An improved geopositioning model of QuickBird high resolution satellite imagery by compensating spatial correlated errors

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Abstract
A lot of studies have been done for correcting the systematic biases of high resolution satellite images (HRSI), which is a fundamental work in the geometric orientation and the geopositioning of HRSI. All the existing bias-corrected models eliminate the biases in the images by expressing the biases as a function of some deterministic parameters (i.e. shift, drift, or affine transformation models), which is indeed effective for most of the commercial high resolution satellite imagery (i.e. IKONOS, GeoEye-1, WorldView-1/2) except for QuickBird. Studies found that QuickBird is the only one that needs more than a simple shift model to absorb the strong residual systematic errors. To further improve the image geopositioning of QuickBird image, in this paper, we introduce space correlated errors (SCEs) and model them as signals in the bias-corrected rational function model (RFM) and estimate the SCEs at the ground control points (GCPs) together with the bias-corrected parameters using least squares collocation. With these estimated SCEs at GCPs, we then predict the SCEs at the unknown points according to their stochastic correlation with SCEs at the GCPs. Finally, we carry out geopositioning for these unknown points after compensating both the biases and the SCEs. The performance of our improved geopositioning model is demonstrated with a stereo pair of QuickBird cross-track images in the Shanghai urban area. The results show that the SCEs exist in HRSI and the presented geopositioning model exhibits a significant improvement, larger than 20% in both latitude and height directions and about 2.8% in longitude direction, in geopositioning accuracy compared to the common used affine transformation model (ATM), which is not taking SCEs into account. The statistical results also show that our improved geopositioning model is superior to the ATM and the second polynomial model (SPM) in both accuracy and reliability for the geopositioning of HRSI.

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1. Introduction

Although the physical sensor models, which account for all the factors that reflect the physical reality, can in principle describe the rigorous geometric relationships between 2D image space and 3D object space for the high resolution satellite sensors, they are still not widely used in practice since they are sensor-dependent and their parameters are kept confidential by some satellite image vendors (Tong et al., 2010). In contrast, the rational function model (RFM), one of the generalized imaging models, has great interests and has been extensively investigated in the last decade. The studies reported that RFM is an ideal replacement for the rigorous physical models due to its high fitting capability, simplicity, and independency of sensors (Li et al., 2007).

The RFM is usually defined as a ratio of two polynomials, whose coefficients (more precisely, the rational polynomial coefficients, RPCs) are supplied by the vendors together with satellite imagery data. The accuracy of the RFM solution was first investigated by Madani (1999) using SPOT images. And then Dowman and Dolloff (2000) investigated the error propagation information and reported that the rational functions can offer effective photogrammetric restitution for a wide range of sensors. Tao and Hu (2001, 2002) made a comprehensive study on the technical issues of the RFM, examined the inverse and forward RFM and derived a 3D reconstruction algorithm.

The RFM can build the satellite image geometry well, but there exist significant systematic biases between the coordinates derived from the RFM and the Earth reference system (ERS). The existing systematic biases can be classified as: (1) the biases introduced by the calibration and the sensor models; (2) the space-variant responses of the sensor (i.e. the spatial intensity variations); (3) the spatial correlated errors (SCEs) caused by the sensor’s focal plane (HPF) and the non-uniformity responses (i.e. the spatial correlated errors, SCEs) caused by the sensor’s focal plane (HPF) and the non-uniformity responses. In the literature, the first two biases are mostly referred to as the deterministic errors (Li et al., 2007). The studies reported that the deterministic errors are small and can be corrected by the common affine transformation models (ATM). However, the last one is a spatially correlated error caused by the characteristics of the sensor’s focal plane (HPF) and the non-uniformity responses, which is not taken into account in the model. This kind of spatial correlated error is not easy to be eliminated. The cause of the SCEs is the non-uniform responses of the sensor’s focal plane (HPF), which will generate spatial correlated errors during the acquisition process.

In the past 10 years, the spatial correlated errors (SCEs) have been considered for the high resolution satellite remote sensing (HRSR). However, most of the previous studies were conducted with a parametric model or a statistical model, which are either not robust or too complicated for practical applications. To date, the effective compensation for the spatial correlated errors is a fundamental work for the geometric orientation and the geopositioning of HRSI (Tong et al., 2010; Liu et al., 2006; Qiao et al., 2010). The spatial correlated errors (SCEs) can be introduced into the geopositioning framework by compensating the biases and the SCEs. In this study, we will present an improved approach for geopositioning QuickBird image by compensating spatial correlated errors (SCEs) in real time.
by using the vendor-provided RPCs and their true ones (Dial and Grodecki, 2002; Fraser and Hanley, 2003; Noguchi and Fraser, 2004; Wang et al., 2005; Aguilar et al., 2007), since the vendor-provided RPCs are usually generated by the terrain-independent methods. Thereby if the high geopositioning accuracy is expected, these systematic biases must be adequately compensated or even eliminated. So far, there are generally two kinds of methods popularly applied to do this task. One is to model these biases with the polynomials in image space or object space, which is referred to as the bias-corrected RFM. But studies showed that the biases modeled in image space is more effective (Fraser et al., 2002). The other is to refine the vendor-provided RPCs, namely, recomputing and regenerating these RPCs with the information of ground control points (Grodecki and Dial, 2001; Tao and Hu, 2001; Tong et al., 2010). The bias-corrected RFM is more appealing than the RPC regeneration because its computation of parameters is easier and more flexible, while the RPC regeneration not only needs to develop multiple layers of 3D grids, but also is numerically illposed during RPCs resolving.

