Mutual Inductance Measurement of the Superconducting Coil for the Joule Balance

Zhengkun Li, Gang Wang, Shisong Li, Jiang Lan, Bing Han, Yunfeng Lu, Zhonghua Zhang, and Qing He

Abstract—The joule balance, an approach for precisely measuring the Planck constant at the National Institute of Metrology (NIM), China, is currently limited by the self-heating of the coil when the measuring current flows. To solve this heating problem, a superconducting coil has been built to replace the existing fixed coil wound with enameled copper wire. The movable coil set is formed by two enameled coils connected oppositely for easy alignment in air. However, the large self-inductance of the superconducting coil brings new challenges for the mutual inductance measurement. Mutual inductance measurement of superconducting coil based on standard square wave compensation method is presented and discussed in this paper.

Index Terms—Cryocooler, impedance, measurement, Moessbauer effect, mutual inductance, Planck constant, superconducting coils.

I. INTRODUCTION

THE KILOGRAM is the last one of the seven basic SI base units that is still represented by physical artifact, known as the international prototype of kilogram (IPK) kept at the Bureau International des Poids et Mesures (BIPM). The relative drift over time between IPK and other official copies was investigated in the last century and a drift of $5 \times 10^{-9}$ kg per year has been observed [1]. The absolute drift value of the IPK cannot be known for lack of a higher level reference. Quantum physics and its application in metrology give us a chance to build quantum primary standards, such as Josephson voltage standard (IVS) and quantized Hall resistance (QHR) standard. Quantum standards provide invariant references linked to natural constants and their reproducibility is several orders of magnitude better than that of physical artifact. It is used worldwide and annually international comparison is not necessary. To replace the last artifact unit in SI and establish a kind of quantum mass standard, a new definition by relating kilogram to basic physical constants, i.e., the Planck constant was proposed and recommended by CIPM-2005 [2].

The joule balance experiment, proposed by B. P. Kibble in 1976 [3], is one of the promising methods to redefine kilogram. Since 1976, the watt balance idea has been realized with different approaches by National Physical Laboratory (NPL), U.K. [4], National Institute of Standard and Technology (NIST), USA [5], Federal Institute of Metrology, Switzerland (METAS) [6], Laboratoire national de métrologie et d’essais (LNE), France [7] and BIPM [8]. In 2010, the Consultative Committee for Mass and Related Quantities (CCM)’s recommendation G1 (2010) gives the requirement on the necessary conditions before redefinition of the kilogram [9].

1) At least three independent experiments, including work both from watt balance and from International Avogadro Coordination projects, yield values of the relevant constants with relative standard uncertainties not larger than five parts in $10^8$.
2) At least one of these results should have a relative standard uncertainty not larger than two parts in $10^9$.
3) For each of the relevant constants, values provided by the different experiments should be consistent at the 95%-degree level of confidence.

Among all the existing watt balance projects, only NPL, NIST and METAS have published results with relative uncertainties of $2 \times 10^{-7}$ [4], $3.6 \times 10^{-8}$ [5] and $2.9 \times 10^{-7}$ [6] separately. In 2009, the NPL watt balance apparatus was shipped to NRC in Canada and has been reassembled with the initial aim of reproducing the U.K. results. The recent result from the International Avogadro Coordination is $3 \times 10^{-8}$ [10]. Thus, the first requirement is not fulfilled so far.

The target uncertainty of the second condition has not been reached in any experiment. The participants of the Avogadro project will continue their work and plan to reduce the uncertainty of the Avogadro constant to two parts in $10^9$ in 2013.

The third condition is violated by the significant discrepancy between the NIST and NRC watt balances and the discrepancy between the NIST watt balance and the Avogadro result. Collaboration between NIST and NRC shall explain the origin of their discrepancy. From the point of view of mass metrology, it is, therefore, too early to make any decision on the redefinition of the kilogram [11].

Since 2006, National Institute of Metrology (NIM), China also started a similar work, known as the joule balance,
The mutual inductance value is demonstrated in [16]. The main uncertainty comes from the self-heating of the conventional coil system when a 250 mA dc current is applied to generate a 2.5 N magnetic force. A superconducting coil is thus built to reduce the influence of the self-heating.

Two approaches to measure the mutual inductance have been developed at NIM: the low frequency compensation method [14] and the standard square wave compensation method [15]. In [16], it has been presented that the mutual inductance of two coils can be expressed by the schematic shown on Fig. 2(a).

\[ M = M_0 - C_{12}R_1R_2*\omega^2 \]  

Here, \( M_0 \) is the mutual inductance at 0 Hz, \( C_{12} \) is the inter-capacitance between primary and secondary, \( R_1 \) and \( R_2 \) are the resistance of the two windings, and \( \omega \) is the angular frequency used to measure the mutual inductor.

