Optimal Auctions for Deregulated Electricity Markets

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1. Introduction: Power System Operation

- A power system has different types of generators

  Thermal Generator  
  Hydro Generator  
  Pumped Storage Unit

  Nuclear Generator  
  Transmission grid  
  Substation

2009  
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Generators are more or less independent, each with its cost function and constraints.

Together, they have to meet the time varying system demand and reserve requirements without much storage.

Unit Commitment and Economic Dispatch: Which units should be on, when, and at what generation level.
Unit Commitment and Economic Dispatch Formulation

- To minimize the sum of the energy and startup costs

\[
\min J, \text{ with } J \equiv \sum_{t=1}^{T} \sum_{i=1}^{I} \{C_i(p_i(t), t) + S_i(t)\}
\]

subject to the additive system demand constraints (additive reserve requirements omitted here):

\[
\sum_{i=1}^{I} p_i(t) = P_d(t), \quad t = 1, 2, \ldots, T
\]

and max/min power level constraints:

\[
P_{i\text{min}} \leq p_i \leq P_{i\text{max}}
\]

- An NP-hard mixed integer optimization problem, but is separable
- Has been solved by Lagrangian relaxation or other methods
Deregulated Electric Markets

- Participants submit **supply bids and demand bids** to the Independent System Operator (ISO)

  Supply Bids and Demand Bids From Participants

  **ISO**

  Generation Levels and MCP’s

  Participant 1  Participant 2  …  Participant I

- ISO conducts market **auctions** to determine
  - Hourly generation/demand levels over a day
  - Market clearing prices (MCP’s)
- The market is then **settled** by using a settlement scheme
6.5 M customers; population 14 M
350+ generators and power plants
300+ participants in marketplace
8,000+ miles of transmission lines
12 interconnections to NY and Canada
Peak demand: 28,130 MW (8/02/06)
32,000 MW of total supply
$10 billion energy market (2007)
www.ISO-NE.org
- Types of market
  - Energy: “Day ahead” and “Real-time”
  - Regulation
  - 10 minute spinning
  - 30 minute non-spinning
  - Capacity
  - Transmission

- Market design is an active research area to reduce uplift payments caused by startup costs and no-load costs
Are there issues for the market?
- No problem. Market will take care of itself!
In This Talk:

- Bid Cost Minimization vs. Payment Cost Minimization
  - A simple but revealing example. Surprise?
- Formulations of the Two Auction Mechanisms
  - A standard separable formulation vs. a non-separable formulation with the cost structure depending on bids selected
- Solution Methodology for Payment Cost Minimization
  - A novel approach combining Augmented Lagrangian relaxation and surrogate optimization
- A Game Theoretic Study of the Two Auction Methods
  - Confirmation of the finding and some empirically observed behaviors
2. Bid Cost Minimization/Payment Cost Minimization

- Two settlement schemes:
  - **Pay-as-Bid**: Selected bids are paid at their bid prices
  - **Pay-at-MCP**: Selected bids are paid at the maximum of selected bid prices
    
    Used by all ISO’s in the US

- Two auction methods
  - **Bid-Cost Minimization**: Select bids to minimize the total bid cost
  - **Payment-Cost Minimization**: Select bids to minimize the total cost that consumers have to pay

  Used by all ISO’s in the US
A Simple Example

One hour with four supply bids and 100 MWh demand

<table>
<thead>
<tr>
<th></th>
<th>$P_{\text{min}}$ (MW)</th>
<th>$P_{\text{max}}$ (MW)</th>
<th>Bid Price ($/\text{MWh}$)</th>
<th>Startup Cost ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unit A</td>
<td>0</td>
<td>45</td>
<td>10</td>
<td>0</td>
</tr>
<tr>
<td>Unit B</td>
<td>0</td>
<td>45</td>
<td>20</td>
<td>0</td>
</tr>
<tr>
<td>Unit C</td>
<td>0</td>
<td>12</td>
<td>100</td>
<td>20</td>
</tr>
<tr>
<td>Unit D</td>
<td>0</td>
<td>80</td>
<td>30</td>
<td>2000</td>
</tr>
</tbody>
</table>

Q. Solution of the bid-cost-minimization auction?


<table>
<thead>
<tr>
<th></th>
<th>Energy Selected (MWh)</th>
<th>Energy Cost ($)</th>
<th>Startup Cost ($)</th>
<th>Sub-Total ($)</th>
<th>Actual Energy Cost ($)</th>
<th>Actual Startup Cost ($)</th>
<th>Sub-Total ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unit A</td>
<td>45</td>
<td>450</td>
<td>0</td>
<td>450</td>
<td>4500</td>
<td>0</td>
<td>4500</td>
</tr>
<tr>
<td>Unit B</td>
<td>45</td>
<td>900</td>
<td>0</td>
<td>900</td>
<td>4500</td>
<td>0</td>
<td>4500</td>
</tr>
<tr>
<td>Unit C</td>
<td>10</td>
<td>1000</td>
<td>20</td>
<td>1020</td>
<td>1000</td>
<td>20</td>
<td>1020</td>
</tr>
<tr>
<td>Unit D</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Total</td>
<td>100</td>
<td>2350</td>
<td>20</td>
<td>2370</td>
<td>10000</td>
<td>20</td>
<td>10020</td>
</tr>
</tbody>
</table>

There is an inconsistency between the minimized total bid cost and total payment cost.

Actual cost that consumers have to pay could be much higher than what is obtained from the auction.
Q. Solution of the payment-cost-minimization auction?

<table>
<thead>
<tr>
<th></th>
<th>Capacity (MW)</th>
<th>Bid Price ($/MWh)</th>
<th>Startup Cost ($)</th>
<th>Energy Selected (MWh)</th>
<th>Energy Cost ($)</th>
<th>Startup Cost ($)</th>
<th>Sub-Total ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unit A</td>
<td>45</td>
<td>10</td>
<td>0</td>
<td>45</td>
<td>1350</td>
<td>0</td>
<td>1350</td>
</tr>
<tr>
<td>Unit B</td>
<td>45</td>
<td>20</td>
<td>0</td>
<td>45</td>
<td>1350</td>
<td>0</td>
<td>1350</td>
</tr>
<tr>
<td>Unit C</td>
<td>12</td>
<td>100</td>
<td>20</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Unit D</td>
<td>80</td>
<td>30</td>
<td>2000</td>
<td>10</td>
<td>300</td>
<td>2000</td>
<td>2300</td>
</tr>
</tbody>
</table>

|       |               |                   |                  |                       | 100            | 3000             | 2000          | 5000          |

- The minimized payment cost is equal to the actual payment cost, and is below what obtained by using bid cost minimization ($10,020)
Q. How to formulate and solve these two problems?
3. Formulation of the Two Auction Problems

Bid-Cost Minimization Auction

- How to formulate the problem?

\[
\min J, \text{ with } J = \sum_{t=1}^{T} \sum_{i=1}^{I} \{C_i(p_i(t), t)) + S_i(t)\}
\]

s.t. system demand and max/min power level constraints

- Same as unit commitment and economic dispatch with generating costs replaced by bid curves

- Separable, and can be solved by Lagrangian relaxation or other methods

- All the old packages can be used

- Would the solution be good if the market is settled by the Pay-at-MCP scheme?
Payment-Cost Minimization Auction

\[ \min_{\{\text{MCP}(t), p_i(t)\}} J, \text{ with } J \equiv \sum_{t=1}^{T} \sum_{i=1}^{I} \{\text{MCP}(t)p_i(t) + S_i(t)\} \]

s.t. system demand and max/min power level constraints

- MCP is defined as the highest price of selected bids (\( \Rightarrow \text{MCP-bid constraints} \)), and is a very subtle part of the optimization
- Cross product terms exist between MCP\( (t) \) and \( p_i(t) \), making the problem inseparable
- Also, if the standard Lagrangian were formed, it will be linear in terms of bid levels causing solutions to oscillate

* The problem is much more difficult, and no solution has been reported previously
  - Need a good methodology to compare the two auction schemes
4. Solution Methodology: Payment Cost Minimization

- **Augmented Lagrangian** to avoid solution oscillation

\[
L_c(\lambda, \eta, \text{MCP}, P) \equiv \sum_{i=1}^{I} \sum_{t=1}^{T} \{\text{MCP}(t)P_i(t) + S_i(t)\}
\]

\[
+ \sum_{t=1}^{T} \left\{\lambda(t) \left( P_d(t) - \sum_{i=1}^{I} P_i(t) \right) + \frac{c}{2} \left( P_d(t) - \sum_{i=1}^{I} P_i(t) \right)^2 \right\}
\]

\[
+ \sum_{i=1}^{I} \sum_{t=1}^{T} \left[ \eta_i(t) \left( c_i^r(t) - \text{MCP}(t) + z_i(t)^2 \right) + \frac{c}{2} \left( c_i^r(t) - \text{MCP}(t) + z_i(t)^2 \right)^2 \right]
\]