Basically three bias-corrected RFM models are frequently employed, i.e., the shift model, the shift and drift model, as well as the ATM (Fraser and Hanley, 2003, 2005). Wang et al. (2005) evaluated the accuracies of different bias-corrected RFMs aided with different configuration of ground control points (GCPs). It is concluded that the high-order polynomials cannot necessarily obtain the significant improvement. In the algorithm aspect, the RPC block adjustment of 3D reconstruction technique was proposed to solve the parameters of the bias-corrected RFM and the ground coordinates simultaneously. This technique can obtain the result as accurate as from the physical camera model for IKONOS imagery, but it is much simpler and more stable (Grodecki and Dial, 2003).

However, the bias-corrected RFMs in the aforementioned literatures are effective for most of the commercial HRSI, i.e., IKONOS, GeoEye-1 and WorldView-1/2 imagery, except for QuickBird imagery. Studies find that QuickBird is the only one that needs that more than a simple shift model to compensate its strong residual systematic errors. The geometric accuracy at the sensor orientation stage of QuickBird Basic images, both in mono and stereo blocks, are improved by using a shift and drift or even better with an ATM bias-compensated RFMs. Moreover, the accuracy results are even better by using the Rigorous model by Toutin included in Geomatica OrthoEngine (Toutin, 2006; Aguilar et al., 2008). It means the QuickBird has the more dynamic nature during scanning imaging and gives rise to a component of higher order error which is not compensated by the shift and drift parameters as reported Fraser et al. (2006). The rest of the high resolution satellites achieve better geopositioning results by using only a translation in space image, i.e., only a shift using one GCP (Toutin, 2006; Fraser et al., 2006; Aguilar et al., 2013). So for QuickBird images, the parameters in the aforementioned bias-corrected RFM models can only capture the lower order systematic errors, but not the stronger spatial correlated errors (SCRs) in the images. To further improve the geopositioning of QuickBird images, these SCRs should be adequately compensated. Shen et al. (2012) has successfully modeled such correlated errors as signals, and solved by least squares collocation (LSC). For the details of LSC, one can refer to Koch (1977) and Tscherning (1978), or recently Yang et al. (2009). Encouraged by the significant improvement of about 15% of geopositioning accuracy in Shen et al. (2012), we will further and comprehensively investigate the effects of SCRs on QuickBird image geopositioning in this paper.

The reminder of this paper is organized as follows. After reviewing the traditional bias-corrected RFM in Section 2, we introduce the SCRs to the bias-corrected RFM and present a method of computing the SCRs at GCPs and unknown points in Section 3. In Section 4 we present the numerical examples to demonstrate the accuracy improvement of our improved geopositioning method. The concluding remarks are summarized in Section 5.

2. Traditional bias-corrected RFM

Since the vendor-provided RPCs of HRSI are calculated by the terrain-independent method, there exist the systematic biases for sensor orientation and geopositioning. In order to estimate and remove such systematic biases, the bias-corrected RFM in the image space was formulated as (OGC, 1999; Tong et al., 2010)

\[
\begin{align*}
\Delta r &= \frac{P_i(x, y, h)}{P_i(x, y, h)} - \frac{r_{\text{scal}} + r_{\text{off}}}{r_{\text{scal}} + c_{\text{off}}} \\
\Delta c &= \frac{P_i(x, y, h)}{P_i(x, y, h)} - \frac{c_{\text{scal}} + c_{\text{off}}}{c_{\text{scal}} + c_{\text{off}}}
\end{align*}
\]

where \(P_i(x, y, h)\) are the polynomials limited to the third order, \(r\) and \(c\) are the image coordinates, \(r_{\text{off}}\) and \(c_{\text{off}}\) are the offsets for the two image coordinates, \(r_{\text{scal}}, c_{\text{scal}}\) are their scale factors, \((\phi, \lambda, h)\) represent the nominal geodetic latitude, longitude and height coordinates in object space, \(\Delta r\) and \(\Delta c\) are the systematic biases of image coordinates.

