven mode). To make our discussion pellucid, several symbols are defined as follows: C and D denote the controller node and the actuator node, respectively, and T and E denote time-driven mode and event-driven mode, respectively. For example, the meaning of C(T) + D(E) is that the controller node works under the event-driven mode and the actuator node under the time-driven mode.

Figure 1 illustrates the block diagram of a NCS. We are assuming that the network communication is a single-packet and single-loop transmission without data packet dropout. The sensor-to-controller delay \( \tau_{sc} \) and the controller-to-actuator delay \( \tau_{ca} \) are two sources of time delays from the network. Generally, the computation time in the controller is included in \( \tau_{ca} \) and the execution time of the plant can be neglected in contrast with network delays. For time-invariant controllers, the sensor-to-controller delay and the controller-to-actuator delay can be collected together as \( \tau = \tau_{sc} + \tau_{ca} \) for analysis purposes.

The controlled system consisting of a continuous-time plant can be modelled as

\[
\dot{x}(t) = Ax(t) + Bu(t)
\]

(1)

where \( x(t) \in \mathbb{R}^n \) is the state of the plant, \( u(t) \in \mathbb{R}^m \) is the input of the plant, and \( A, B \) are the matrices of appropriate dimensions. Discretising equation (1) with the sampling period \( T \), gives

\[
x(k + 1) = \Phi x(k) + \Gamma u(k)
\]

(2)

where

\[
\Phi = e^{AT}, \quad \Gamma = \int_0^T e^{\alpha s} ds B
\]

(3)

and \( u(k) \) denotes the control signal based on the state information \( x(k) \) at the time instant \( kT \).

Obviously, if time delay is longer than one sampling period, then zero, one, or more than one control signal(s) will arrive at the actuator in a single sampling period, which makes the analysis and design of a NCS more complex (Zhang et al., 2001). So we will take account of a special delay range, i.e., \( dT < \pi (k) < (d + 1)T \), \( d = 0, 1, 2, \ldots \), which can guarantee that one and only one control signal is received at the actuator node every sample period for \( k > d \). Given plant equation (2), the regularity of the plant models of the NCS under the four driven modes can be got as follows.

The timing of signals for the system nodes under the \( C(E) + A(T) \) driven mode is shown in Figure 2. The feedback information from the sensor is sent to the network at the instant \( (k - (d + 1))T \) and arrives at the controller after the delay \( \tau_{sc} \). The control signal is computed and sent out right now due to the event-driven controller and then it arrives at the actuator after the delay \( \tau_{ca} \). According to the given span of delays, the arrival time of the control signal
falls within the time interval \(((k-1)T, kT)\). The time-driven actuator samples the control input at the instant \(kT\) and supplies it to the plant by zero-order holding. Therefore, the discrete-time controlled system model using the \(C(E) + A(T)\) driven mode can be given as
\[
x(k+1) = \Phi x(k) + \Gamma u(k-(d+1)).
\]
(4)

**Figure 2** Timing diagram under the \(C(E) + A(T)\) mode

Figure 3 indicates the timing of signals under the \(C(E) + A(E)\) mode. This case is similar to \(C(E) + A(T)\); the difference is that the control signal as the plant input is not constant but piecewise continuous in each sampling period. Referring to the analysis (Nilsson et al., 1998), the plant model with the \(C(E) + A(E)\) driven mode can be written as
\[
x(k+1) = \Phi x(k) + \Gamma_0(\tau(k))u(k-d) + \Gamma_1(\tau(k))u(k-(d+1))
\]
where
\[
\Gamma_0(\tau(k)) = \int_0^{T-\tau(k)-dT} e^{\nu s} dsB, \quad \Gamma_1(\tau(k)) = \int_T^{T-\tau(k)-dT} e^{\nu s} dsB.
\]
(6)

