Singularity Analysis and Nonlinear Tracking Controller Design for a Five-bar Parallel Manipulator

Weiwei Shang, Shuang Cong and Yuan Ge
University of Science and Technology of China, Hefei, Anhui, 230027, P. R. China
{wwshang & scong}@ustc.edu.cn, ygetoby@mail.ustc.edu.cn

Abstract— The singularity of a five-bar parallel manipulator is analyzed and a nonlinear tracking controller is designed in this paper. Three types of the singularity are analyzed and the singularity curve equations are formulated by using the Jacobian matrix of the parallel manipulator. The dynamic model is established in the active joint space, and the active joint friction is described with the Coulomb + viscous friction model. Based on the dynamic model, a nonlinear tracking controller eliminating the tracking error with the power function is proposed. The trajectory tracking experiment of an actual five-bar parallel manipulator is carried out. The performances between the proposed controller and the traditional augmented PD (APD) controller are compared.

Index terms— Parallel manipulator, nonlinear tracking controller, singularity, dynamic

I. INTRODUCTION
Parallel manipulators are characterized as having closed-loop kinematic chains. Compared to the serial ones which have open-loop structure, parallel manipulators have many advantages in terms of accuracy, stiffness and ability to manipulate heavy loads [1]. These advantages avoid the drawbacks of serial ones and make parallel manipulators be potential high performance motion platforms. However, the numerous singularities which divide the workspace into multiple regions make a bad impact on motion performance of parallel manipulators. When resting at a singular configuration, parallel manipulators will lose their precision and stiffness completely. So it is of great importance to obtain the distribution of singularity loci before we design the controllers for parallel manipulators.

Up to now, many researchers have devoted to the study of singularity. One of the first works to address singularity of general parallel manipulators is that of Gosselin and Angeles[2]. In [2], the Jacobian matrix between the input and output velocity coordinates of parallel manipulators is used to analyze the singularity. Using this approach, three types of singularity can be identified: (i) actuator singularity—configuration in which the parallel manipulator loses one or more degrees of freedom (DOF) as a result of the choice of actuated joints; (ii) end-effector singularity—the end-effector frame loses DOF of available motion; (iii) configuration space singularity—singularity of the joint configuration space. Actuator singularity can be sub-classified into degenerate and non-degenerate types [3]. The singularity analysis with the Jacobian matrix between the input and output velocity coordinates has been further applied to several actual parallel manipulators[4],[5]. Besides the above method, the Jacobian matrix of the closed-loop constraint equation has been proposed to analyze the singularity, especially, the configuration space singularity[6],[7]. At the singularity configuration, the dynamic performances of parallel manipulators are bad, and the motion is uncontrollable. Thus the singularity should be avoided in the design and application of parallel manipulators. Using the differential properties of the Jacobian matrix, an approximate method of singularity avoidance is proposed for parallel manipulators by Bhattacharya et. al. [8]. And a general method for the singularity avoidance of parallel manipulators at a given configuration is developed in [9]. Now, there are large numbers of references about singularity analysis and singularity avoidance, but how to design the dynamic controller for parallel manipulators with singularities are interesting and challenging.

Current controllers for parallel manipulators can be divided into two categories. The first kind is kinematic controller, including PD controller [10], nonlinear PD (NPD) controller [11], and iterative learning controller [12], etc. The dynamics of parallel manipulators are not considered in these controllers, so the complex computation of dynamics can be avoided and the controller design can be simplified greatly. However, the performances of these controllers are always limited, and the stability of these controllers at high-speed are not guaranteed. The second kind is dynamic controller. Unlike the kinematic controller, full dynamic model of parallel manipulators are taken into account in dynamic controller. The traditional dynamic controllers include the augmented PD (APD) controller [13],[14], and the computed-torque (CT) controller [15],[16]. Recently, some novel controllers with performance indicators were presented for parallel manipulators [17],[18],[19]. These novel controllers can improve the performances of the traditional dynamic controllers further, but also require more accurate dynamic model. However, it’s very difficult to get the accurate values of the dynamic parameters of parallel manipulators. The nonlinear dynamics and friction in parallel manipulators, and the un-known trajectory disturbance decrease the tracking accuracy. Especially the nonlinear friction, decrease the position control accuracy of parallel manipulators obviously [20]. In order to overcome these nonlinear and uncertain factors, a novel nonlinear tracking controller with the friction compensation and dynamic compensation is designed for a five-bar parallel manipulator in this paper.