The most frequently used bias-corrected RFM describes the systematic errors as polynomials of line and sample coordinates (Grodecki and Dial, 2003).

\[
\begin{align*}
\Delta r &= \xi_1 + \xi_2 r + \xi_3 c + \xi_4 r^2 + \xi_5 r c + \xi_6 c^2 \\
\Delta c &= \zeta_1 + \zeta_2 r + \zeta_3 c + \zeta_4 c^2 + \zeta_5 r c + \zeta_6 c^2
\end{align*}
\]

where \(\xi_i\) and \(\zeta_i\) are the bias-corrected parameters or bias compensation parameters. In real applications, more than second order of the polynomials is not recommended. Formula (2) is called the second-order polynomial model (SPM), which will be used to compare with our proposed method. Based on the SPM above, three widely used bias-corrected RFMs reduced from it, i.e.

(i) Shift model with parameters \(\xi_1\) and \(\xi_2\).
(ii) Shift and drift model with parameters \(\xi_i\) (\(i = 1, 2, \ldots, 4\)).
(iii) Affine transformation model (ATM) with \(\xi_i\) (\(i = 1, 2, \ldots, 6\)).

The parameters of the bias-corrected RFM are estimated by least squares adjustment. Taking the ATM as an example, if \(m\) GCPs are collected, the observation equation of estimating the bias-corrected parameters is formed by substituting (2) into (1)

\[
L = AX + E
\]

where \(X = [\xi_1, \xi_2, \xi_3, \xi_4, \xi_5, \xi_6]^{T}\) is the vector of the bias-corrected parameters; \(E = [e_{i1}^{T} \cdots e_{im}^{T}]^{T}\) and \(e_{il}^{T} = [e_{il}, e_{il}]^{T} (i = 1, \ldots, m)\) indicates the observational error vector. \(L = [l_1^{T} \cdots l_m^{T}]^{T}\)

\[
\begin{align*}
\Delta r_i &= \frac{P_i(x, y, h)}{P_i(x, y, h)} - \frac{r_{\text{scal}} + r_{\text{off}}}{r_{\text{scal}} + c_{\text{off}}} - r_i \\
\Delta c_i &= \frac{P_i(x, y, h)}{P_i(x, y, h)} - \frac{c_{\text{scal}} + c_{\text{off}}}{c_{\text{scal}} + c_{\text{off}}} - c_i
\end{align*}
\]

The design matrix reads

\[
A = \begin{bmatrix}
1 & 0 & r_1 & 0 & c_1 & 0 \\
0 & 1 & 0 & r_1 & 0 & c_1 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
1 & 0 & r_m & 0 & c_m & 0 \\
0 & 1 & 0 & r_m & 0 & c_m
\end{bmatrix}
\]

Assume the covariance matrix of the observation errors \(e\) as

\[\Sigma_e = \sigma^2_e Q_{ee}\]

Here \(Q_{ee}\) is the cofactor matrix and \(\sigma^2_e\) is the variance of unit weight. The least squares solutions of parameter \(X\) is
\[ \mathbf{X} = (\mathbf{A}^t \mathbf{Q}_{st}^{1} \mathbf{A})^{-1} \mathbf{A}^t \mathbf{Q}_{st}^{1} \mathbf{L} \] (5)

and the posterior variance of unit weight is computed as

\[ \sigma^2 = \frac{(\mathbf{L} - \mathbf{AX}) \mathbf{Q}_{st}^{1} (\mathbf{L} - \mathbf{AX})}{2m - 6} \] (6)

where the terms with hat symbol indicate the estimated values. Furthermore, the covariance matrix \( \Sigma \) of the estimated parameters \( \hat{\mathbf{X}} \) is derived with

\[ \Sigma = (\mathbf{A}^t \Sigma^{-1} \mathbf{A})^{-1} = \sigma^2 (\mathbf{A}^t \mathbf{Q}_{st}^{1} \mathbf{A})^{-1} \] (7)

If \( \sigma^2 \) is unknown, one can use its posterior estimate in Eq. (6) replace it. Once the bias-corrected parameters of all the images are resolved with Eqs. (3-6), the image coordinate biases at unknown points can be calculated using the ATM. Then one is able to carry out geopositioning with the bias-corrected satellite images by linearize Eq. (1). For more details of this process, refer to Wang et al. (2005) and Grodecki and Dial (2001, 2003).

3. Improved model and its solution

3.1. Improved bias-corrected RFM

The improved bias-corrected RFM takes the SCEs into account and model them as signals, its observation equation reads

\[ \mathbf{L} = \mathbf{AX} + \mathbf{S} + \mathbf{E} \] (8)

where \( \mathbf{S} = [s_{1}^t; \ldots; s_{n}^t]^t \) and \( s_{i}^t = [s_{i1}; s_{i2}; \ldots; s_{im}](i = 1, \ldots, m) \) is the ith vector of SCEs. It is emphasized here that \( \mathbf{S} \) is assumed random rather than deterministic. The statistics of \( \mathbf{S} \) are

\[ \mathbf{E}(\mathbf{S}) = \mathbf{0}, \quad \Sigma_{ss} = \sigma^2 \mathbf{Q}_{ss} \] (9)

where \( \mathbf{Q}_{ss} \) is the corresponding cofactor matrix and \( \sigma^2 \) is its variance of unit weight.