**Figure 3** Timing diagram under the \(C(E) + A(E)\) mode

If the time-driven controller is used, the respective spans of \(\tau_c\) and \(\tau_{ac}\) should be taken into account. It is assumed that \(dT < \tau_c < (d+1)T\) and \(dT < \tau_{ac} < (d+1)T\), where \(d, d_1, d_2 = 0, 1, 2, \ldots\). Figure 4 reveals the timing diagram under the \(C(T) + A(T)\) mode. The sensor signal being sent out at the instant \((k-(d_1+d_2+2))T\) arrives at the controller in the time interval \([(k-(d_2+1))T, (k-(d_2+1))T)\) after \(\tau_{ac}\). Since the controller is time-driven, it will sample the sensor data at \((k-(d_1+d_2+1))T\). And the control information will arrive at the actuator within the interval \((k-(d_1+d_2+1))T\) after \(\tau_{ac}\) and be sampled at the instant \(kT\). So we can obtain the system model representation under the \(C(T) + A(T)\) driven mode as follows:
\[
x(k+1) = \Phi x(k) + \Gamma u(k-(d_1+d_2+2)).
\]
(7)

**Figure 4** Timing diagram under the \(C(T) + A(T)\) mode

From the ranges of \(\tau_c\) and \(\tau_{ac}\), the range of the whole network delay \(\tau\) can be expressed as
\[(d_1 + d_2)T < \tau < (d_1 + d_2 + 2)T.\]

To compare the different kinds of driven modes within the same span of time delays, i.e., \(dT < \tau < (d+1)T\), we need to rewrite equation (7). The condition \((d_1 + d_2)T < \tau < (d_1 + d_2 + 2)T\) can be divided into two parts:
\[(d_1 + d_2)T < \tau < (d_1 + d_2 + 1)T\]
and
\[(d_1 + d_2 + 1)T < \tau < (d_1 + d_2 + 2)T.\]

If
\[(d_1 + d_2)T < \tau < (d_1 + d_2 + 1)T,\]
then we define \(d = d_1 + d_2\), as, thus, equation (7) can be changed into
\[
x(k+1) = \Phi x(k) + \Gamma u(k-(d+1)).
\]
(8)

And if
\[(d_1 + d_2 + 1)T < \tau < (d_1 + d_2 + 2)T,\]
defining \(d = d_1 + d_2 + 1\), then equation (7) becomes the following form
\[
x(k+1) = \Phi x(k) + \Gamma u(k-(d+1)).
\]
(9)

Figure 4 indicates the timing of signals under the \(C(T) + A(T)\) mode.

The similar analysis can be made for another time-driven-controller mode \(C(T) + A(E)\). The relevant timing diagram is shown in Figure 5. The controlled system model using this driven mode is
\[
x(k+1) = \Phi x(k) + \Gamma_0(\tau_{ac}(k))u(k-(d_1+d_2+1)) + \Gamma_1(\tau_{ac}(k))u(k-(d_1+d_2+2))
\]
(10)
Table 1 System models under different driven modes when 
\((d_1 + d_2)T < \tau < (d_1 + d_2 + 1)T\)

<table>
<thead>
<tr>
<th>Driven modes</th>
<th>Controlled system model</th>
</tr>
</thead>
<tbody>
<tr>
<td>((T) + A(T))</td>
<td>(x(k + 1) = \Phi x(k) + \Gamma u(k - (d + 2)))</td>
</tr>
<tr>
<td>((E) + A(T))</td>
<td>(x(k + 1) = \Phi x(k) + \Gamma u(k - (d + 1)))</td>
</tr>
<tr>
<td>((T) + A(E))</td>
<td>(x(k + 1) = \Phi x(k) + \Gamma_0 (\tau_m(k))u(k - \tau) + \Gamma_1 (\tau(k))u(k - (d + 1)))</td>
</tr>
<tr>
<td>((E) + A(E))</td>
<td>(x(k + 1) = \Phi x(k) + \Gamma_0 (\tau_m(k))u(k - \tau) + \Gamma_1 (\tau(k))u(k - (d + 1)))</td>
</tr>
</tbody>
</table>

3 Effects of different driven modes on system performance

By analysing Tables 1 and 2, some results about the different effects of the above four kinds of driven modes on the performance of the NCS can be got as follows.