In this paper, the singularity and dynamic controller are studied for the motion control of a five-bar parallel manipulator, and a novel nonlinear tracking controller is proposed to overcome the nonlinear dynamics and friction. The theory
analysis and design are validated by the trajectory tracking control of an actual five-bar parallel manipulator. The actuator singularity, end-effector singularity, and the configuration space singularity of a five-bar parallel manipulator are analyzed using the Jacobian matrix between the active joints and the end-effector coordinates. The singularity curve equations are formulated, and the distribution of singularity loci is plotted.

The dynamic model is established in the active joint space, and the friction of two active joints is described with the Coulomb + viscous friction model. Based on the dynamic model established, a nonlinear tracking controller which eliminates the tracking error with the power function is proposed. The trajectory tracking experiments of an actual five-bar parallel manipulator are carried out, and the experimental results of the proposed controller are compared with the traditional APD controller. Our experimental results indicate that, compared with the APD controller, the nonlinear tracking controller can get better trajectory tracking accuracy of the end-effector. The singularity curve equations are formulated, and the distribution of singularity loci is plotted.

The paper is organized as follows. In section II, the curve equations of the three types of singularity are formulated, and the figure of the singularity distribution loci is shown. In section III, the dynamic model is established in the active joint space. In section IV, the nonlinear tracking controller is designed based on the dynamic model. In section V, the trajectory tracking experiments of an actual parallel manipulator are carried out, and the experiment results are compared with the conventional APD controller. Finally, several important remarks are concluded.

II. SINGULARITY ANALYSIS

The coordinates of the five-bar parallel manipulator studied in this paper is shown in Fig. 1. From Fig. 1, one can see that the parallel manipulator consists of two kinematic chains which meet at a common point. In the parallel manipulator, two active joints are connected to the base A1, A2, and the others are passive joints B1 and B2. The end-effector is mounted at the common point O, where the two chains meet. Coordinates of the two bases are A1 (0, 0.25), and A2 (0.433, 0), and all of the links have the same length 0.244m. The definitions of the joint angles are shown in Fig. 1, where \(q_{a1}\) and \(q_{a2}\) are active joint angles; \(q_{b1}\) and \(q_{b2}\) are passive joint angles.

Next, we will analyze the actuator singularity, end-effector singularity, and the configuration space singularity for the five-bar parallel manipulator. The analysis begins by choosing the active joints and the end-effector coordinates, denoted by \(q_e\) and \(q_o\). Then the relation between the active joints and the end-effector coordinates is written as

\[
f(q_e, q_o) = 0,
\]

(1)

where \(f\) is an 2-dimensional implicit function of \(q_e\) and \(q_o\), and \(0\) is the 2-dimensional zero vector. Differentiating Eq. (1) with respect to time leads to the relationship between the active joints and the end-effector velocity as follows:

\[
A\dot{q}_e = B\dot{q}_o,
\]

(2)

where \(A\) and \(B\) are both 2×2 Jacobian matrices, which can be defined as

\[
A = \frac{\partial f}{\partial q_e}, \quad B = -\frac{\partial f}{\partial q_o}.
\]

(3)

For the parallel manipulator, the end-effector singularity occurs when \(B\) becomes singular, and the actuator singularity occurs when \(A\) becomes singular. The configuration space singularity can only occur when both \(A\) and \(B\) become singular simultaneously. Next, we will analyze the three types of singularity for our five-bar parallel manipulator.

In Fig. 1, the active joint coordinate is denoted by \(X_{a1} = [x_{a1} \ y_{a1}]^T\), the passive joint coordinate is denoted by \(X_{b1} = [x_{b1} \ y_{b1}]^T\), the end-effector coordinate is denoted by \(q_e = [x \ y]^T\). Here, subscript \(i = 1,2\) refers to the \(i\) th kinematic chains. Furthermore, let \(q_{ai}\) be the angles of the active joints, \(q_{bi}\) be the angles of the passive joints, and \(l\) be the length of the links. From Fig.1, one can obtain

\[
\begin{align*}
x &= x_{a1} + l\cos(q_{a1}) + l\cos(q_{b1}) \\
y &= y_{a1} + l\sin(q_{a1}) + l\sin(q_{b1}).
\end{align*}
\]

(4)