In following, it is assumed that: (I) the SCEs and observation errors are independent, namely, \( \mathbf{Q}_{ss} = \mathbf{0} \); (II) the SCEs at the GCPs are correlated with those at the unknown points (denoted as \( \mathbf{S}' \)), namely, \( \mathbf{Q}_{ss} \neq \mathbf{0} \); (III) the SCEs in line direction (\( s_{i} \)) are independent with those in sample direction (\( s_{j} \)) in the pushbroom HRSI, namely \( \mathbf{Q}_{ss} = \mathbf{0} \).

The cofactor matrices of SCEs \( \mathbf{Q}_{ss} \) and \( \mathbf{Q}_{sr} \) are constructed by

\[ \begin{align*}
\mathbf{Q}_{ss} &= \mathbf{I}_{m \times m} \otimes \mathbf{Q}_{ss}, \quad \text{for} \ i, j = 1, \ldots, m \\
\mathbf{Q}_{sr} &= \mathbf{I}_{m \times (n-m)} \otimes \mathbf{Q}_{sr}, \quad \text{for} \ i \in \mathbf{P}_s, j \in \mathbf{P}_r, (i = 1, \ldots, m; j = m+1, \ldots, n+m) 
\end{align*} \] (10)

where \( \mathbf{I}_{m \times m} \) and \( \mathbf{I}_{m \times (n-m)} \) are the ones-matrix of \( m \times m \) and \( m \times (n-m) \) dimension; the symbol \( \otimes \) represents Kronecker product; \( \mathbf{P}_s \) and \( \mathbf{P}_r \) indicate GCP set and unknown point set. \( m \) is the number of GCPs and \( n \) is number of all the ground points. \( \mathbf{Q}_{ss} \) is a two-dimension cofactor matrix of the SCEs at the ith and jth point, whose form as

\[ \mathbf{Q}_{ss} = \begin{bmatrix} q_{ii}^* & 0 \\ 0 & q_{jj}^* \end{bmatrix} \]

where \( q_{ii}^* \) is covariance of SCEs of the ith and jth point in the line direction. \( q_{jj}^* \) is the corresponding cofactor in sample direction. When \( i = j \), they are the corresponding variance components. Their values are commonly computed with a prior empirical covariance function (ECF), which will be discussed in Section 3.3.3.

3.2. Solution of improved bias-corrected RFM

The observational Eq. (8) can be solved by LSC, which minimizes the following cost function

\[ \varepsilon^t \mathbf{X} + \mathbf{S}^t \mathbf{X}^t \mathbf{S} = \min \] (11)

The solutions of the bias-corrected parameters and SCEs of the control points are get by

\[ \begin{align*}
\hat{\mathbf{X}} &= (\mathbf{A}^t \Sigma^{-1} \mathbf{A})^{-1} \mathbf{A}^t \Sigma^{-1} \mathbf{L} \\
\hat{\mathbf{S}} &= \Sigma_{ss}^{-1} \mathbf{L} - \mathbf{AX}
\end{align*} \] (12)

where \( \Sigma = \Sigma_{ss} + \Sigma_{sr} = \sigma^2 \mathbf{Q}_{ss} + \sigma^2 \mathbf{Q}_{sr} \). For the details of derivation, one can refer to Koch (1977). Then with the stochastic relation of the SCEs at the control points and the unknown points, the SCEs of the unknown points are predicted as follows

\[ \hat{\mathbf{S}} = \Sigma_{ss} \Sigma_{ss}^{-1} \hat{\mathbf{S}} \] (13)

Using the bias-corrected parameters and the predicted SCEs, we can compute the corrections of systematic biases and SCEs as

\[ \begin{bmatrix} \Delta \mathbf{r}'^t_i \\ \Delta \mathbf{c}'_i \end{bmatrix} = \begin{bmatrix} 1 & 0 & r'_i & 0 & c'_i & 0 \\ 0 & 1 & 0 & r'_i & 0 & c'_i \end{bmatrix} \mathbf{X} + \hat{\mathbf{S}}_i \] (14)

where \( [\Delta \mathbf{r}'^t, \Delta \mathbf{c}'^t_i] \) denotes the corrections for the ith unknown point; \( [r'_i, c'_i] \) and \( \hat{\mathbf{S}}_i = [s'_i, s'_j] \) are its corresponding image coordinates and predicted SCEs of line and sample direction, respectively.