- \( z_i^2 \) can be analytically solved

- \( L_c \) has different structures for different bid selections
Traditional Lagrangian relaxation methods require $L_c$ to be fully optimized w.r.t. all the bids

- Problem inseparability and structural difference of $L_c$ over sub-regions $\Rightarrow$ Each sub-region needs to be optimized
- Combinatorial number of sub-regions

Surrogate optimization allows approximate optimization of $L_c$ if the following condition is satisfied

$$L_c(\lambda^{k+1}, x^{k+1}) < L_c(\lambda^{k+1}, x^k)$$

- If the condition is satisfied, then the “surrogate” subgradient obtained is a “good direction” forming an acute angle with the direction toward optimal multiplier
– Optimize $L_c$ w.r.t. one bid at a time and w.r.t. MPC, with other variables adjusted as needed to satisfy the condition

- Multipliers are updated by using the surrogate subgradient

$$g^k_\lambda = \left( P_d(t) - \sum_{i=1}^{I} p_i(t) \right)$$  Surrogate Subgradient

$$\lambda(t)^{k+1} = \lambda(t)^k + s^k \cdot g^k_\lambda$$  Step size

$$0 < s^k < \frac{2(L^* - L^k)}{\|g^k\|^2}$$  Physical meaning?

- The other set of multipliers $\{\eta_i\}$ is adjusted in a similar way
- Simple heuristics are used to obtain feasible solutions
Overall Structure of the Algorithm

1. Original problem

2. Relax system demand and MCP-bid constraints by using Lagrange multipliers

3. Update multipliers

4. Solve one or a few bid subproblems, or MCP subproblems until the surrogate optimization condition is satisfied

5. Meet Stopping criteria?
   - Yes: Construct a feasible solution, and compare for the best one
   - No: Construct a feasible solution every few iterations
Example. A Medium-Sized Problem

- **Given** 25 bids over 24 hours
- The bids were designed similar to a typical auction day of ISO New England Market

<table>
<thead>
<tr>
<th>Methods\Costs</th>
<th>Pay-as-Bid Cost</th>
<th>Actual Payment Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bid Cost Minimization</td>
<td>$3,403,999</td>
<td>$5,262,480</td>
</tr>
<tr>
<td>Payment Cost Minimization</td>
<td>N/A</td>
<td>$5,127,535</td>
</tr>
<tr>
<td>SAVINGS</td>
<td></td>
<td>$134,945 (2.56%)</td>
</tr>
</tbody>
</table>

- In view that the annual energy market even for ISO-NE runs in the tens of billions of dollars, even 0.1% savings represents significant amount of dollars annually
MCPs under payment cost minimization are usually less than those under offer cost minimization, and this translates to lower actual payment costs.
In This Talk:

- Bid Cost Minimization vs. Payment Cost Minimization
  - A simple but revealing example. Surprise?
- Formulations of the Two Auction Mechanisms
  - A standard separable formulation vs. a non-separable formulation with the cost structure depending on bids selected
- Solution Methodology for Payment Cost Minimization
  - A novel approach combining Augmented Lagrangian relaxation and surrogate optimization
- A Game Theoretic Study of the Two Auction Methods
  - Confirmation of the finding and some empirically observed behaviors
5. A Game Theoretic Study of Auctions

- From the above results, payment cost minimization yields significant reduction in consumer payments as compared to bid cost minimization for the same set of supply bids.
- Suppliers may bid differently under the two auction schemes.
- Each supplier must consider the strategies of other participants in determining its own strategy.
  - How to study multiple suppliers’ decisions?
    \[ \Rightarrow \text{Within a game theoretic context} \]
- How would you expect the result to look like?
A Matrix Nash Game

- Assume each supplier has a **finite** number of strategies
  - Suppliers maximize their own profit in a **non-cooperative** way
  - Given a combination of strategies of all the suppliers, payoffs (or profits) of suppliers are obtained by solving the auction
  ⇒ **Matrix game** is formed for each auction method
- **Nash equilibrium** is used as the solution concept
  - A solution where no single supplier can profit from unilateral deviation

\[
J_i(\gamma_1^*, \gamma_i^*, \ldots, \gamma_J^*) \geq J_i(\gamma_1^*, \gamma_i, \ldots, \gamma_J^*), \quad \forall \, i
\]

Nash equilibrium

**Q.** How to solve the problem?
Overall Structure of the Algorithm

Start

Run auctions to obtain payoffs for all combinations of supplier strategies

Examine a strategy combination for Nash equilibrium

N

All strategy combinations examined?

Y

Remove dominated Nash solutions

End
A Two-Supplier Simple Example

- **Unit Characteristics**

<table>
<thead>
<tr>
<th>Supplier</th>
<th>Unit</th>
<th>Pmin (MW)</th>
<th>Pmax (MW)</th>
<th>Incremental Cost ($/MWh)</th>
<th>Startup Cost ($/Start)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Supplier 1</td>
<td>Unit 1</td>
<td>5</td>
<td>30</td>
<td>25</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>Unit 2</td>
<td>2</td>
<td>15</td>
<td>80</td>
<td>10</td>
</tr>
<tr>
<td>Supplier 2</td>
<td>Unit 1</td>
<td>3</td>
<td>45</td>
<td>40</td>
<td>1000</td>
</tr>
</tbody>
</table>

- **Bidding Strategies**

<table>
<thead>
<tr>
<th>Supplier</th>
<th>Unit</th>
<th>Low (L)</th>
<th>Middle (M)</th>
<th>High (H)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Supplier 1</td>
<td>Unit 1</td>
<td>(26, 0)</td>
<td>(35, 10)</td>
<td>(40, 20)</td>
</tr>
<tr>
<td></td>
<td>Unit 2</td>
<td>(81, 10)</td>
<td>(100, 20)</td>
<td>(200, 50)</td>
</tr>
<tr>
<td>Supplier 2</td>
<td>Unit 1</td>
<td>(41, 1000)</td>
<td>(52, 1200)</td>
<td>(80, 1500)</td>
</tr>
</tbody>
</table>

- **System Demand = 33MWh**
Matrix Game for Payment Cost Minimization

<table>
<thead>
<tr>
<th>S1 / S2</th>
<th>L</th>
<th>M</th>
<th>H</th>
</tr>
</thead>
<tbody>
<tr>
<td>LL</td>
<td>480, 3</td>
<td>1683, 0</td>
<td>1683, 0</td>
</tr>
<tr>
<td>LM</td>
<td>480, 3</td>
<td>750, 230</td>
<td>2320, 0</td>
</tr>
<tr>
<td>LH</td>
<td>480, 3</td>
<td>750, 230</td>
<td>1650, 620</td>
</tr>
<tr>
<td>ML</td>
<td>0, 33</td>
<td>1693, 0</td>
<td>1693, 0</td>
</tr>
<tr>
<td>MM</td>
<td>0, 33</td>
<td>0, 530</td>
<td>2330, 0</td>
</tr>
<tr>
<td>MH</td>
<td>0, 33</td>
<td>0, 530</td>
<td>0, 1820</td>
</tr>
<tr>
<td>HL</td>
<td>0, 33</td>
<td>1703, 0</td>
<td>1703, 0</td>
</tr>
<tr>
<td>HM</td>
<td>0, 33</td>
<td>0, 530</td>
<td>2340, 0</td>
</tr>
<tr>
<td>HH</td>
<td>0, 33</td>
<td>0, 530</td>
<td>0, 1820</td>
</tr>
</tbody>
</table>

- The unique Nash solution: S1 bids “LL” and S2 bids “L”
  - MCP = $41/MWh, $p_{S1-U1} = 30MWh, $p_{S2-U1} = 3MWh
## Comparison of Two Auction Methods

<table>
<thead>
<tr>
<th>Auction Games</th>
<th>Nash Solution</th>
<th>MCP $/MWh</th>
<th>Selected MWs</th>
<th>Total Payment ($)</th>
<th>Total Bid Cost ($)</th>
<th>Total Prod. Cost ($)</th>
<th>Total Generation Profit ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Payment Cost Min.</td>
<td>(LL, L)</td>
<td>41</td>
<td>30-0-3</td>
<td><strong>2353</strong></td>
<td>1903</td>
<td>1870</td>
<td>483</td>
</tr>
<tr>
<td>Bid Cost Min.</td>
<td>(HH, X)</td>
<td>200</td>
<td>30-3-0</td>
<td><strong>6670</strong></td>
<td>1870</td>
<td>1000</td>
<td>5670</td>
</tr>
</tbody>
</table>

