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A Random Effects Sensitivity Analysis for Patient Pathways Model

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Abstract

In this paper, we present a random effects approach to modelling of patient pathways with an application to the neonatal unit of a large metropolitan hospital. This approach could be used to identify pathways such as those resulting in high probabilities of death/survival, and to estimate cost of care or length of stay. Patient-specific discharge probabilities could also be predicted as a function of the random effect. We also investigate the sensitivity of our modelling results to random effects distribution assumptions.

1. Introduction

One of the key elements in improving efficiency in health care services delivery is patient flow. Clinically, patient flow represents the progression of a patient’s health status. Therefore, a good understanding of patient flow can offer knowledge and insight to health care providers, managers, administrators, and patients about the health care needs associated with medical concerns like disease progression or recovery status. Also, a good understanding of patient flow is also needed to support a health care facility’s operational activities. From an operational perspective, patient flow can be thought of as the movement of patients through a set of locations in a health care facility. Effective resource allocation and capacity planning are contingent upon patient flow because patient flow, in the aggregate, is a good measure of demand for health care services [1]. Modelling patients flow in health care systems is considered to be vital in understanding the operational and clinical functions of the system and has proved to be useful in improving the functionality of the health care system. There has been a great deal of work on modelling patients flow in healthcare system. These works have used different techniques to model patient flow in health care systems with several different assumptions imposed on the system. Markov, compartmental models and Coxian Phase-type distributions have been used extensively to capture probabilistic laws that govern the dynamics of patients between states in the healthcare system [2, 3, 4].

Modelling patient pathways was initiated with the work in [5] in which a semi Markov model was proposed for an individual patient’s experience during a visit to a doctor’s office. The stochastic model is based on tracking the visit of patients at a local family practice clinic. Also [6] developed a Markov models approach to show how patients’ pathways may be extracted and describe an algorithm based on branch and bound developed to efficiently extract some interesting pathways. Though the work of previous researchers on patient pathways modellers are novel, short comings of their approaches include the fact that these are based on virtual states rather than physical states [6], inference are based on the population rather than on individual patient and patient history through the system is not incorporated hence they do not really model individual patient’s experience in the process of care and discharge is to absorption (death). To deal with the shortcomings of these methods we initiate and develop a random effect modelling framework for patient pathways through an artificial system [7]. This approach could predict the probability of discharge from the system, as well as identify interesting pathways.

In this paper, we have proposed a random effect model for patient pathways with specific application to the neonatal system. We also investigate effect of different random effects distributions assumptions on the result of our model.

2. Context

We introduce, in this section, the neonatal system since the data used in this work concerns the neonatal unit of the University College of London Neonatal Unit.

2.1. Neonatal System

In the neonatal care unit, infants are taken into one of three units depending on some characteristics.

The Special Care Unit (SCU) provides a basic level of newborn care to infants at low risk. They have the capabilities to perform neonatal resuscitation at every delivery and to evaluate and provide routine postnatal care of healthy newborn infants.

The High Dependency Unit (HDU) can provide care to infants who are ill with problems that are expected to resolve rapidly.
The Intensive Care Unit (ICU) is defined as having continuously available personnel (neonatologists, neonatal nurses, respiratory therapists) and equipment to provide life support for as long as needed. This system is depicted in the figure below. Discharge from each unit could be by death, transfer to other hospitals, or home.

\[ p(y_{pi} \mid \beta, \theta_p) = \frac{\exp(X_{pi}\beta + Z_{pi}\theta_p)}{\sum\exp(X_{pi}\beta + Z_{pi}\theta_p)}; y_{pi} = 0, 1, \ldots, m - 1. \]  

(3.1)

Figure 1: The Neonatal System

A restriction imposed on this system is that there is no backward flow of babies, though this is possible clinically.