In the low frequency compensation method [14], the mutual inductance is measured at several different frequencies. With (1), we can get the mutual inductance at 0 Hz. If the standard square wave compensation method is used, we can get the dc mutual inductance directly [15]. However, the measurement results of both approaches include the effect from distributed parameters. A correction of \( C_{12}R_1R_2 \) shown in (1) should be considered. When the superconducting coil is used, an obvious advantage is \( R_2 \) becomes zero, thus the correction from distribution parameters is not necessary. However, the big self-inductance of the superconducting coil brings new trouble for the current source of the mutual inductance measurement system described in [17]. The solution for measuring the mutual inductance of the superconducting coil is presented and discussed. The outline of the rest of this paper is organized as follows. Section II presents details of the superconducting coil. In Section III, the solution for measuring the mutual inductance of the superconducting coil is presented and discussed. The measurement results are shown in Section IV. A conclusion is drawn in Section V.

II. SUPERCONDUCTING COIL SYSTEM

In the NIST watt balance, liquid helium is used to cool the superconducting coil. Here, we try to use a pulse tube (PT) cryocooler to cool the superconducting coil for the joule balance to save the expensive liquid helium and avoid the trouble from injecting liquid helium to the Dewar. Since there is no moving part in cold head, the pulse tube cooler has several advantages, such as high reliability, low mechanical vibration, low magnetic noise, and long lifetime. A cryogenic cooling system using a two-stage cryocooler for the superconducting magnet has been designed, fabricated, and tested. The superconducting magnet is composed of NbTi solenoid coils. The NbTi solenoid is wound around a fiber reinforced plastics (FRP) frame. Even through the mechanical vibration is very low, it still needs to be isolated or damped for the high precision measurement application here.

Three aspects of the superconducting coil system used in joule balance should be considered with great attention.

1) Every part close to the superconducting coil should be made of nonmagnetic material to avoid possible disturbance during the mutual inductance measurement.
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2) The vibration from the cryocooler should be isolated to avoid the disturbance to the length measurement.
3) To avoid eddy current effect, the Dewar for the superconducting coil is made of FRP and the cooling-shield is made of nonmagnetic copper strips.

The structure of the superconducting coil is shown in Fig. 3 (all the units are in millimeters) and the arrangement of the new coil system is shown in Fig. 4.

The fixed coil is the superconducting coil. It is different from the arrangement in Fig. 1 in that the fixed coil is the single superconducting coil and two enamelled coils connected in opposite act as movable coil. The advantage of this design is to avoid the difficult alignment of two coils in a low temperature environment. In contrast, the alignment of the two coils in air is much easier. The superconducting coil is wounded with NbTi wire and its parameters are shown in Table I.

When a 10 A dc current is applied to the superconducting coil, a magnetic field of 0.1 T can be generated, which is higher than previous cooper wire coil.

### Table I

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Superconducting coil</th>
<th>Movable coil set</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inner radius, ( R_1 ) (mm)</td>
<td>86.8</td>
<td>150</td>
</tr>
<tr>
<td>Outer radius, ( R_2 ) (mm)</td>
<td>110</td>
<td>170</td>
</tr>
<tr>
<td>Axial dimensions, ( J ) (mm)</td>
<td>160</td>
<td>20</td>
</tr>
<tr>
<td>Turns of winding, ( N )</td>
<td>12348</td>
<td>3500 × 2</td>
</tr>
<tr>
<td>Current, ( I ) (A)</td>
<td>10</td>
<td>0.025</td>
</tr>
<tr>
<td>Wire diameter, ( d ) (mm)</td>
<td>0.5</td>
<td>0.3</td>
</tr>
<tr>
<td>Self-inductance, ( L ) (H)</td>
<td>20.85</td>
<td>6.084</td>
</tr>
</tbody>
</table>

In the mutual inductance measurement mode, the superconducting coil, the outer movable coil, is configured as direction and used as the secondary of the mutual inductor. Thus, no current is applied to the movable coil and only the induced voltage across its terminals is measured.

Using the superconducting coil brings the advantage of no heating with current. Another great advantage is that the magnetic field can be changed according to the current through it. Thus, the electromagnetic force between the fixed coil and the movable coil can be changed to balance the gravity of different masses easily, such as 1 kg, 500 g, etc. In these cases, the current going through the movable coil does not change, and the self-heating of the movable coil is almost the same.

However, to measure the mutual inductance of the coils, a slope current should be applied to the superconducting coil, for which self-inductance is almost 20 H and brings some difficulty for the current source. Details for the measurement of the mutual inductance are presented in the next section.

### III. MUTUAL INDUCTANCE MEASUREMENT OF THE SUPERCONDUCTING COIL

A standard square wave compensation method (SSCM) has been proposed by authors in [15]. The principle of the SSCM method is introduced briefly in Fig. 5.

Here \( i(t) \) is the exciting current ramp injected in the primary of the mutual inductor, \( I_1 \) and \( I_2 \) are stable currents at the start and the end of the ramp, \( u(t) \) is the voltage induced across the secondary winding, and \( E(t) \) is a standard square wave.
wave acting as a compensation voltage for the secondary of the mutual inductor. Its amplitude and width can be known precisely. \( \Delta S \) is the residual area between the induced voltage and compensation voltage. \( T_s \) is the starting point, and \( T_e \) is the end point for the digital acquiring of \( \Delta S \).