Substituting Eq. (14) into the linearized equation of Eq. (1), the geopositioning for the unknown points can be easily carried out referencing to Wang et al. (2005) and Grodecki and Dial (2001, 2003).

3.3. Empirical covariance function

The most important and critical aspect to extract and remove the SCEs from the image point observations is to ensure that the covariance matrix \( \Sigma_{ss} \) is known beforehand. But actually the covariance matrix \( \Sigma_{ss} \) is unknown in most cases, so the key problem comes to make a choice of an appropriate empirical covariance function (ECF) and determine its parameters to compute the elements of \( \Sigma_{ss} \).

The ECF should satisfy the following mathematical properties:

(1) The covariance of any two points is the continuous function of the distance, which is isotropic and only distance-dependent.

(2) The longer the distance between two points, the smaller the covariance for them. Hence if the distance is sufficiently long, the covariance should be close to 0.

(3) The covariance matrix computed by ECF should be positive-definite.

According to the three mathematical properties, the Gaussian function (Moritz, 1980; Yang et al., 2009) is most commonly used in practice, which is employed in this study as

\[ C(d_{ij}) = C_0 e^{-k d_{ij}^2} \] (15)

where \( C(d_{ij}) \) denotes the covariance between the ith and jth points with the distance \( d_{ij} \), and \( C_0 \) and \( k \) are the constants to be determined. According to Yang et al. (2009) we compute the constants \( C_0 \) and \( k \) with following three steps.

Firstly, take the residuals \( \mathbf{v} = \mathbf{L} - \mathbf{AX} \) of Eq. (3), compute the distance \( d_{ij} \) and its corresponding covariance component \( C(d_{ij}) = v_i v_j \) of two arbitrary ith point and jth point.

Secondly, sort the covariance components according to their distances in ascending order, and classify the covariance components into \( t \) groups based on the given distance intervals \( d_k \) \( (k = 1, \ldots, t) \), and then compute the covariance component \( C(d_k) \) at distance \( d_k \) with all the possible products \( v_i v_j \) in the kth group as follows:
\[ C(d_k) = \frac{1}{n_k} \sum_{d_i \in d_k} C(d_i) \]  

(16)

where \( n_k \) is the number of covariance components and \( S_k \) is the set of distances in the \( k \)th group.

Thirdly, estimate the constants \( C_0 \) and \( k \) with non-negative \( C(d_k) \) by using least squares adjustment, since \( C(d_k) \) is not allowed to be negative.

### 3.4. Variance component estimation of the SCEs and observation errors

The covariance matrices \( \Sigma_{xx}, \Sigma_{ss} \) and \( \Sigma_{xs} \) are relevant to not only their cofactor matrices \( Q_{xx}, Q_{ss} \) and \( Q_{xs} \), but also the variance components \( \sigma_x^2 \) and \( \sigma_s^2 \) of the SCEs and observation errors, which are also unknown. The cofactor matrices \( Q_{xx} \) and \( Q_{ss} \) of the SCEs are constructed in terms of Eq. (8) while \( Q_{xs} \) of the observation errors is generally taken as an identity matrix as the observing accuracies of image points are usually the same. The variance components are empirically given at advance. However, if these empirical variance component values are not adequately correct, which will result in systematic errors in the parameter estimates (Yang et al., 2009). In this paper, we employ the Helmert type of VCE (Variance Component Estimation) method to recover the realistic variance components. The iteration formula of Helmert VCE reads

\[
\begin{bmatrix}
n_0 - 2tr(N^{-1}N_{ss}) + tr(N^{-1}N_{ss}N^{-1}N_{ss}) \\
tr(N^{-1}N_{ss}) \\
n_0 - 2tr(N^{-1}N_{ss}) + tr(N^{-1}N_{ss})^2
\end{bmatrix}
\left[
\begin{bmatrix}
n_0 - 2tr(N^{-1}N_{ss}) + tr(N^{-1}N_{ss}N^{-1}N_{ss}) \\
tr(N^{-1}N_{ss}) \\
n_0 - 2tr(N^{-1}N_{ss}) + tr(N^{-1}N_{ss})^2
\end{bmatrix}
\right]^{-1}
\]

\[
\begin{bmatrix}
\frac{\sigma_x^2}{\sigma_s^2} = S^\top Q_{ss} S
\end{bmatrix}
\]

(17)

where \( n_0 \) is the number of image observations, \( n \) denotes the number of SCEs at GCPs and in this paper \( n = k \). \( p = L - AX - S \)

\[ N_{ss} = \begin{bmatrix} A^\top Q_{xx}^{-1} A & A^\top Q_{xs}^{-1} \\ Q_{xx}^{-1} A & Q_{xs}^{-1} \end{bmatrix}, \quad N_{ss} = \begin{bmatrix} 0 & 0 \\ 0 & Q_{ss}^{-1} \end{bmatrix} \quad \text{and} \quad N = N_{ss} + N_{xx}. \]

One can refer to Yang et al. (2009) for other VCE methods in the collocation model and to Li et al. (2011) for the efficient computation techniques.