2.2. Data

The data used is a UCLH, Neonatal Unit dataset of 1002 neonates admitted into the Neonatal Unit during the year 2006. There are 264 neonates admitted into the ICU, 93 to the HDU and 884 to the SCU. 102 neonates use facilities in the ICU only, 1 HDU only and 681 SCU only. There are 2 neonates that use facilities in ICU and HDU only, 43 in HDU and SCU only while we have 113 neonates using facilities in the ICU and SCU only. 47 neonates use facilities in ICU, HDU and SCU. There are 3 neonates we do not have information about and these are dropped from our analyses. In this paper, we propose a random effects model for patient pathways and apply same to the neonatal data as well as investigate the sensitivity of the model to different random effects distribution assumptions. In most studies, random effects are taken to be normally distributed, an assumption often made both for their appropriateness and for mathematical convenience. We apply a computational method based on probability integral transform as developed in [8] to obtain the maximum likelihood estimates (MLEs) in non-linear random effects models where the random effects are non-normally distributed. The MLEs are obtained from the marginal likelihood, integrating over the random effects. Because the model is non-linear in the random effects, the marginal likelihood has no closed form, and a numerical integration approximation must be used. We have applied the method developed which is based on probability integral transform in which probability integral transform is used to transform a normal random effect to a non-normal random effect. For all these, we have applied the method using the adaptive Gaussian quadrature and implemented in SAS PROC NMLIXED.

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3. Methods and Application

In this section, we introduce our proposed random effects pathways model.

3.1. The Proposed Pathways Model

Denote by \( y_p = \{y_{p1}, y_{p2}, \ldots, y_{pm}\} \) the vector of response variables for the \( p \)-th patient, \( p = 1, 2, \ldots, n \). The components or elements of \( y_p \) could be continuous, binary or (censored) failure times. However, in our case, these are categorical outcomes. Outcomes in the same unit induce some correlation due to various shared features, introduced by a shared random effect \( \theta_p \) for the \( p \)-th patient. The probability of passing through each state, via any of the pathway is modelled as

\[ P(y_{pi} \mid \beta, \theta_p) = \frac{\exp(X_{pi}\beta + Z_{pi}\theta_p)}{\sum\exp(X_{pi}\beta + Z_{pi}\theta_p)}; y_{pi} = 0, 1, \ldots, m - 1. \]  

This is a random effects multinomial pathways model proposed in [7]. We assume that the random effects, \( \theta_p \), has a density \( f(\theta_p \mid \psi) \), with parameter \( \psi \). \( X_p, Z_p \) are the fixed and random effects design matrices respectively and \( \beta \) is the fixed effects parameter. In most of these studies, random effects distribution \( f(\theta_p \mid \psi) \) are taken to be normally distributed, an assumption often made both for their appropriateness and for mathematical convenience. As a result, most available user-friendly softwares focus on normal random effects models. However, results could be improved with appropriate random effects distribution assumptions. In this paper, to assess the sensitivity of inferences to assumptions about the distribution of the random effects \( \theta_p \), we assume: (1) \( \theta_p \) is normal with mean 0 and variance \( \sigma^2 \); (2) \( \theta_p \) is gamma, (3) \( \theta_p \) is exponential with mean \( \lambda \), (4) \( \theta_p \) is lognormal parameters \( \mu = 0 \) and \( \sigma^2 = 1 \) and (5) \( \theta_p \) is uniform on the unit interval.

3.2. Application

We apply a computational method based on probability integral transform as developed in [8] to obtain the maximum likelihood estimates (MLEs) in non-linear random effects models where the random effects are non-normally distributed. The MLEs are obtained from the marginal likelihood, integrating over the random effects. Because the model is non-linear in the random effects, the marginal likelihood has no closed form, and a numerical integration approximation must be used. We have applied the method developed which is based on probability integral transform in which probability integral transform is used to transform a normal random effect to a non-normal random effect. For all these, we have applied the method using the adaptive Gaussian quadrature and implemented in SAS PROC NMLIXED.