Differentiating Eq.(4) yields

\[
\begin{align*}
\dot{x} &= -l\sin(q_{a1})\dot{q}_{a1} - l\sin(q_{b1})\dot{q}_{b1} \\
\dot{y} &= l\cos(q_{a1})\dot{q}_{a1} + l\cos(q_{b1})\dot{q}_{b1},
\end{align*}
\]

(5)

and solving the above equation yields

\[
\begin{align*}
\cos(q_{b1})\dot{x} + \sin(q_{b1})\dot{y} &= l\sin(q_{b1} - q_{a1})\dot{q}_{a1} \\
\cos(q_{a1})\dot{x} + \sin(q_{a1})\dot{y} &= -l\sin(q_{b1} - q_{a1})\dot{q}_{b1}.
\end{align*}
\]

(6)

Express Eq. (6) with matrix form as

\[
A_a\dot{q}_e = B_a\dot{q}_b,
\]

(7)

where \(q_a = [q_{a1} \ q_{a2}]^T\), \(q_b = [q_{b1} \ q_{b2}]^T\), \(\dot{q}_e = [\dot{x} \ \dot{y}]^T\); and matrix \(A_a\), \(A_b\), \(B_a\), and \(B_b\) are defined as

\[
A_a = \begin{bmatrix}
\cos(q_{b1}) & \sin(q_{b1}) \\
\cos(q_{a1}) & \sin(q_{a1})
\end{bmatrix}, \quad
B_a = \begin{bmatrix}
l\sin(q_{b1} - q_{a1}) & 0 \\
0 & l\sin(q_{b2} - q_{a2})
\end{bmatrix}.
\]

(8)
Eliminate $1 \pm \frac{1}{\sqrt{2}}$ in Eq. (17), the ' is ' when $0 = \pm \pi$; else the ' is '. Thus, the points $(x, y)$ satisfy Eq. (18) is the actuator singularity of the parallel manipulator. Using the function `ezplot` in Matlab 7.0, one can draw the curves of Eq. (18). As shown in Fig. 2, the blue dotted curves are the actuator singularity loci of the parallel manipulator.

For the parallel manipulator, the configuration space singularity occurs when the following condition is held:
\[
\det(\mathbf{A}_a) = 0 \quad \text{and} \quad \det(\mathbf{B}_a) = 0. \quad (19)
\]
From Eqs. (11) and (16), one can have
\[
q_{b2} = q_{b1} + k\pi \cap (q_{b1} = q_{a1} + k\pi \cup q_{b2} = q_{a2} + k\pi) \quad k = -1,0,1. \quad (20)
\]
From Eq.(20) one can find that, in the configuration space singularity position, three links of the parallel manipulator will be in the same line. Thus, this type of singularity will not exist for our parallel manipulator. And one can calculate the common solutions of Eqs. (12), (13) and (18), also one can find the common solutions do not exist, that is to say the configuration space singularity will not exist for our parallel manipulator.

### III. Dynamic Modeling

In this section, we will formulate the dynamic model for the five-bar parallel manipulator in the active joint space. Cutting the parallel manipulator at the common point $O$ in Fig. 1, one can have an open-chain system including two independent 2-DOF serial manipulators, each of which contains an active joint and a passive joint. The dynamic model of the parallel manipulator equals to the model of the open-chain system plus the closed-loop constraints, thus the dynamics of the parallel manipulator can be established by combining the dynamics of the two serial manipulators under the constraints.

As well as we know, the dynamic model of each 2-DOF serial manipulator can be formulated as
\[
\mathbf{M}_i\ddot{q}_i + \mathbf{C}_i\dot{q}_i = \mathbf{f}_i, \quad (21)
\]
where $q_a = [q_{a1} \ q_{b1}]^T$, $q_{ai}$ and $q_{bi}$ are the active joint and passive joint angle respectively; $M_a$ is inertia matrix, and $C_i$ is Coriolis and centrifugal force matrix.

In Eq. (21), $f_i = [f_{ai} \ f_{bi}]^T$, where $f_{ai}$ and $f_{bi}$ are the active and passive joint friction torque, respectively. In order to simplify the dynamic model, $f_{bi}$ is ignored. So the active joint friction torque $f_{ai}$ can be formulated as

$$f_{ai} = \text{sign}(q_{ai})f_{ci} + f_{vi}q_{ai},$$

where $f_{ci}$ represents the Coulomb friction, and $f_{vi}$ represents the coefficient of the viscous friction.