### 4. Experiments and analysis

#### 4.1. Dataset description

A stereo pair of cross-track QuickBird Basic images acquired in February (called left image in our experiments) and May (right image) 2004 are used to test the presented method, their characteristics are show in Table 1. The images cover about 18 km \( \times \) 18 km area over the center city of Shanghai. A total of 141 ground points (road roundabouts and building corners) are precisely measured by DGPS with an accuracy of better than 5 cm, of which, 50 evenly distributed points are used as GCPs and the rests are used as check points (CKPs). The image coordinates of the 141 points are carefully measured to nominal accuracy of 0.5 pixels with the ERDAS LPS tool. The elevations are from 12 m to 18 m for most of these points except very few higher than 100 m. Fig. 1 shows the layout of all the GCPs and CKPs, where the triangles and dots denote the GCPs and CKPs, respectively.

#### 4.2. Estimating two constants of ECF

We use the normalized distance in estimating the parameters of ECF. The normalizing formula is

\[ d_i = \sqrt{(r_{ij} - r_{ii})^2 + (e_{ij} - e_{ii})^2} \]  

(18)

where \( r_i \) and \( e_i \) are the normalized image coordinates of the QuickBird image, with the expression as

\[ r_i = \frac{r - r_{off}}{r_{scal}}, \quad e_i = \frac{e - e_{off}}{e_{scal}} \]  

(19)

Theoretically, 3 image control points are enough to estimate the 2 constants, but the more control points are used, the results are more precise. We compute the constants \( C_0 \) and \( k \) of the Gauss covariance function using the method described in Section 3.3 based on the image coordinate residuals of all the 50 GCPs. The results presented in Table 2 show that the correlation is higher in the line than the sample direction for both QuickBird images, which can surmise that the facts contributing to the SCEs have more impact on along-track (line direction) than cross-track direction (sample direction) for QuickBird forward-scanning imagery.

#### 4.3. Experimental results and analysis

The geopositioning process of HRIS with the improved ATM (IATM) by correcting the SCEs is carried out with the following steps:

- **The first step:** compute the cofactor matrices of SCEs \( Q_{xx} \) and \( Q_{ss} \) by using the covariance components of Gauss covariance function (15) with the used GCPs and CKPs.

Fig. 1. Layout of GCPs (the triangles) and CKPs (the dots) demonstrated in the left image.
The second step: estimate the bias-corrected parameters $X$ and the SCEs $S$ at GCPs with Eq. (12), meanwhile, the variance components of the SCEs and the observation errors should also be estimated by Eq. (17) in this step.

The third step: predict the SCEs $\hat{S}$ at the CKPs with Eq. (13).

The last step: calculate systematic biases of all the CKPs using the estimated bias-corrected parameters $X$ and the SCEs $\hat{S}$ with Eq. (14), and then carry out geopositioning using linearized equation of (1).

In the following, three experiment schemes are designed to test the performance, stability and reliability of IATM, and without special note, the short notations $lat$, $lon$ and $hg$ denote the latitude, longitude and height, respectively.

### 4.3.1. Performance of IATM

In this subsection, we test the geopositioning accuracy of ATM, SPM and IATM in 5 scenarios specified by the number of the used GCPs. The GCPs in all scenarios are evenly distributed in the image plane and the elevation. For each experiment scenario, we implement three geopositioning methods to compute the geo-coordinates of all 91 CKPs. Then the RMS accuracy is computed by

$$\text{RMS}(X) = \sqrt{\frac{\sum_{i=1}^{91} AX_i^2}{91}}$$  \hspace{1cm} (20)$$

where $X$ stands for either the ground coordinate component of latitude, longitude, height or the image coordinate components of line, sample of the left and right images; $AX_i$ is the difference between the computed coordinate and measured coordinate. Table 3 lists the computed RMSs of these differences in line, sample, latitude, longitude and height directions derived by different models. In addition, the relative accuracy improvement percentages of SPM and IATM relative to ATM are calculated by

$$p = \frac{\text{RMS}_{ATM} - \text{RMS}_{SPM/IATM}}{\text{RMS}_{ATM}} \times 100\%$$  \hspace{1cm} (21)$$

where the subscripts, ATM and SPM/IATM, indicate the models by which the RMS is computed.

Based on the results in Table 3, the following research findings are summarized:

1. With the more GCPs, the accuracies in height direction are improved for all the geopositioning methods, while the accuracies in latitude and longitude directions are almost invariant. It implies that the accuracy in height is more sensitive to the number of GCPs.

2. In the image space, both SPM and IATM can achieve the accuracy improvements for the left and right images relative to ATM, especially in the line direction, but the IATM is more significant, the relative improvement can reach 22.5% and 18.2% for the line direction in the left and right images with 10 GCPs.