When the actuator uses the time-driven mode, such as \((T) + A(T)\) and \((E) + A(T)\), the whole time delay of the system is an integral multiple of the sampling period. It can be indicated that the system performance depends on the integer \(d\) and is independent of the concrete value of the delay; that is, the output response characteristic of the system will be determined as long as the integer \(d\) relating to the delay range is fixed. Consequently, the control system will become much easier without the requirement of considering the randomness of network delays.

Using the event-driven-actuator mode, the actual values of the time delays in the system should be taken into account. As shown in the above tables, the delay \(\tau\) is included in the coefficient matrices \(\Gamma_0\) and \(\Gamma_1\) under the \((E) + A(E)\) mode and the delay \(\tau_m\) included in \(\Gamma_0\) and \(\Gamma_1\) under the \((T) + A(E)\) mode, which will result in the variant system dynamic response with the delay varying.

In addition, when \((d_1 + d_2 + 1)T < \tau < (d_1 + d_2 + 2)T\), both the state equations of the controlled system are the same expression under the \((T) + A(T)\) mode and under the \((E) + A(T)\) mode, i.e., these two driven modes can be thought to be identical in the condition. Similarly, the two state equations corresponding to \((T) + A(E)\) and \((E) + A(E)\) largely overlap in form, but their output response curves will not be of superposition as a result of their coefficient matrices \(\Gamma_0\) and \(\Gamma_1\) depending on the different delays.

It can also be seen from the above tables that the \((T) + A(T)\) mode makes the strongest impact on the system performance and the \((E) + A(E)\) mode has the weakest effect among the four different modes. From the standpoint of cybernetics, if the controller and the actuator both use the
event-driven mode, the plant will receive the control information in the fastest manner, hence the system will achieve the best performance; on the contrary, the control signal will be used at the latest if all the network nodes adopt the time-driven mode, which means that the system will work in the open-loop state for the longest time, thus, the control system will perform the worst. The $C(E) + A(T)$ mode and the $C(T) + A(E)$ mode will be in the medium degree. When the network delay $\tau$ satisfies the condition $(d_1 + d_2)T < \tau < (d_1 + d_2 + 1)T$ with $d = d_1 + d_2$, from Table 1, the plant state at the instant $(k + 1)T$ depends on the one at $(k - (d + 1))T$ under the $C(E) + A(T)$ mode, while the system state with the $C(T) + A(E)$ mode at $(k + 1)T$ is subject to the states not only at $(k - (d + 1))T$ but also at $(k - (d + 2))T$, which shows that the system will be affected by the longer delay under the $C(T) + A(E)$ mode and then $C(E) + A(T)$ seems superior to $C(T) + A(E)$. In the same way, when $(d_1 + d_2 + 1)T < \tau < (d_1 + d_2 + 2)T$ and given $d = d_1 + d_2 + 1$, the system state equation with the $C(E) + A(T)$ mode does not change, but under the $C(T) + A(E)$ mode the system state at $(k + 1)T$ relies on the ones at the instants of both $(k - d)T$ and $(k - (d + 1))T$. So, in the mentioned condition $C(T) + A(E)$ can give the better result in system performance to the contrary.

4 Features and suitable application of each driven mode

This section will discuss the features including the advantages and the disadvantages of each driven mode and propose their respective suitable areas of practical applications.