- Payment Cost Minimization achieves significant reduction in consumer payment cost ($6670–$2353), at relatively small increases of total bid cost ($1903–$1870) and total production cost ($1870–$1000) ~ A concern
- Under Bid Cost Minimization, S1 bids $40/MWh for U1 and $200/MWh for its small U2 (“Hockey-Stick Bidding”), resulting in a high MCP
6. Concluding Remarks

- The inconsistence issue is hard to recognize and difficult to solve
- A novel approach has been developed for PCM with good results
  - The method leads to significant reduction in consumer payments as compared to Bid Cost Minimization for the same set of bids
  - Opens up a new direction to solve problems w/ complicated structures
- Game theoretic study verifies that the above holds even when strategic behaviors of participants are considered
  - PCM also discourages malicious behaviors, e.g., hockey-stick bidding
- Transmission capacities can be handled w/i the same framework
- Further economic implications are under study
- When considering auctions, one should seriously think about what to optimize – PMC is a prudent auction mechanism
References


Scientific methods and technologies that improve efficiency, productivity, quality, and reliability

Special Issues

- Nano-scale Manipulation and Assembly
- Automation for the Life Sciences
- Distributed Sensing for Quality & Productivity Improvement
- Automation for the Home Environment
- eManufacturing in the Semiconductor Industry
- Drug Delivery Automation
Bidding Strategies in Overlapping Internet Auctions

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National Central University

Chien-Fu Chou
National Taiwan University

August 14, 2009
presented at Mini-workshop on Design of Auctions and Contracts
Overview

- Why do we care about overlapping auctions?
  - Identical items are often sold on an Internet auction site at the same time.
  - Most previous studies on Internet auctions treat them as isolated of each other.

- How to bid on an item if you can also participate in other auctions selling the same object?
Overview

• Why do we care about overlapping auctions?
  ◦ Identical items are often sold on an Internet auction site at the same time.
  ◦ Most previous studies on Internet auctions treat them as isolated of each other.

• How to bid on an item if you can also participate in other auctions selling the same object?

• In this paper, we
  ◦ propose a model for overlapping auctions,
  ◦ analyze a buyer’s optimal bidding strategy, and
  ◦ perform an empirical test to verify model predictions.
Preview of the Main Results

- Buyers only bid on the auction which is the first to end among all current auctions.

- Because of the option value of bidding in future auctions, the submitted bid is less than a buyer’s valuation except in the final auction.

- The expected transaction price is ex ante identical for all auctions. (In general?)

- Our results can be viewed as an explanation of last minute bidding.
Plan of the Talk

• **Literature**
• Theoretical Model
• Empirical Evidence
• Conclusion
There have been many researches on Internet auctions, but most of them treat auctions as isolated. (Bajari and Hortaçsu, 2004 JEL; Ockenfels, Reiley, and Sadrieh, 2006)
Literature

- There have been many researches on Internet auctions, but most of them treat auctions as isolated. (Bajari and Hortaçsu, 2004 JEL; Ockenfels, Reiley, and Sadrieh, 2006)

- **Simultaneous auctions:**
  - Peters and Serinov (2006, JET): simultaneous ascending second-price auctions
    - bid the lowest possible price at the auction with the lowest standing price as long as the price is less than a buyer’s valuation.
  - Anwar, McMillan, and Zheng (2006, EER): 14% to 20% of eBay buyers **cross-bid** on auctions ending simultaneously.
Literature

- There have been many researches on Internet auctions, but most of them treat auctions as isolated. (Bajari and Hortaçsu, 2004 JEL; Ockenfels, Reiley, and Sadrieh, 2006)

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- **Sequential auctions:**
Plan of the Talk

- Literature
- **Theoretical Model**
- Empirical Evidence
- Conclusion
Timeline of Overlapping Auctions

- \( T \) second-price auctions. Each lasts for two periods.
• $T$ second-price auctions. Each lasts for two periods.
• At least one buyer arrives in period one. A new buyer arrives in each subsequent period, $t = 2, 3 \ldots, T + 1$.
• The valuation of a buyer $v$ is drawn i.i.d from $[0, \bar{v}]$ according to the distribution $F$.
• Each buyer demands one unit of the good.
Auction Rules

• At the beginning of a period, the starting price of a new auction is at 0. \( p_t^t = 0 \)

• The starting price of an old auction is the final standing price in the previous period. \( p_{t+1}^t = p_t^{t*} \)
Auction Rules

• At the beginning of a period, the starting price of a new auction is at 0. \( p_t^t = 0 \)

• The starting price of an old auction is the final standing price in the previous period. \( p_{t+1}^t = p_t^{t*} \)

• Buyers make their bids simultaneously in each period.

• A buyer can choose to bid on both auctions, one of the auctions, or none of them.

• When a buyer bid on an auction, she sends a number (greater than the standing price of that period) to eBay.
Auction Rules (Continued)

- The highest bidder of auction $k$ in period $t$ ($B^k_t$) is the buyer who submits the highest number for that auction.
  - When no one submits, $B^k_t \equiv \emptyset$. 
Auction Rules (Continued)

- The **highest bidder** of auction $k$ in period $t$ ($B_t^k$) is the buyer who submits the highest number for that auction.
  - When no one submits, $B_t^k \equiv \emptyset$.

- The **final standing price** of auction $k$ in period $t$ ($p_t^k\ast$) is the second highest number submitted by buyers in that period.
  - When none or only one of the buyers submits, $p_t^k\ast \equiv p_t^k$. 
Auction Rules (Continued)

• The **highest bidder** of auction $k$ in period $t$ ($B^k_t$) is the buyer who submits the highest number for that auction.
  ◦ When no one submits, $B^k_t \equiv \emptyset$.

• The **final standing price** of auction $k$ in period $t$ ($p^{k*}_t$) is the **second** highest number submitted by buyers in that period.
  ◦ When none or only one of the buyers submits, $p^{k*}_t \equiv p^k_t$.

• The **winner** of an auction is the highest bidder in the last active period of the auction ($B^t_{t+1}$). The winner pays the final standing price in the last period of the auction to the seller ($p^{t*}_{t+1}$).
A Perfect Bayesian Equilibrium

We want to find out

• Buyers’ bidding strategies;

• $p_{t}^{i*}$: Final Standing price of auction $i$ in period $t$. 
A Perfect Bayesian Equilibrium

We want to find out

- Buyers’ bidding strategies;
- $p_t^i$: Final Standing price of auction $i$ in period $t$.

- In each period $t = 2, 3, \ldots, T + 1$, there is one bidder who wins and leaves in equilibrium.
Bidding Strategy in the Last Period $t = T + 1$

- Auction $T$ is a standard single-period sealed-bid second-price auction with the starting bid $\underline{p}_{T+1}^T = p_T^*$. 

- A buyer’s dominant strategy is to submit her own valuation $v$. 

  \[ \beta_{T+1,i}^T (v, \underline{p}_{T+1}^T) = \max \left\{ v, \underline{p}_{T+1}^T \right\}. \]

- The strategy is independent of a buyer’s information.
Expected Surplus at the Beginning of the Last Period

- The expected surplus for a buyer who is not the highest bidder of auction \( T \) at the end of period \( T \):

\[
\pi_{T+1}(v, p_{T+1}^T, w_T) = \int_0^{\max\{w_T, p_{T+1}^T\}} \left[ v - \max\{w_T, p_{T+1}^T\} \right] dF_{T+1}(x) + \int_v^{\max\{w_T, p_{T+1}^T\}} (v - x) dF_{T+1}(x)
\]

\[
= \int_{\max\{w, p_{T+1}^T\}}^{\max\{w, p_{T+1}^T\}} F_{T+1}(x) dx,
\]

where \( w_T \) is the highest valuation among other buyers who entered but have not won in earlier periods.
Expected Surplus at the Beginning of the Last Period

- The expected surplus for a buyer who is the highest bidder of auction $T$ at the end of period $T$:

$$\pi_{T+1}^H(v, p_{T+1}^T, w_T) = \begin{cases} 
\int_{p_{T+1}^T}^{v} F_{T+1}(x) \, dx, & \text{if } p_{T+1}^T \geq w_T; \\
\int_{w_T}^{v} F_{T+1}(x) \, dx, & \text{if } p_{T+1}^T \leq w_T \leq v; \\
0, & \text{if } w_T \geq \max\{p_{T+1}^T, v\}.
\end{cases}$$

where $w_T$ is the highest valuation among other buyers who entered but have not won in earlier periods.
Expected Surplus at the Beginning of the Last Period

- The expected surplus for a buyer who is the highest bidder of auction $T$ at the end of period $T$:

$$
\pi^H_{T+1}(v, p_{T+1}^T, w_T) = \begin{cases} 
\int_{p_{T+1}^T}^{v} F_{T+1}(x) dx, & \text{if } p_{T+1}^T \geq w_T; \\
\int_{w_T}^{v} F_{T+1}(x) dx, & \text{if } p_{T+1}^T \leq w_T \leq v; \\
0, & \text{if } w_T \geq \max\{p_{T+1}^T, v\}.
\end{cases}
$$

where $w_T$ is the highest valuation among other buyers who entered but have not won in earlier periods.