Combine the dynamic models of two 2-DOF serial manipulators, and unite the active joint equation and the passive joint equation respectively; then the dynamic model of the open-chain system can be expressed as

$$M \ddot{q}_a + C \dot{q}_a = \tau_a - f_a,$$

where $q_a$ and $q_b$ are the active and passive joint vector respectively; $f_a$ and $f_b$ are the friction torque vector of the active and passive joints respectively; $\tau_a$ and $\tau_b$ are the torque vector of the active and passive joints respectively; $M$ is inertia matrix, and $C$ is Coriolis and centrifugal force matrix.

Considering the five-bar parallel manipulator moves in the workspace without singularities, from the analysis of the three types of singularity, one can know that matrix $A_a$ and $B_a$ are both nonsingular. Also from Eq. (8), one can note that $B_b = -B_a$, thus matrix $B_b$ is nonsingular. From Eq. (7), the velocity relation between the active joints and passive joints can be written as

$$\dot{q}_b = (B_b)^{-1}A_a (A_a)^{-1}B_a \dot{q}_a.$$  \hspace{1cm} (24)

Let $S = (B_b)^{-1}A_a (A_a)^{-1}B_a$, Eq.(24) can be briefly expressed as

$$\dot{q}_b = S \dot{q}_a.$$  \hspace{1cm} (25)

Differentiating Eq.(25), the acceleration relation between the active joints and passive joints can be calculated as

$$\ddot{q}_b = \dot{S} \dot{q}_a + S \ddot{q}_a.$$  \hspace{1cm} (26)

Substitute Eqs. (25) and (26) into Eq. (23), and let $\frac{S}{\dot{S}} = J$ be the Jacobian velocity matrix, the dynamic model of the open-chain system in the active joint space can be written as

$$MJ \ddot{q}_a + (MJ + CJ) \dot{q}_a = \begin{bmatrix} \tau_a \\ \tau_b \\ \text{sign}(q_{ai})f_{ci} + f_{vi}q_{ai} \end{bmatrix}.$$  \hspace{1cm} (27)

Based on Eq. (27) and the constraint forces due to the closed-loop constraints, the dynamic model of the parallel manipulator can be formulated as

$$MJ \ddot{q}_a + (MJ + CJ) \dot{q}_a = \begin{bmatrix} \tau_a \\ \tau_b \\ \text{sign}(q_{ai})f_{ci} + f_{vi}q_{ai} \end{bmatrix} + E^T \lambda.$$  \hspace{1cm} (28)

In Eq. (28), $E^T \lambda$ represents the constraint force vector, where constraint matrix $E$ is the differential of the closed-loop constrained equation, and $\lambda$ is a multiplier representing the magnitude of the constraint forces. The closed-loop constraint differential equations of the parallel manipulator can be determined by

$$E \dot{q}_a = 0$$

where $q = [q_{a1} \ q_{a2} \ q_{b1} \ q_{b2}]^T$. The velocity vector $\dot{q}_a$ contains independent angles of the two active joints, so one can obtain $EJ = 0$, or equivalently $J^T E'F = 0$. Multiply both sides of Eq.(28) by $J^T$, one can obtain

$$J^T MJ \ddot{q}_a + J^T (MJ + CJ) \dot{q}_a = \tau_a - f_a.$$  \hspace{1cm} (30)

Define $M_a = J^T MJ$, $C_a = J^T (MJ + CJ)$, then the dynamic model of the parallel manipulator can be briefly expressed as

$$M_a \ddot{q}_a + C_a \dot{q}_a = \tau_a - f_a.$$  \hspace{1cm} (31)

IV. NONLINEAR TRACKING CONTROLLER DESIGN

Let $q_a^d(t)$ be the desired trajectory vector of the active joints, then the actual tracking error vector is computed as

$$e = q_a^d - q_a.$$  \hspace{1cm} (32)

Then, one can define the combined error as

$$s = \dot{e} + Q \ddot{e}.$$  \hspace{1cm} (33)

where $Q$ is a symmetric positive definite matrix. Indeed $s \equiv 0$ represents a group of linear differential equations whose unique solution is $e \equiv 0$, given the initial conditions $q_a^d(0) = q_a(0)$. Thus the convergence problem of $e$ and $\dot{e}$ can be reduced to that of keeping the vector $s$ at zero. With Eq. (33), $\dot{s}$ can be written as

$$\dot{s} = \ddot{e} + Q \dddot{e}.$$  \hspace{1cm} (34)