The measurement procedure is explained shortly as following. When a ramp current \( i(t) \) is applied to the primary of the mutual inductor, the induced voltage of the secondary is a square wave. The output of the secondary of the mutual inductor is

\[
u(t) = M \frac{di(t)}{dt}
\]

(2)

With the integral of both sides of (2), we obtain

\[
M = \int_{T_1}^{T_2} \frac{\nu(t)dt}{E_r - E_i} = \frac{S}{E_r - E_i}
\]

(3)

If \( T_1, T_2 \) and \( S \), the area of the output of the secondary of the mutual inductor, can be measured precisely, the mutual inductance can be accurately determined. It is, however, difficult to measure \( S \) directly with high accuracy due to performance of the voltage integrator. The area \( S \) is compensated by a standard square wave area \( S_0 \) with the same width and known voltage as

\[
S_0 = (E_0 - E_i) \cdot (T_2 - T_1) = (E_0 - E_i) \Delta T
\]

(4)

The voltage of the standard square wave can be traced to the Josephson voltage standard, and only the residual area \( \Delta S \) needs to be measured. With (3) and (4), we get

\[
M = S_0 + (S - S_0) \cdot (E_0 - E_i) \Delta T
\]

\[
= \frac{(E_0 - E_i) \Delta T}{E_2 - E_1} \left( 1 + \frac{S - S_0}{S_0} \right)
\]

(5)

\( E(t) \) is generated from a dc voltage standard and a switch circuit, and the area of \( S_0 \) can be determined with an uncertainty less than 1 part in \( 10^6 \). The dc voltage standard is a Fluke 732B for its good performance on short time stability. A digital integrator is used to acquire \( \Delta S \), and \( \Delta S/S_0 \) is measured at several points in \( 10^7 \). Thus, if \( \Delta S \) can be measured with an uncertainty of \( 10^{-3} \) level, the uncertainty of several parts in \( 10^6 \) for the mutual inductance measurement could be obtained.

However, for the mutual inductance measurement of superconducting coil, the 20 H self-inductance brings some challenge for the current source in the measurement system, i.e., there is a limitation of the voltage output of the buffer to excite such high impedance load.

In the SSCM method, the 1.018 V output of a 732B controlled by high-speed switches is used to compensate the induced voltage of the secondary of the mutual inductor. For a 1 H mutual inductance of the new superconducting coil system, the \( (E(t)/dt) \) should be approximately 1 A/s. The voltage drop on the self-inductance is about 20 V, which is beyond the capability of the buffer (LT1010) used at present. Two approaches have been tried to solve this problem as explained below.
The Meissner effect of superconducting wire. In [18], a similar inductance phase in joule balance because of the diamagnetic weighting phase and velocity phase in watt balance or mutual. Furthermore, superconducting wire will behave differently in different paths when the superconducting state disappears. The current applied to the superconducting coil flows in quite a degree change of mutual inductance is observed. It shows that stopped. Moreover, the change of the superconducting coil has been tested by stopping the cryocooler for a moment and mutual inductance change greatly after the cryocooler is stopped. Fig. 9 shows the resistance change after the cryocooler is stopped. Of the superconducting coil and Fig. 10 shows the mutual inductance change just after the cryocooler is stopped. From Figs. 9 and 10, it can be seen that both resistance and mutual inductance change greatly after the cryocooler is stopped. Moreover, the change of the superconducting coil has a 200 s delay compared with the resistance. By further analysis on Figs. 9 and 10, the almost 16-degree change of mutual inductance is observed. It shows that the current applied to the superconducting coil flows in quite different paths when the superconducting state disappears. Furthermore, superconducting wire will behave differently in weighting phase and velocity phase in watt balance or mutual inductance phase in joule balance because of the diamagnetic (Meissner effect) of superconducting wire. In [18], a similar phenomenon has been also observed. Thus, Meissner effect should be investigated further when the superconducting coil is used in both watt balance and joule balance.

V. CONCLUSION

The mutual inductance measurement is one of several key techniques for the joule balance at NIM. In the first stage, enamelled copper wire has been used to form the fixed coil and movable coil. The self-heating of the coils brings some trouble. To solve this problem, a superconducting coil has been designed and built to act as the fixed coil. In addition, there are some other advantages for the joule balance method, such as stronger magnetic field, less error from the coil resistance in the mutual inductance mode, etc. Measurements of the mutual inductance between the superconducting coil and the movable coil are presented here. In 30 seconds, the relative standard deviation of the mutual inductance measurement results is $6 \times 10^{-7}$. To test the feasibility to totally avoid the mechanical vibration, the cryocooler was stopped for a moment. However a big change of mutual inductance was observed. This experiment shows that Meissner effect should be investigated further when the superconducting coil is used in both watt balance and joule balance.

REFERENCES

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