- Comparing the expected surpluses:

$$
\pi_{T+1}(v, p_{T+1}^T, w_T) \geq \pi^H_{T+1}(v, p_{T+1}^T, w_T) \quad \forall (v, p_{T+1}^T, w_T),
$$
Expected Surplus at the Beginning of the Last Period

• $w_T$ is only partially revealed from the information available at the end of period $T$, $\Omega_{Ti}$.

• The expected surplus of a non-highest bidder of auction $T$ is greater than that of a highest bidder:

$$S_{T+1} \left( v, p_{T+1}^{T} ; \Omega_{Ti} \right) \equiv E_{wT} \left[ \pi_{T+1}(v, p_{T+1}^{T}, w_T) | \Omega_{Ti} \right]$$

$$\geq S_{T+1}^{H} \left( v, p_{T+1}^{T} ; \Omega_{Ti} \right) \equiv E_{wT} \left[ \pi_{T+1}^{H}(v, p_{T+1}^{T}, w_T) | \Omega_{Ti} \right]$$

$$\forall v, p_{T+1}^{T}, \Omega_{Ti}.$$
Other Periods

Use induction to show the following results holds for each period $t$:

- The expected surplus of a non-highest bidder of auction $t$ in the previous period is greater than the highest bidder.
  \[ S_t(v, p_{t-1}^t; \Omega_{t-1}, i) \geq S_H^t(v, p_{t-1}^t; \Omega_{t-1}, i) \]

- The optimal bid for auction $t$ is zero.

- The optimal bid for auction $t - 1$ is
  \[ \beta_{t-1}^t (v, p_{t-1}^t; \Omega_{t-1}, i) \equiv \max \{ v - E [S_{t+1}(v, 0; \Omega_{t}, i)|\Omega_{t-1}, i], p_{t-1}^t \} \]

- The final price of auction $t$ in period $t$ is zero.

- The final price of auction $t$ in period $t + 1$ is determined by the second highest bid for it. It is the second highest value in the set
  \[ \{ E [v_i - S_{t+1}(v_i, 0; \Omega_{t}, i)|\Omega_{t-1}, i] : i \in B_t, i \neq B_{k+1}^t \forall k < t \} \]
The Induction Assumption

- Assume the inequality holds in period $t + 1$.

$$S_{t+1}(v, p^t_{t+1}; \Omega_{t,i}) \geq S_{t+1}^H(v, p^t_{t+1}; \Omega_{t,i})$$

- Claim the inequality holds in period $t$ as well.
The optimal bid for auction \( t \)

- Compare the **expected surplus** for four possible outcomes of buyer 1.
  1. The highest bidder in both auctions:
     
     \[
     (v_1 - \max_{i \neq 1} b_{ti}^{t-1}) + S_{t+1}^{H}(0, \max_{i \neq 1} b_{ti}^{t}; \Omega_{t1})
     \]
  2. The highest bidder of auction \( t - 1 \) but not auction \( t \):
     
     \[
     v_1 - \max_{i \neq 1} b_{ti}^{t-1}
     \]
  3. The highest bidder of auction \( t \) but not auction \( t - 1 \):
     
     \[
     S_{t+1}^{H}(v_1, \max_{i \neq 1} b_{ti}^{t}; \Omega_{t1})
     \]
  4. Not the highest bidder in either auction:
     
     \[
     S_{t+1}(v_1, \max_{i \neq 1} b_{ti}^{t}; \Omega_{t1})
     \]

- Given other buyers’ bids, case (4) is better than case (3), and case (2) is better than case (1).

- So, optimal bid for auction \( t \) is **zero**.
The optimal bid for auction \( t - 1 \)

- Given the belief that other buyers bid zero in auction \( t \), the bid for auction \( t - 1 \) is determined by

\[
\max_{b \geq p_{t-1}^{t-1}} E \left[ 1 \left\{ b > \max_{i \neq 1} b_{ti}^{t-1} \right\} \left( v - \max_{i \neq 1} b_{ti}^{t-1} \right) + 1 \left\{ b \leq \max_{i \neq 1} b_{ti}^{t-1} \right\} S_{t+1}(v_1, 0; \Omega_{t1} \mid \Omega_{t-1,1}) \right].
\]

- The optimal bid is

\[
\beta_t^{t-1}(v, p_{t-1}^{-1}; \Omega_{t-1,i}) \equiv \max \left\{ v - E \left[ S_{t+1}(v, 0; \Omega_{t,i}) \mid \Omega_{t-1,i} \right], p_{t}^{t-1} \right\}.
\]

- In general, it is complicated to write down a closed-form expression for the conditional expectation because it involves inferring the distribution of other buyers’ valuations from the observed prices in the previous periods.
Prove the Induction Assumption for period $t$

- The bidding strategy is the same regardless a buyer is the highest bidder of auction $t - 1$ at the end of period $t - 1$.
- The buyer $B_{t-1}$ would win auction $t - 1$ if and only if (a) her submitted bid is greater than any other buyer’s bid or (b) every buyer submits the starting price $p_{t-1}$. 

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- The buyer $B_{t-1}^{t-1}$ would win auction $t - 1$ if and only if (a) her submitted bid is greater than any other buyer’s bid or (b) every buyer submits the starting price $p_{t}^{t-1}$.
  - In case (a), the surplus, which is the valuation minus the highest bid among others, is the same whether a buyer has been the highest bidder in the period $t - 1$ or not.
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  - In case (a), the surplus, which is the valuation minus the highest bid among others, is the same whether a buyer has been the highest bidder in the period $t - 1$ or not.
  - In case (b), the buyer $B_{t-1}^{t-1}$ wins auction $t - 1$ at the starting price $p_{t-1}^{t-1}$, and the surplus of winning it is less than that of losing it. (Otherwise the buyer would have chosen to submit a bid greater than the starting price.)
Prove the Induction Assumption for period $t$

- The bidding strategy is the same regardless a buyer is the highest bidder of auction $t - 1$ at the end of period $t - 1$.

- The buyer $B_{t-1}^{t-1}$ would win auction $t - 1$ if and only if (a) her submitted bid is greater than any other buyer’s bid or (b) every buyer submits the starting price $p_{t-1}^{t-1}$.
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  - In case (b), the buyer $B_{t-1}^{t-1}$ wins auction $t - 1$ at the starting price $p_{t-1}^{t-1}$, and the surplus of winning it is less than that of losing it. (Otherwise the buyer would have chosen to submit a bid greater than the starting price.)

- Buyers other than $B_{t-1}^{t-1}$ would win auction $t - 1$ if and only if case (a) occurs.
Prove the Induction Assumption for period $t$

- When a buyer loses auction $t - 1$, the expected surplus is the same whether she is the highest bidder in the previous period ($B_{t-1}$) or not.
- Consequently, $S_t(v, p_t^{t-1}; \Omega_{t-1,i}) \geq S_t^H(v, p_t^{t-1}; \Omega_{t-1,i})$. 
An Example with Two Buyers in Each Period

• Suppose only one buyer enters in all periods.
• Each period \( (t = 2, \ldots, T + 1) \) consists of a new buyer and an old buyer.
An Example with Two Buyers in Each Period

- Suppose only one buyer enters in all periods.
- Each period \((t = 2, \ldots, T + 1)\) consists of a new buyer and an old buyer.
- Closed form formula for the optimal bid of auction \(t - 1\) in period \(t\):

\[
\beta_{t-1}^t(v, 0; \Omega_{t-1,i}) = \int_0^v [1 - F(x)]^{T-t+1} dx.
\]
An Example with Two Buyers in Each Period

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  \[
  \beta_{t-1}^t(v, 0; \Omega_{t-1,i}) = \int_0^v [1 - F(x)]^{T-t+1} dx.
  \]
- The auctions are efficient (selling goods to the \(T\) highest-valuation buyers).
- The ex-ante expected transaction price is identical across periods.
  \[
  E(p_{t-1}^t) = \int_0^\bar{v} (1 - F(v))^{T+1} dv \quad \forall t.
  \]
Properties of the Equilibrium

- Buyers only bid on the auction which is the first to end among all current auctions.
- Because of the option value of future auctions, the submitted bid is less than a buyer’s valuation except in the final auction.
- Efficient in general (?)
- Expected transaction prices (?)
Last Minute Bidding

- Many previous researches find buyers tend to submit their bids to an online auction only in the very last moment of the pre-specified duration of an auction.