4.1 $C(T) + A(T)$ driven mode

Since the system model is invariant under the $C(T) + A(T)$ driven mode when the network delays meet the requirement $(d_1 + d_2)T < \tau < (d_1 + d_2 + 2)T$, the system performance will not be affected by the variance of the delays within the comparatively wide range of the delays, i.e., two sampling periods. Hence, in this condition we can regard such a NCS as the traditional control system with the constant delay. Many thorny matters introduced by time-varying network delays can be avoided and the complexity of analysis and design of the system can be reduced largely. The drawback of the $C(T) + A(T)$ mode is that the whole delay is prolonged to the integral multiples of the sampling period, which makes the feedback information transmitted to the plant slowest and becomes the cause of the worst system performance. Besides, for the different delay ranges the system response curve will change by leaps rather than by gradual transitions, that is, if the time delay varies from one range to another even though the variance is very small, a large transformation of the system response will occur.

4.2 $C(E) + A(E)$ driven mode

The controlled system with the $C(E) + A(E)$ mode can receive the fastest feedback control information, by comparison, so it possesses the best performance, which is a remarkable advantage for this driven mode. However, the time-varying characteristic of the network delays will induce the coefficient matrices of the system to be also time varying, which causes the theoretical analysis and the practical control of the NCS to be very complicated.

4.3 $C(E) + A(T)$ driven mode

When network delays satisfy the condition $(d_1 + d_2)T < \tau < (d_1 + d_2 + 1)T$, the system response under the $C(E) + A(T)$ driven mode is closer to the one under the $C(E) + A(E)$ mode, i.e., in this case the system performance from $C(E) + A(T)$ stands second among all the driven modes. Similar to $C(T) + A(T)$, the analysis of the system and the design of the controller become simpler in this driven mode due to the constant matrices $\Phi$ and $\Gamma$ within the fixed delay range. The disadvantage of this driven mode is also similar to $C(T) + A(T)$, that is, there is a relatively large difference among the system response curves for different ranges of the network delays. So, these two driven modes are not suitable to be used when network delays change frequently among two or more delay ranges.

4.4 $C(T) + A(E)$ driven mode

This case is alike with the $C(E) + A(E)$ mode. The dynamic response of the controlled system using this driven mode is nearer to the best status from $C(E) + A(E)$ if time delays satisfy the condition $(d_1 + d_2 + 1)T < \tau < (d_1 + d_2 + 2)T$. And the hindrance to study is also the randomness of the coefficient matrices owing to the random network delays.

4.5 Appropriate choice of driven modes

The $C(E) + A(E)$ driven mode is very suitable for the controlled system sensitive to time since it can reduce the delays to the lowest bound and meet the demands of the system for time as best as possible. But the analysis and the control of the system are not easy as a result of the time-varying coefficient matrices in the plant model. When the complexity makes it hard to proceed with the study, other driven modes can be considered for approximate analysis. If $(d_1 + d_2)T < \tau < (d_1 + d_2 + 1)T$, the $C(E) + A(T)$ mode can be tried to substitute for the $C(E) + A(E)$ mode during the theoretical analysis. The system performance from the $C(E) + A(T)$ mode is closest to the one from the $C(E) + A(E)$ mode in the above condition, and the time-invariant coefficient matrices will also make for the much easier analysis and control. Of course, the smaller the time delays are, the exacter the result after the substitution is.
If the controlled system has no strict requirement of time, either the \( C(T) + A(T) \) or \( C(E) + A(T) \) mode can be selected. The system models with these two driven modes are independent of the delay variance, which is favourable for the analysis and the control of the system. Furthermore, since the \( C(E) + A(T) \) mode excels \( C(T) + A(T) \) in system performance, it will be the better choice in this case.

Under the \( C(T) + A(E) \) driven mode, the system performance is medium, and the control law is comparatively complex due to the system model dependent on time-varying delays. Thus, this mode is rarely used.

5 Numerical examples

A DC motor controlled system model is used to prove the above theoretically analytic results. Sampling the system with the period \( T = 0.05 \) s, we obtain the discrete-time state-space plant model as follows

\[
\begin{bmatrix}
1 & 0.04961 \\
0.9843 & 0.00014
\end{bmatrix}
\begin{bmatrix}
x(k) \\
u(k)
\end{bmatrix}
+ \begin{bmatrix}
0.00571
\end{bmatrix} u(k)
\]

\[
y(k) = [1 \ 0] \begin{bmatrix}
x(k)
\end{bmatrix}
\]

where \( y \) is the output of the system. The LQR state feedback controller is \( u = -Kx \), where \( K = [148.6965 \ 49.1209] \).