The referenced tracking velocity $\dot{q}_a^r$ and acceleration $\ddot{q}_a^r$ can be defined as

$$\dot{q}_a^r = \dot{q}_a + s = \dot{q}_a^d + Q \dot{e},$$  \hspace{1cm} (35)

$$\ddot{q}_a^r = \ddot{q}_a + s = \ddot{q}_a^d + Q \ddot{e}.$$  \hspace{1cm} (36)

Based on the dynamic model Eq.(31), the control law of the active joints is designed as

$$\tau_a = M_a \ddot{q}_a^r + C_a \dot{q}_a^r + f_a + K_c(s)s,$$  \hspace{1cm} (37)

where $f_a$ is the friction compensation term defined in Eq.(22), and $K_c(s)$ is a symmetric positive definite matrix with time-varying gains. Matrix $K_c(s)$ can be expressed as

$$K_c(s) = \text{diag} \left[ k_c(1) |y_1|^d \right],$$  \hspace{1cm} (38)

where the variables $y_i$, $i = 1,2$ can be determined by the following rules: if $|y_i| > \delta$, then $y_i = s_i$, else $y_i = \delta$. $k_c$, $\lambda$ and $\delta$ are the designed parameters which should be tuned in practice, and $\lambda$ can be selected in the interval $[0.5, 1.0]$. This choice makes the nonlinear gain matrix $K_c(s)$ with the
following characteristics [11],[21]: large gain element for the small combined error and small gain element for the large combined error. Such variations of the gains result in a rapid transition of the systems with favorable damping. In addition, the nonlinear tracking controller is robust against the changes of system parameters and external disturbances.

V. ACTUAL EXPERIMENTS

The actual system platform is shown in Fig. 3. The parallel manipulator is equipped with two permanent magnet synchronous motors with harmonic gear drives. The active joint angles are measured with the absolute optical-electrical encoders. In the actual experiments, the control algorithms ran on a Pentium III CPU at 733MHz, and sampling period is 2ms.

In the trajectory tracking experiments, both the circle and line in the workspace are selected as the desired trajectory. For the circular trajectory, the center is (0.006, 0.0785), the radius is 0.02m, the starting point is (0.026, 0.0785), and the constant speed is 0.2m/s. For the linear trajectory, the starting point is (0.0657, 0.084), and the ending point is (-0.026, 0.07). The desired velocity profile of the line is a trapezoidal curve, the maximum velocity is 0.5m/s and the acceleration is 10m/s². The nonlinear tracking controller parameters are tuned as follows: $k_c = 100$, $Q = \text{diag}(10, 10)$, $\lambda = 0.7$, and $\delta = 0.003$. Moreover, to demonstrate that the nonlinear tracking controller can improve the tracking accuracy, experiments of using the APD controller [13], [14] have been introduced as comparison. We choose the APD controller is because it has similar nonlinear dynamics compensation and friction compensation.

The experiment results of the linear trajectory tracking using the nonlinear tracking controller and the APD controller are shown in Fig. 4. Fig. 4a and Fig. 4b are the tracking errors of the end-effector on the X-direction and Y-direction respectively. From the experiment curves, one can see that the nonlinear tracking controller can decrease the tracking errors during the whole motion process obviously, and the maximum error in the motion process is smaller. The experiment results of the circular trajectory tracking are shown in Fig. 5. From the experiment curves, one can see that the tracking accuracy is improved and the maximum error in the motion process is much smaller, with the nonlinear tracking controller.

VI. CONCLUSION

In order to implement the trajectory tracking control for an actual five-bar parallel manipulator, singularity analysis and dynamic controller design are studied. The actuator singularity, end-effector singularity, and the configuration space singularity

Fig. 3. The prototype of the five-bar parallel manipulator.

Fig. 4 Tracking errors of the circular trajectory (a) X-direction; (b) Y-direction.
of the parallel manipulator are analyzed using the Jacobian matrix, and the singularity curve equations are formulated. Considering the singularity distribution, a nonlinear tracking controller which eliminates the tracking error with the power function is proposed based on the dynamic model. Our experiment results show that, compared with the APD controller, the tracking accuracy of the end-effector are improved obviously with the proposed nonlinear tracking controller. The proposed nonlinear tracking controller is also suitable to other motion control fields, such as robots and CNC machine tools.

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