- When the arrival of new auctions is stochastic, our theoretical findings imply last-minute-bidding because
  1. Buyers only want to bid on the first-to-end auction.
  2. The submitted bid needs to account for the future option value.
Plan of the Talk

- Literature
- Theoretical Model
- **Empirical Evidence**
- Conclusion
The regression equation is

\[ p_{ij} = \beta_1 t_{ij} + \beta_2 [t_{ij} - F_i]_+ + \beta_3 [t_{ij} - (T_i - \Delta)]_+ + \xi_i + \varepsilon_{ij}, \]

\[ j = 0, 1, 2, \ldots J_i \]

- \( p_{ij} \): Standing price
- \( t_{ij} \): Time since the beginning of the auction (days)
- \([t_{ij} - F_i]_+ \equiv \max\{t_{ij} - F_i, 0\}\): Time since the auction becomes the first-to-end auction
- \([t_{ij} - (T_i - \Delta)]_+ \equiv \max\{t_{ij} - (T_i - \Delta), 0\}\): Time in the last moment of an auction
- \( \xi_i \): Fixed effect of auction \( i \)
Testable Implications

- Buyers place bids only the first-to-end auction.
- When the arrival of new auctions is stochastic, buyers delay submitting bids until the last moment of an auction.
- Expect $\beta_1 = 0$, $\beta_2 > 0$, and $\beta_3 > 0$. 
Data


• All transactions on an identical item: Casio G-Shock World Time Data Memory Watch G2900F.

• 178 auctions (the index $i$)

• For each auction, the observations (the index $j$) are starting/ending time and all bidding times.
## Descriptive statistics

<table>
<thead>
<tr>
<th>Variable</th>
<th>Meaning</th>
<th>Mean</th>
<th>Std. Dev.</th>
<th>Min.</th>
<th>Max.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p_{i0}$</td>
<td>Starting price</td>
<td>15.70</td>
<td>22.80</td>
<td>0.01</td>
<td>66.00</td>
</tr>
<tr>
<td>$p_{iJ_{i}}$</td>
<td>Final price</td>
<td>33.22</td>
<td>12.80</td>
<td>0.01</td>
<td>66.00</td>
</tr>
<tr>
<td>$T_{i}$</td>
<td>Duration</td>
<td>4.38</td>
<td>2.15</td>
<td>0.42</td>
<td>10.00</td>
</tr>
<tr>
<td>$F_{i}$</td>
<td>Duration of first-to-end</td>
<td>3.05</td>
<td>2.53</td>
<td>0.00</td>
<td>9.85</td>
</tr>
<tr>
<td>$J_{i}$</td>
<td>No. of bids</td>
<td>5.00</td>
<td>4.20</td>
<td>0.00</td>
<td>21.00</td>
</tr>
</tbody>
</table>

*Notes*: The number of auctions is 178.
## Regression Result

<table>
<thead>
<tr>
<th>Parameter</th>
<th>A ((\Delta = 1) hour)</th>
<th>B ((\Delta = 1) minute)</th>
<th>C ((\Delta = 1) second)</th>
<th>D ((\Delta = 1) second)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\beta_1)</td>
<td>0.587* (0.279)</td>
<td>-0.225 (0.293)</td>
<td>-0.0154 (0.295)</td>
<td>0.221 (0.292)</td>
</tr>
<tr>
<td>(\beta_2)</td>
<td>9.544** (0.754)</td>
<td>8.305** (0.727)</td>
<td>9.080** (0.740)</td>
<td>9.407** (0.752)</td>
</tr>
<tr>
<td>(\beta_3)</td>
<td>187.064** (19.189)</td>
<td>8502.464** (984.145)</td>
<td>314883** (41577)</td>
<td></td>
</tr>
</tbody>
</table>

**within-\(R^2\)** | 0.4082 | 0.470 | 0.440 | 0.419 |

*Notes:* Huber-White robust standard errors are in parentheses. The superscripts * and ** represent significance at 5% level and 1% level, respectively.
Plan of the Talk

- Literature
- Theoretical Model
- Empirical Evidence
- Conclusion
Conclusion

• A theoretical model of overlapping auctions:
  ◦ Bidding strategies
  ◦ Equilibrium prices (??)
• Bids are submitted only to the first-to-end auctions.
• Our model provide an important explanation for last minute bidding.
• Data from eBay.com confirm the theoretical implications.
Conclusion

• A theoretical model of overlapping auctions:
  ◦ Bidding strategies
  ◦ Equilibrium prices (??)
• Bids are submitted only to the first-to-end auctions.
• Our model provide an important explanation for last minute bidding.
• Data from eBay.com confirm the theoretical implications.

• Future work:
  ◦ Arrival of buyers may be stochastic.
  ◦ The supply side problem: When to post an auction?
A Contract of Purchase Commitments on Shared Yields as A Risk-Sharing Mechanism among Fabless-Foundry Partnership

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outline

- Introduction
- Contracts in coordinating supply chain
- A model of purchase commitments on shared yield
- Analysis in the supply chain with risk-sharing
- Concluding remarks
INTRODUCTION

- Semiconductor manufacturing cost continues to rise
  - The average capital investment of 90nm and 65nm fab is above US$5 billion
  - What about the next generations?

- In the uncertain demand side
  - consumers require IC products for
    - higher variety
    - shorter life cycle
    - lower cost
  - the business cycle and economy downturn
Alliance for mitigating risks

- IDM with pure-play foundries
- Joint ventures
- The fab-lite model
  - outsourcing 40 percent to 50 percent of manufacturing operations (Shelton 2003)
    e.g., Motorola, ADI, Sony, Renesas
  - retaining some control of their own process technology while gaining access to leading-edge foundry technology
  - joint process technology and/or fab capacity developments
    - IBM, Chartered and Samsung or ICIS
    - Qimonda/Micron and Nanya
A contract of purchase commitments on shared yields

- Under this contract as in forming an alliance
  - the design house commits to purchase $\alpha$ yields produced by the foundry fab

- the design house
  - in addition to a traditional "money-for-chip" relationship...
  - contributes its proprietary semiconductor process technology and related know-how
Contracts in coordinating supply chain

- Coordination in supply chain
  - to maximize the profit of the chain system, in stochastic environments.

- Most of the previous papers focus on
  - contracts achieving coordination in a decentralized supply chain
  - for example,
    - buy-back contracts (Pasternack 1985, Bernstein and Federgruen 2005),
    - price-discount contracts, revenue-sharing contracts (Dana and Spier 2001, Cachon and Lariviere 2005),
    - quantity-flexibility, and/or mixed of them (Cachon and Lariviere 2005)
Previous papers on contracts in SC

- in a non-cooperative game
  - Contracts of coordinating distributed decision-makers in a non-cooperative game
    - offering a scheme to align incentives of decentralized partners
    - guiding distributed decisions on polices of pricing, inventory, and capacity planning
  - one-shot game/short-term perspectives

- Coordination based on a non-cooperative game are attractive... but...

- in a game of alliance/long-term perspectives?
Repeated interactions among supply chain partners

- In an alliance,
  - the partners of the supply chain actually engage in **repeated interactions** rather in one-shot game.
  - The incentive to deviate from the long-run cooperative outcome should be low.

- Some recent papers has raise this need and analyzed

- This paper can be viewed as
  - a study on the long-run outcome of behaviors in an alliance
Risk-neutral vs risk-averse

Most of the previous papers
- models risk factors in “mean” payoffs of firms
- this approach implicitly assumes risk-neutral members of the supply chain
- suitable for markets with stable demand/cost, low risks.

Some of the recent papers
- Van Mieghem (2003, MSOM)
  - the articles incorporating variability in payoffs of firms are surprisingly small
For allied members in SC

- Taking the supply chain as a whole, the long-term objectives are jointly to...
  - create/increase the value of the chain
  - improve efficiency/reduce costs of the chain
  - how about risks of the chain?

- Optimal allocation of the risk among partners?
  - The newsvendor model’s implications...
  - Think about the traditional "money-for-chip" relationship of the design house and the foundries...
The typical newsvendor model

- The buyer pay a (pre-specified, typically fixed) wholesale price to the supplier
- The buyer alone bear all possible price variations arising from the uncertain market (money-for-the-good)
- When market uncertainty overwhelmingly increases the buyer could
  - withdraw from the market, or…
  - has to shift the risk to someone else … this possibly leads to information distortion
How and consequences of risk transmission through the chain?