3. In the object space, the accuracies of SPM in latitude direction are obviously better than those of ATM in all the 5 tests, but in other two directions the accuracy improvements are very marginal or even worse in the cases of using 10 and 15 GCPs; The proposed IATM exhibits an apparent accuracy improvement relative to ATM and SPM in all the three latitude, longitude and height directions though relatively smaller in longitude, which means that it has a better performance than both ATM and SPM in geopositioning.

4. With sufficient number of GCPs ($\geq 25$), the larger than 20% improvements relative to ATM are achievable in latitude and height for IATM, which are corresponding to 15 cm and 18 cm, whereas, if the number of GCPs is more than 25, the further improvement is not significant anymore.

Fig. 2 shows the extracted SCEs at CKPs with different number of GCPs. The subplots from first to last rows correspond to the 5 experimental scenarios. The results indicate that the extracted SCEs of line are markedly bigger than that of sample for both left and right images, and meanwhile, the SCEs are indeed significant, even more 1 pixel at some point, therefore it is unreasonable to ignore these SCEs in HRSI.

### Table 2

Estimated constants of Gauss covariance function for the left and right images.

<table>
<thead>
<tr>
<th>Direction</th>
<th>Constants</th>
<th>Left image</th>
<th>Right image</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$C_0$</td>
<td>$K$</td>
<td>$C_0$</td>
</tr>
<tr>
<td>Line</td>
<td>1.1530</td>
<td>4.6660</td>
<td>0.2264</td>
</tr>
<tr>
<td>Sample</td>
<td>0.0699</td>
<td>14.7403</td>
<td>–</td>
</tr>
</tbody>
</table>

*Note: ‘‘–’’ means the parameters of Gauss covariance function in this direction cannot be estimated, that is, there is no SCEs or very little SCEs in this direction.*

### Table 3

The RMS values of CKPs calculated by different models.

<table>
<thead>
<tr>
<th>Model scheme</th>
<th>Number of GCPs</th>
<th>RMS values of CKPs in the image space (pixel)</th>
<th>RMS values of CKPs on the ground (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Line Sample Line Sample Latitude Longitude Height</td>
<td>Line Sample Left image Right image</td>
</tr>
<tr>
<td>ATM</td>
<td>10</td>
<td>1.02 0.73 0.88 0.64</td>
<td>0.583 0.463 0.940</td>
</tr>
<tr>
<td></td>
<td>15</td>
<td>1.09 0.80 0.83 0.69</td>
<td>0.579 0.534 0.976</td>
</tr>
<tr>
<td></td>
<td>25</td>
<td>1.00 0.75 0.84 0.67</td>
<td>0.585 0.503 0.949</td>
</tr>
<tr>
<td></td>
<td>35</td>
<td>1.00 0.78 0.84 0.67</td>
<td>0.584 0.506 0.897</td>
</tr>
<tr>
<td></td>
<td>50</td>
<td>1.00 0.74 0.85 0.65</td>
<td>0.595 0.487 0.874</td>
</tr>
<tr>
<td>SPM</td>
<td>10</td>
<td>0.86 0.74 0.86 0.62</td>
<td>0.564 0.488 1.056</td>
</tr>
<tr>
<td></td>
<td>15</td>
<td>0.93 0.84 0.64 0.75</td>
<td>0.431 0.546 1.093</td>
</tr>
<tr>
<td></td>
<td>25</td>
<td>0.72 0.74 0.68 0.68</td>
<td>0.461 0.485 0.912</td>
</tr>
<tr>
<td></td>
<td>35</td>
<td>0.73 0.77 0.69 0.67</td>
<td>0.458 0.497 0.906</td>
</tr>
<tr>
<td></td>
<td>50</td>
<td>0.72 0.74 0.67 0.64</td>
<td>0.454 0.477 0.869</td>
</tr>
<tr>
<td>IATM</td>
<td>10</td>
<td>0.79 0.73 0.72 0.64</td>
<td>0.528 0.455 0.786</td>
</tr>
<tr>
<td></td>
<td>15</td>
<td>0.69 0.69 0.78 0.69</td>
<td>0.542 0.519 0.791</td>
</tr>
<tr>
<td></td>
<td>25</td>
<td>0.67 0.75 0.61 0.67</td>
<td>0.448 0.479 0.709</td>
</tr>
<tr>
<td></td>
<td>35</td>
<td>0.65 0.77 0.64 0.67</td>
<td>0.465 0.487 0.707</td>
</tr>
<tr>
<td></td>
<td>50</td>
<td>0.63 0.74 0.62 0.65</td>
<td>0.448 0.470 0.700</td>
</tr>
</tbody>
</table>
Fig. 2. The predicted SCEs at CKPs for left and right images using IATM with different number of GCPs, 10, 15, 25, 35 and 50 for subplots from (a) to (e), respectively. The dash and solid lines indicate the results for the line and sample directions, respectively.
the weights of SCEs and the observation errors. The experiment results showed that our IATM can greatly improve the geopositioning accuracy of QuickBird image compared to the commonly used ATM or even the SPM.