Consider the step responses of the given controlled system in the cases of four different driven modes. We choose the time delay \( \tau \) to be within the interval \((0, T)\) or \((T, 2T)\), respectively. For simplification of computation, the delay is assumed to be constant. Figures 6 and 7 reveal the system response curves under the four driven modes when \( \tau = 0.03 \) s and \( \tau = 0.08 \) s, independently.

Figure 6  System step responses with \( \tau = 0.03 \) s

As we can see from the figures, the \( C(E) + A(E) \) driven mode surpasses all others in the response performance of the system, which can reduce the impact of the delays on the controlled system to the minimum. And it can be easily understood that \( C(T) + A(T) \) mode is the direct reason for the worst system performance. \( C(E) + A(T) \) and \( C(T) + A(E) \) are the medium driven modes. When \( \tau = 0.03 \) s, i.e., the delay falls within \((0, T)\), from Figure 6, the system response performance from the \( C(E) + A(T) \) mode is better than the one from the \( C(T) + A(E) \) mode.

However, Figure 7 indicates that the \( C(T) + A(E) \) mode is preferable to the \( C(E) + A(T) \) mode if \( \tau = 0.08 \) s, i.e., the delay range is \((T, 2T)\), and the response curve dependent on \( C(E) + A(T) \) is completely superposed with that one dependent on \( C(T) + A(T) \).

Figure 7  System step responses with \( \tau = 0.08 \) s

In order to prove the results regarding the respective features of the driven modes for different time delay intervals, we choose \( \tau = 0.01 \) s or \( \tau = 0.04 \) s from \( \tau \in (0, T) \) and \( \tau = 0.06 \) s or \( \tau = 0.09 \) s from \( \tau \in (T, 2T) \), to compare their step response curves.

When the actuator uses the time-driven mode, the system response curves hold invariant as the time delay \( \tau \) varies in one sampling period by \( C(E) + A(T) \), shown in Figure 8, or in two sampling periods by \( C(T) + A(T) \), which greatly simplifies the complex problems introduced by random delays. But we can also see that under the \( C(E) + A(T) \) mode even the slight change from one delay range to another, such as the delay changing from \( \tau = 0.04 \) s to \( \tau = 0.06 \) s, can bring on a notable variance of the response curve. And there will be a more noteworthy change from the \( C(T) + A(T) \) mode between \( \tau = 0.09 \) s and \( \tau = 0.11 \) s. By contrast with the above two driven modes, if the event-driven mode is applied to the actuator, such as \( C(E) + A(E) \) and \( C(T) + A(E) \), the system responses can be seen to vary smoothly with the delays varying, as indicated in Figure 9. It can be seen from the above figures that the simulation results are consistent with the expected.
6 Conclusion

In this paper, we have built the models of the NCS under the various driven modes for the network nodes in a special range of time delays. Based on the given models, we analysed the effects of driven modes on system performance in detail, and subsequently proposed the application areas of different driven modes by comparing their respective features. It was found that the $C(E) + A(E)$ mode is the best choice with a view to system performance, but it requires more computation. And the $C(E) + A(T)$ mode as well as the $C(T) + A(T)$ mode are also worthy of consideration due to their common advantage of a simple system model independent of random network delays. The simulation study was done to support the theoretical results. At present, we are making an experiment on remote controlling a two-dimensional robot arm system via a network, and the system setup with the $C(E) + A(E)$ driven mode chosen is of benefit to a good result.

In practical NCSs, the range of network delays will be extended to more than one sampling period, which may introduce many new problems, including vacant sampling, multiple sampling, data disorder, and so on, so further researches are needed.

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