- If all risks fall on *only* the downstream of the chain
  - down-stream firms will be driven out of the *game*
  - risks are transmitted to the up-stream firms

- Information distortion due to risks transmission
  - For instance, in the semiconductor equipment supply chain "soft" vs "hard" orders orders
    - \[\Rightarrow\] inflated forecast/insufficient capacity supply
  - Equipment buyers inflate forecasts/suppliers respond in turn, delayed capacity supply
  - Empirically, cancellation/holding costs 2-3 times higher than delayed costs (Cohen et al. 2003).
  - The classical prisoner’s dilemma in a non-cooperative game.
The cause of the problem

 If taking the supply chain as a whole, the problem is due to
  • distributing the market risk in a wrong way

 The solution should consider
  • how to appropriately allocate risk into chain partners
  • how to gauge the information sent by the buyers and received by the supplier

 In summary,
  • the risk cannot be assumed away in the objective of member firms
Our simple model

- reconsiders a long-run prospect over contracting semiconductor supply chains
  - in line with the work by Wilson (1968) and Stiglitz (1974)
- proposes a novel business model,
  - a contract of purchase commitments on shared yields
- focuses on a static cooperative outcome
  - which might be considered as a long-run equilibrium ultimately reached through repeated interactions among partners of the supply chain.
The model settings

- Consider an aggregated market demand of IC products
  - for two risk-averse SC members, a design house (DH) and a foundry
  - stochastic market price: $p$
  - the demand quantity: $q$

- DH owns know-how of semicond. processes/IPS
  - $M$: the stock of know-how contributed by DH

- Foundry contributes production capacity
  - $K$: the capacity

- a contract of purchase commitments on shared yields
  - DH commits to purchase $\alpha \in \alpha \in [0,1]$
Assumptions

- Production function (in total yields)
  \[ Y = Y(K, M) \]
  where \( Y_K > 0, \ Y_M > 0, \ Y_{KK} < 0, \) and \( Y_{MM} < 0. \)

- Costs
  \( C_K \) and \( C_M \) represent the unit cost of \( K \) and \( M \)

- The risk-averse firm’s utility
  - to maximize risk-adjusted expected payoffs

  \[
  \max. \quad EU = E(\Pi) - \frac{1}{2} r \sigma^2_{\Pi}
  \]

  \( r > 0, \) the degree of risk aversion
The objectives of the SC members

- The design house's problem is to maximize
  \[
  EU_D = \alpha (\mu_D - w_T) Y - \frac{1}{2} r_D \alpha^2 Y^2 \sigma^2_p - C_M M
  \]

- The foundry fab's problem is to maximize
  \[
  EU_F = (1 - \alpha)(\mu_F + w_T) Y - \frac{1}{2} r_F (1 - \alpha)^2 Y^2 \sigma^2_p - C_K K
  \]

- \( r_i, \mu_i \) denotes firm i’s degree of risk-aversion and expected payoff, respectively.
- \( w_T \): wholesale price paid by DH to the foundry
The benchmark of optimal risk-sharing

- the allied chain maximizes the joint expected utility of the two firms

\[
\text{Max } \alpha(\mu_D - w_T)Y - \frac{1}{2}r_D \alpha^2 Y^2 \sigma_p^2 - C_MM \\
+ (1 - \alpha)(\mu_F + w_T)Y - \frac{1}{2}r_F (1 - \alpha)^2 Y^2 \sigma_p^2 - C_KK
\]

- FOCs

\[
Y_M \left[ \alpha H_D + (1 - \alpha)H_F \right] = C_M, \quad (3)
\]
\[
Y_K \left[ \alpha H_D + (1 - \alpha)H_F \right] = C_K, \quad (4)
\]
\[
YH_D - YH_F = 0, \quad (5)
\]

where \( H_D = \mu_D - r_D \alpha Y \sigma_p^2 \) and \( H_F = \mu_F - r_F (1 - \alpha) Y \sigma_p^2 \)
The optimal input levels

- Eq. (5) implies in the optimum,
  - This indicates the chain can achieve utility maximization by equating two firms' risk-adjusted prices.

- Substitute $H_D = H_F$ into (4) and (5) to get

  \[ Y_M H_D = C_M , \quad (7) \]
  \[ Y_K H_D = C_K . \quad (8) \]

- Therefore, this suggests $Y_M / Y_K = C_M / C_K$ to obtain optimal $M^*$ and $K^*$ of the chain’s optimal input levels.
The optimal share ($\alpha^*$) for the design house is

- Solve $H_D = H_F$ to get the optimal committed share ($\alpha^*$)

  $$\alpha^* = \frac{\mu_D - \mu_F + r_F Y \sigma_p^2}{(r_D + r_F) Y \sigma_p^2}.$$  

- Note that

  - $\alpha^*$ is cost-invariant
    i.e. is also optimal for any cases of high/low costs (with certain value)
  
  - $\alpha = 1$ (hitting upper bound) for the typical newsvendor model
    i.e.,

    $$\mu_D = \mu_F + r_D Y \sigma_p^2 \Rightarrow \mu_D > \mu_F$$
A special case when $\mu_D = \mu_D$

- $\mu_D = \mu_D$ implies
  if DH and foundry have the same expected payoffs, then
  \[
  \alpha^* = \frac{r_F}{r_D + r_F}
  \]

$\alpha^*$ only depends on
  - two parties' degree of risk-aversion and
  - is independent of total yields $Y$ and price risk.
  - That is,
    $\alpha^*$ will be always optimal no matter the price risk is
Bargaining for wholesale price $w_T$

- Once the optimal output of the supply chain are achieved given $\alpha^*$
  - a rule for allocating the chain's profit between the design house and foundry is needed in practice, i.e.,
  - $w_T$ determines the distribution of the chain’s payoffs

- the Nash fixed threat bargaining solution proposed
  - by (Chatterjee et al. 2002, Taylor and Plambeck 2007b)
  - to maximize the product of DH and the foundry's payoffs
the Nash fixed threat bargaining solution $w_N$

- Maximize

$$\text{Max } \left( \alpha \mu_D Y - \frac{1}{2} r_D \alpha^2 Y^2 \sigma_p^2 - C_M M - w_N Y \right) \times \left( (1 - \alpha) \mu_F Y - \frac{1}{2} r_F (1 - \alpha)^2 Y^2 \sigma_p^2 - C_K K + w_N Y \right).$$  \hspace{1cm} (13)

- The solution is

$$w_N = \frac{1}{2} \left( \frac{\text{EU}_D - \text{EU}_F}{Y} \right).$$  \hspace{1cm} (14)

- This implies

$$\text{EU}_D - w_N Y = \text{EU}_F + w_N Y = \frac{1}{2} \left( \text{EU}_D + \text{EU}_F \right).$$
Features of the contract of purchase commitments

- Serving as a risk-sharing mechanism
  - since the chain risk is diversified into partners.

- Partially mitigating the double-marginalization problem
  - (though this point is not directly addressed in our model)

- Providing a novel type of in-between orders
  - between the soft and hard order.
  - The problem of soft order's variability and inflation can be alleviated.
  - The foundry will not be too conservative to invest in capacity.
Design of multi-tier supply chains: Forecasting, delegation, and information

Ying-Ju Chen¹

¹UC Berkeley

2009 Summer
Principal-agent model

- Adverse selection (hidden information)
- Moral hazard (hidden action)

<table>
<thead>
<tr>
<th>Principal</th>
<th>Contract</th>
<th>Agent</th>
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<tbody>
<tr>
<td>Proceeds (revenue)</td>
<td>Signal</td>
<td>Sales effort</td>
</tr>
<tr>
<td>Market demand</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

- Promotion
- Shelf space rearrangement
- Response to customers’ inquiry
Applications to supply chain management

- production planning
  - Iyer et al. [2005], Netessine and Taylor [2007], Chen [2001], Taylor and Xiao [2009]

- revenue management
  - Akan et al. [2008], Liu and Xiao [2008], Zhang and Zenios [2008],

- procurement and auctions:
  - Beil and Wan [2009], Cachon and Zhang [2006], Chu and Shen [2006], Yang et al. [2009]

- manufacturing/marketing interface: Chen [2005], Porteus and Whang [1991],

- health care: Li and Lovejoy [2005], Su and Zenios [2006],
Multi-tier supply chains

- Multi-tier supply chains with information asymmetry:
  - channel structure (middleman?)
  - two-layer principal-agent model?
  - multiple sources of asymmetric info
  - information precision
  - operational decisions

- Collaborators:
  - Mingcherng Deng (Minnesota), Ke-Wei Huang (NUS), Ling-Chieh Kung (Berkeley), Wenqiang Xiao (NYU), some more here?
Motivation

- 2007 war among SAP, Oracle, and IBM;
  - Acquiring Business Objects, Hyperion, and Cognos
  - Business Intelligence (BI) systems:
    - Demand forecasting and performance measurement
    - SAS for Autozone and Brooks Brothers, Business Objects for Tesco, Microsoft for Inchcape, ...