We have presented, in Table 1, parameter estimates and standard errors of our model for different random effects distributions assumptions. We have also presented fit statistics, -2loglikelihood, Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC) to assess goodness of fit for the models. The results suggest that significance of the fixed effects parameters is not sensitive to the choice of the random effects distribution except in result based on exponential random effects.
Table 1: Estimation results

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Normal with variance=0.0003(0.037)</th>
<th>Exponential with Lambda=2.8249(0.4702)</th>
<th>Uniform on the unit interval</th>
<th>Lognormal</th>
<th>Gamma, lambda=0.00005(0.000094)</th>
</tr>
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<tbody>
<tr>
<td></td>
<td>Estimate</td>
<td>SE</td>
<td>Estimate</td>
<td>SE</td>
<td>Estimate</td>
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<tr>
<td>ICU – OH</td>
<td>2.9140</td>
<td>0.5016</td>
<td>0.6075</td>
<td>0.0284</td>
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<td>HDU – OH</td>
<td>0.8033</td>
<td>1.6013</td>
<td>0.3129</td>
<td>1.2140</td>
<td>0.0690</td>
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<td>SCU – OH</td>
<td>0.8260</td>
<td>0.1021</td>
<td>0.4707</td>
<td>0.1032</td>
<td>0.6922</td>
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<td>ICU – HDU – OH</td>
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<td>1.2792</td>
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<td>1.2836</td>
<td>-0.9978</td>
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<td>ICU – SCU – OH</td>
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<td>HDU – SCU – OH</td>
<td>-0.0758</td>
<td>1.1560</td>
<td>0.1977</td>
<td>1.2053</td>
<td>0.0529</td>
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<td>ICU – HDU – SCU – OH</td>
<td>0.8139</td>
<td>1.3664</td>
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<td>ICU – HDU</td>
<td>2.5722</td>
<td>0.4582</td>
<td>0.2586</td>
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<td>HDU – SCU</td>
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<td>0.6960</td>
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<td>ICU – SCU</td>
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<td>0.4630</td>
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<td>ICU – HOME</td>
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<td>0.4558</td>
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<td>3.3658</td>
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<td>SCU – HOME</td>
<td>1.8884</td>
<td>0.0809</td>
<td>1.5332</td>
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<td>SCU – HOME</td>
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<td>HDU – SCU – HOME</td>
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<td>-0.3975</td>
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</table>

Fit Statistics
-2 Loglikelihood: 33410.3, 32272.7, 32889.9, 32535.0, 32553.5
AIC: 33750.0, 32612.1, 32910.9, 33289.5, 32878.7
BIC: 34583.5, 33445.4, 34102.0, 33644.8, 3370.0

Table 1: Estimation results

The MLEs of parameters in all the models, except the model with exponential random effects, are identical especially in the conclusion derived from them. These models yield significant estimates for the pathways except in HDU - OH; ICU - HDU - OH; ICU – HDU – SCU – OH; HDU - SCU – OH; ICU - HDU – SCU; ICU – SCU; HDU – SCU – Home and ICU – HDU – SCU – Home. In the exponential random effects model ICU – OH is also not significant, which suggests that the random effects distribution assumptions may affect the fixed effects parameters.

In terms of model fit, all the models describe the activities within the neonatal system satisfactorily since the non-significant pathways are those with few or no flow however, the model with exponential random effects distribution seems the most plausible model, considering the fit statistics (smaller is better). An explanation could be that the random effects captures the severity such that as babies progress through the system there is a drop in the severity because they get better which can be described by an exponential function.

4. Conclusion

We have shown that fixed effects parameters, sometimes, could be affected by random effects distribution assumptions. This is evident from the model with exponential random effects. The normal random effects model is out performed by all other models, which implies that assuming normal random effects raise many questions in practical applications of random effects modelling. The most plausible model is the one with exponential random effects. This study has also extended the application of the probability integral transform [8] to a multinomial logit random effects model. The pathways model proposed here could be applied in any context, though we have demonstrated its use in with a UCLH neonatal system however it can be used in other contexts where progressions are physical rather than virtual.

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5. References