The main conclusions can be summarized as follows:

1. The SCEs indeed exist in the vendor provided RPCs of QuickBird HRSI, especially in the line direction. Our proposed IATM can successfully recover and then adequately compensate the SCEs from the image coordinate observations to achieve the high accuracy geopositioning.

2. The configuration of the used GCPs affects the geopositioning accuracy of QuickBird image. If the used GCPs are in very poor geometry, the accuracy of IATM may be lower than that of ATM, but if they are evenly distributed, the probability that the accuracy is improved by IATM can reach 99.57% or even 100% with only 10 or 15 GCPs.

3. The determination of weight scale of observation errors and SCEs is very important to IATM. That may be a problem as the exactly variance components cannot be prior known in operational environments. But experiments find that if we have 10 or more GCPs, we can estimate the variance components well.

4. Theoretically, the method proposed in this paper can apply to any satellite imagery that has significant SCEs, but studies find that the high-order errors (SCEs belong to high-order errors) in IKONOS, GeoEye-1, WorldView-1/2 imagery are tiny except QuickBird imagery. So our method is more useful to deal with QuickBird imagery.

### Acknowledgements

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### References


### Table 4

<table>
<thead>
<tr>
<th>Number of GCPs</th>
<th>RIM (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>IATM</td>
</tr>
<tr>
<td>10</td>
<td>79.8</td>
</tr>
<tr>
<td>15</td>
<td>94.4</td>
</tr>
<tr>
<td>25</td>
<td>99.8</td>
</tr>
<tr>
<td>35</td>
<td>100</td>
</tr>
<tr>
<td>45</td>
<td>100</td>
</tr>
</tbody>
</table>

4.3.2. Reliability and stability of IATM

In order to test the reliability and stability of our proposed method, that is, less sensitive to the numbers and configurations of GCPs, in this experiment, we randomly select a certain number of points (10, 15, 25, 35 and 45) as control information by computer from 50 GCPs to carry out geopositioning using ATM, SPM and IATM respectively. Then their corresponding RMs of CKPs are computed as

\[
RMS = \sqrt{\text{RMS(lat)}^2 + \text{RMS(long)}^2 + \text{RMS(hg)}^2} \quad (22)
\]

If the RMs for SPM/IATM are smaller than those for ATM, it means their geopositioning accuracies are improved. This procedure is repeated 10,000 times for each scenario. Then the number of times \(N_{\text{RM}}\), for which the accuracy is improved by SPM or IATM relative to ATM is counted. The ratio \(R_{\text{IM}}\) of accuracy improvement times out of total 10,000 is computed with

\[
R_{\text{IM}} = \frac{N_{\text{RM}}}{10000} \times 100\% \quad (23)
\]

Since the GCPs are randomly selected, their distribution may be poor or good. Thereby, 10,000 computations can statistically reflect the reliability and stability of SPM/IATM method. Large \(R_{\text{IM}}\) means that the SPM/IATM method is reliable, otherwise not. Table 4 shows the \(R_{\text{IM}}\) values of SPM and RATM with varying number of GCPs. It clearly demonstrates that the \(R_{\text{IM}}\) of SPM is always smaller than that of IATM, namely, our IATM is definitely better than SPM. Particularly, if the number of GCPs is less than 15, only 47% computation times of SPM are better than ATM, whereas the IATM performance is much better than SPM. Its \(R_{\text{IM}}\) with respect to ATM is overall very larger even 79.8% with only 10 GCPs.

After further investigation, we find that the cases that the accuracies of IATM are worse than that of ATM in the above 10,000 times computation are due to either the divergence of variance components estimation or the poor geometry configuration of the GCPs. If we fix five GCPs (four in the image corners and one in the middle) and the variance components, which are estimated with all 50 GCPs and assumed prior known, the results show that the \(R_{\text{IM}}\) of IATM can be further improved to 99.97% and 100% only with 10 and 15 GCPs, while only 42.37% and 62.08%, respectively, for SPM when we fix the same five GCPs. Therefore it is concluded that as long as the variance components of observation errors and SCEs are reasonably determined and the geometric configuration of GCPs is good, the presented IATM is confidently better than ATM and SPM.

5. Concluding remarks

In this paper, we introduced spatial correlated errors in the affine transformation model to realize the high accurate geopositioning of QuickBird HRSI. Based on the theory of least squares collocation, the formulated model was solved. The formulation of computing the bias-corrected parameters, the SCEs at GCPs as well as the predicting of the SCEs at the unknown points were derived. In addition, the VCE method was employed to optimally balance