- Information asymmetry
  - Demand forecasting – AS; Performance measurement – MH

- The trend of “going global”
  - AmBev, the National Distributing Company, Hyundai Car (UK), U.S. Lumber...
  - BI systems for resellers rather than manufacturers
Research questions

- Reseller’s problem:
  - Two polar cases: demand forecasting or performance measurement?
  - IS versus Accounting
    - Customer Relationship Management (CRM), Activity-Based Costing (ABC), Balance Scorecard, Advanced accounting systems
  - Resource allocation problem: focus or balance?
- Manufacturer’s partner selection problem
Road map

- Model setting
- Partner selection problem
- Resource allocation problem
- Conclusions
Model setup

- Manufacturer (M), reseller (R), and salesperson (S).
- “make-to-order” (MTO) manner
  - production cost: 0, selling price: 1.
- Sales outcome $x = \theta + a + \epsilon$,
  - market condition $\theta \sim F, f$; mean $\mu \equiv \mathbb{E}\theta$;
  - IFR: $H(\theta) \equiv \frac{f(\theta)}{1 - F(\theta)}$ is decreasing in $\theta$
- salesperson’s sales effort $a$; cost of effort $\frac{1}{2}a^2$; $[\frac{1}{2}ka^2]$
- random noise $\epsilon \sim \mathcal{N}(0, \sigma^2_{\epsilon})$
- Adverse selection and moral hazard in the direct channel
Reseller: two polar cases

- Knowledgeable reseller (K)
  - observes $\theta$; eliminate adverse selection

- Diligent reseller (D)
  - monitors $a$; eliminates moral hazard

- Partner selection problem: *Knowledgeable or diligent?*
Model

- **Risk attitudes:**
  - Manufacturer & Reseller: risk neutral
  - Salesperson: risk averse: \( U(z) = -e^{-\rho z} \)
    - \( \rho > 0 \): coefficient of absolute risk aversion; \( z \) : net income

- **Linear contracts**
  - **S:** \( \alpha + \beta x \)
    - \( \alpha \): fixed payment, \( \beta \): commission rate
  - **R:** \( u + v x - (\alpha + \beta x) \)
    - \( u \): fixed payment; \( v \): commission rate (from M)
Road map

- Model setting
- **Partner selection problem**
  - Knowledgeable reseller
  - Diligent reseller
  - Extensions (risk attitude, contract form, ...)
- Resource allocation problem
- Conclusions
Sequence of events with the knowledgeable reseller

- M announces the contract \((u, v)\);
- R decides whether to accept the contract;
  - If rejects, the game ends, every channel party receives 0
- Both R and S observe the demand signal \(\theta\);
- R offers the contract \((\alpha, \beta)\) to S
- S decides whether to accept the contract from R
  - If accepts, S determines \(a\); then sales \(x\) is realized;
  - If rejects, the game ends.

- **Two-stage** contract design: M-R and R-S
Contract design: Knowledgeable reseller

- **Salesperson’s compensation:**
  \[ x = \theta + a + \epsilon \sim N(\theta + a, \sigma^2_\epsilon), \]
  \[ z(a) \equiv \alpha + \beta x - \frac{1}{2}a^2 \sim N\left(\alpha + \beta(\theta + a) - \frac{1}{2}a^2, \sigma^2_\epsilon \beta^2\right), \]

- **S’s utility and certainty equivalent:**
  \[ \mathbb{E}[-e^{-\rho z(a)}] = -e^{-\rho CE_S(\theta|a)}, \]
  \[ CE_S(\theta|a) = \alpha + \beta(\theta + a) - \frac{1}{2}a^2 - \frac{1}{2}\rho \sigma^2_\epsilon \beta^2 \]
  \[ \max_a \mathbb{E}[-e^{-\rho z(a)}] \equiv \max_a CE_S(\theta|a) \Rightarrow a = \beta \]

- **R’s payoff:** \((u, v)\) given
  \[ \max_{\alpha \text{ urs}, \beta \geq 0} \left\{ u - \alpha + (v - \beta)(\theta + \beta) \ \bigg| \ \alpha + \beta \theta + \frac{1}{2} \beta^2 (1 - \rho \sigma^2_\epsilon) \geq 0 \right\} \]
Contract design: Knowledgeable reseller

- Optimal effort level: \( a^K(\theta) = \frac{1}{1 + \rho \sigma^2_\epsilon} \nu \)
- R’s payoff: \( R^K(\theta) = u + \nu \theta + \frac{1}{2(1 + \rho \sigma^2_\epsilon)} \nu^2. \)
- M’s contract design problem:

\[
\max_{u \text{ urs, } \nu \geq 0} \left\{ (1 - \nu)(\mu + \frac{1}{1 + \rho \sigma^2_\epsilon} \nu) - u \middle| u + v \mu + \frac{1}{2(1 + \rho \sigma^2_\epsilon)} \nu^2 \geq 0 \right\}
\]

- \( M^K = \mu + \frac{1}{2(1 + \rho \sigma^2_\epsilon)}, \ a^K(\theta) = \frac{1}{1 + \rho \sigma^2_\epsilon}, \) for all \( \theta, \)
- Selling the business: \( v^K = 1; \ M \) fully extracts R’s surplus.
Contract design: Diligent reseller

- Observes the effort level but not the demand signal
- Adverse selection in R-S relationship
  - menu of contracts \( \{ \alpha(\theta), \beta(\theta) \} \) and effort \( a(\theta) \).
- Sequence of events
  - S observes the market condition \( \theta \);
  - M announces the contract \((u, v)\)
  - R decides whether to accept the contract or not; if rejects, the game ends
  - If R accepts, R offers \( \{ \alpha(\cdot), \beta(\cdot), a(\cdot) \} \) to S; if rejects, the game ends
  - If S accepts, S reports \( \theta \) (chooses a contract \((\alpha(\theta), \beta(\theta), a(\theta))\)); sales \( x \) realizes.
Contract design: Diligent reseller

- S’s problem: if market condition is $\theta$ but reports it as $\tilde{\theta}$:

$$CE_S(\tilde{\theta}, \theta) = \alpha(\tilde{\theta}) + \beta(\tilde{\theta})(\theta + a(\tilde{\theta})) - \frac{1}{2}[a(\tilde{\theta})]^2 - \frac{1}{2}\rho\sigma^2[\beta(\tilde{\theta})]^2.$$  

- R’s problem: $CE_S(\theta) \equiv CE_S(\theta, \theta)$

$$R^D(\theta) = \max \mathbb{E}_\theta \{u - \alpha(\theta) + (v - \beta(\theta))(\theta + a(\theta))\}$$

s.t. $CE_S(\theta) \geq CE_S(\tilde{\theta}, \theta)$ $\forall \theta, \tilde{\theta}$

$CE_S(\theta) \geq 0$ $\forall \theta$,

$\alpha(\theta)$ urs, $\beta(\theta) \geq 0$, $a(\theta) \geq 0$

- $a^D(\theta) = v$, $R^D = u + v\mu + \frac{1}{2}v^2$. 

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Design of multi-tier supply chains: Forecasting, delegation, and information
Contract design: Diligent reseller

- M’s contract design problem: \([a^D(\theta) = \nu]\)

\[
M^D = \max_{u \leq u_\text{urs}, \nu \geq 0} \left\{ (1 - \nu)(\mu + \nu) - u \left| u + \nu \mu + \frac{1}{2} \nu^2 \geq 0 \right. \right\}
\]

- \(a^D(\theta) = 1 \geq \frac{1}{1 + \rho \sigma^2_\epsilon} = a^K(\theta), \) for all \(\theta\).
- \(M^D = \mu + \frac{1}{2} \geq \mu + \frac{1}{2(1 + \rho \sigma^2_\epsilon)} = M^K. \) D dominates K!
Extensions

- Risk averse resellers: $U(y) = -e^{-ry}$,
  - $y$: payoff and $r \in (0, \rho)$: risk aversion coefficient
  - $r = 0$: risk neutral

- Multiplicative form of sales outcome: $x = \theta a + \epsilon$

- Timing of contracting
  - when $K$ observes the demand signal before contracting with $M$.

- One-part tariff
Direct sales: Manufacturer-Salesperson relationship

- S’s choice: if market condition is $\theta$ but reports it as $\tilde{\theta}$

$$CE_S(\tilde{\theta}, \theta | a) = \alpha(\tilde{\theta}) + \beta(\tilde{\theta})(\theta + a) - \frac{1}{2}a^2 - \frac{1}{2}\rho\sigma^2[\beta(\tilde{\theta})]^2;$$

$$M = \max \mathbb{E}_\theta \left[ \theta + \beta(\theta) - \alpha(\theta) - \beta(\theta)[\theta + \beta(\theta)] \right]$$

s.t. $$CE_S(\tilde{\theta}, \theta) = \max_a CE_S(\tilde{\theta}, \theta|a),$$

$$CE(\theta) \geq CE(\tilde{\theta}, \theta) \quad \forall \theta, \tilde{\theta}$$

$$CE(\theta) \geq 0 \quad \forall \theta,$$

$$\alpha(\theta) \text{ urs, } \beta(\theta) \geq 0.$$
Resellers’ private expertise

\[
\begin{align*}
\text{max} & \quad p \left[ (1 - v_P^K) \left( \mu + \frac{1}{1 + \rho \sigma^2} v^K \right) - u^K \right] \\
& + (1 - p) \left[ (1 - v^D) (\mu + v^D) - u^D \right] \\
\text{s.t.} & \quad u^K + v^K \mu + \frac{1}{2(1 + \rho \sigma^2)} [v^K]^2 \geq u^D + v^D \mu + \frac{1}{2(1 + \rho \sigma^2)} [v^D]^2, \\
& \quad u^D + v^D \mu + \frac{1}{2} [v^D]^2 \geq u^K + v^K \mu + \frac{1}{2} [v^K]^2, \\
& \quad u^K \text{ urs}, v^K \geq 0, u^D \text{ urs}, v^D \geq 0. \text{ D dominates K, } v^K \text{ distorted}
\end{align*}
\]
Road map

- Model setting
- Partner selection problem
- Resource allocation problem
  - Model
  - Analytical results
  - Numerical experiments
- Conclusions
Resource allocation problem

- Two estimators for demand and effort:
  - $\eta = \theta + \tau$, $\tau \sim N(0, \sigma^2_\tau)$ is independent of $\theta$
  - $q = a + \xi$, $\xi \sim N(0, \sigma^2_\xi)$ is independent of $a$ and $\theta$.

- Resource constraints:
  - $\sigma^2_\tau \geq K_\tau$, $\sigma^2_\xi \geq K_\xi$, physical restrictions
  - $g(\sigma^2_\tau, \sigma^2_\xi) \geq B$
    - $g$ jointly concave, $B$ budget
    - $g(\sigma^2_\tau, \sigma^2_\xi) = c_\tau \sigma^2_\tau + c_\xi \sigma^2_\xi$, or $g(\sigma^2_\tau, \sigma^2_\xi) = c_\tau \sigma_\tau + c_\xi \sigma_\xi$.

- Assume $\theta \sim N(\mu_\theta, \sigma^2_\theta)$: normal conjugates
Resource allocation problem

- **Monitoring accuracy:** \( \eta = \theta + \tau \)

\[
\eta \sim N(\mu_\theta, \sigma^2_\theta + \sigma^2_\tau) \iff \eta = \mu_\theta + \sqrt{\sigma^2_\theta + \sigma^2_\tau} S
\]

- \( S \sim N(0, 1) \) : standard normal
- One-to-one mapping between \( \eta \) and \( S \)

- **Posterior distribution of \( \theta \):**

\[
\theta|_{S=s} \sim N(\mu_\theta + \sqrt{1 - \sigma^2 \sigma_\theta s}, \sigma^2_\theta \sigma^2)
\]

\( \sigma^2 \triangleq \frac{\sigma^2_\tau}{\sigma^2_\tau + \sigma^2_\theta} \in [0, 1] \) : R’s monitoring accuracy.
Sequence of events (Reseller-Salesperson relationship)

- R decides $\sigma_T^2$ and $\sigma_\xi^2$;
- $\theta$ realizes, observed by S, and demand signal $s$ is observed by R and S;
- R updates belief $\theta|s$ and announces $\{\alpha(\cdot), \beta(\cdot), w(\cdot)\}$ to S;
  - $w$: input compensation (regarding $q$);
- S chooses a contract $(\alpha, \beta, w)$;
- S exerts $a$, R gets estimator $q$, and sales $x$ realizes.
Analysis

- **S’s problem:**

\[
CE_S(\theta, \tilde{\theta} | a) = \alpha(\tilde{\theta}) + \beta(\tilde{\theta})(\theta + a) + w(\tilde{\theta}) a - \frac{1}{2}a^2
\]

\[
- \frac{1}{2} \rho[\beta(\tilde{\theta})]^2 \sigma_\epsilon^2 - \frac{1}{2} \rho[w(\tilde{\theta})]^2 \sigma_\xi^2
\]

\[
a(\tilde{\theta}) = \beta(\tilde{\theta}) + w(\tilde{\theta}), \quad CE_S(\theta, \tilde{\theta}) \triangleq \max_a CE_S(\theta, \tilde{\theta} | a).
\]

- **R’s problem: contract design**

\[
\max \quad \mathbb{E}_\theta \left[ (1 - \beta(\theta))(\theta + a(\theta)) - w(\theta) a(\theta) - \alpha(\theta) | s \right]
\]

s.t.

\[
CE_S(\theta) \geq CE_S(\theta, \tilde{\theta}) \quad \forall \theta, \tilde{\theta},
\]

\[
CE_S(\theta) \geq 0 \quad \forall \theta,
\]

\[
\alpha(\theta) \text{ urs}, \beta(\theta) \geq 0, w(\theta) \geq 0
\]
Reseller’s problem: monitoring accuracies

\[
\max_{\sigma^2_\tau, \sigma^2_\xi} \mathbb{E}_S \left\{ \mathbb{E}_\theta \left[ (1 - \beta(\theta))(\theta + a(\theta)) - w(\theta)a(\theta) - \alpha(\theta) \right| s \right\} \\
\text{s.t.} \quad \sigma^2_\tau \geq K_\tau, \sigma^2_\xi \geq K_\xi, g(\sigma^2_\tau, \sigma^2_\xi) \geq B.
\]

- Closed form expression based on normal conjugates
- Expected profit decreases in \( \sigma^2_\xi \),
- Expected effort decreases in \( \sigma^2_\tau \).
Numerical experiments: effort

- $\tau$: demand monitoring; $\xi$: effort monitoring
- Being diligent is more profitable than being knowledgeable
Numerical experiments: profit

- $\tau$: demand monitoring; $\xi$: effort monitoring
- *Being diligent is more profitable than being knowledgeable*
Numerical experiments: The impact of resource constraints

- \( g(\sigma_T^2, \sigma_\xi^2) = c_T \sqrt{\sigma_T^2} + c_\xi \sqrt{\sigma_\xi^2} \): strictly concave form

- \( K_T = 0.1, c_\xi = 1 \), 1620 experiments (27 scenarios, 60 combinations of \( K_\xi \) and \( c_T \))

- Focusing vs balancing strategies:
  - type-E: \( \sigma_T^2 > K_T, \sigma_\xi^2 = K_\xi \), [focusing]
  - type-D: \( \sigma_T^2 = K_T, \sigma_\xi^2 > K_\xi \), [focusing]
  - type-B: \( \sigma_T^2 > K_T, \sigma_\xi^2 > K_\xi \), [balancing] – never appears!
  - \( g \) non-binding: never appears

- *Focusing typically dominates balancing*
Switch preferences as $K_\xi$ increases
Reducing the marginal cost of monitoring demand may induce more effort monitoring.
Summary

- Multi-tier supply chain design
- M’s partner selection problem:
  - D dominates K
  - ... Accounting versus IS?
- R’s resource allocation problem:
  - focusing dominates balancing
  - effort monitoring dominates demand monitoring
Multi-tier supply chain: Kayis et al. [2008], Guo et al. [2006]

Economics of organization:
- Baron and Besanko [1992], Melumad et al. [1992], Mookherjee [2006] ...

Principal-agent model: thousands (if not millions)
- Holmstrom [1982], Maskin and Riley [1984], Mishra and Prasad [2004], Moorthy [1984], Moorthy and Png [1992], Salanie [2003], ...
- Applications to SCM ...

Monitoring: Holmstrom and Milgrom [1991], Lazear [2000], Prendergast [2002], Zhao [2009], ...
Other related problems

- Channel selection in other scenarios
- Impact of contract forms
- Capacity allocation
- Bargaining
- Signaling