Tracking and Disturbance Rejection in Non-minimum Phase Systems

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Abstract: This paper concerns with the problems of tracking control and disturbance rejection for two types of non-minimum phase (NMP) systems associated with the right half plane (RHP) zeroes and with time delay, respectively. As the achievable closed-loop bandwidth for such systems is usually very limited, obtaining good tracking and disturbance rejection becomes great challenges in the control design. For disturbance rejection, this paper mainly focuses on the method of active disturbance rejection control (ADRC), where modifications in tuning, in ESO and in ADRC structure are made to make the solutions more effective for the NMP systems; for fast tracking, a unique feedforward design is combined with ADRC solution to overcome the bandwidth limitation. The proposed methods are validated in simulation with satisfactory performance and give practitioners a set of tools to deal with the problems of the NMP systems.

Key Words: non-minimum phase system, time delay, active disturbance rejection control, feedforward control, combined design

1 Introduction

In this paper, by “non-minimum phase (NMP) systems” we follow the definition [1, pp. 452] that reads the “range of non-minimum phase curve is greater than the minimum possible for the given amplitude curve”. By this definition, systems with right half plane (RHP) zeroes and systems with time delay are all NMP systems. For example, the following three systems

\[ G_1(s) = \frac{(s-z_1)}{(s-p_1)(s-p_2)} \]  
\[ G_2(s) = -\frac{(s-z_1)}{(s-p_1)(s-p_2)} \]  
\[ G_3(s) = \frac{(s-z_1)e^{-\tau t}}{(s-p_1)(s-p_2)} \]

where \( p_1 = -2, p_2 = -5, z_1 = -1, z_2 = 1, \tau = 2 \), all have the same amplitude curve, but different phase plots, as shown in Figure 1. The system (1) has the minimum possible phase range and is therefore minimum phase (MP), compared to which the systems (2) and (3) are NMP systems.

The control design for NMP systems is generally challenging due to the additional phase lags, which limit the achievable closed-loop bandwidth. In particular, as a rule of thumb, the closed-loop bandwidth \( \omega_{cl} \) is limited to \( \omega_{cl} < z/2 \) for systems with RHP zero \( z \), and \( \omega_{cl} < 1/\tau \) for systems with time delay \( \tau \) [2].

The fundamental goal in a control system design, as shown in Fig. 2, is to keep the error \( e \) between the system output \( y \) and the reference input \( r \) to be zero or very small. In other words, the error \( e \) is to be kept small and invariant in the presence of the changes in both the reference input \( r \) and the disturbance input \( d \), as well as to the uncertainties in the system dynamics. This is considered as the problem of disturbance under the disturbance rejection paradigm (DRP) [3] and it is central to all control system design. From the traditional perspective, the design might be separated as two goals which are tracking the reference (also known as servo design) and rejecting disturbances (also known as regulator design).

To achieve these goals, different methods can be used. The error driven feedback controller can achieve the two goals at the same time, but it is passive since it only reacts to the error. Also its performance is limited in the context of NMP systems. The model based feedforward control, however, can achieve better tracking performance, because it does not have to wait for the error to occur. Similarly, better disturbance rejection performance can also be achieved by canceling the disturbance based on direct measurement or estimation. Both feedforward control and disturbance cancellation are active compared to the feedback control, hence both can be seen in the same framework of active disturbance rejection.

Among various disturbance estimation methods, such as the unknown input observer (UIO) [4], the disturbance observer (DOB) [5], etc., the extended state observer (ESO) aims to estimate both the external and internal dynamics, and to cancel them in real time, reducing the system dynamics to ideal integrators. This forms the basis of the active
disturbance rejection control (ADRC) [6] as a practical solution for nonlinear, time varying and uncertain systems.

![Diagram](image.png)

**Fig. 2:** The diagram of a general control design.

The research work on ADRC, however, has been mainly focused on MP systems with few exceptions [7-8]. Recall that the feedforward control does not help with the disturbance rejection and the disturbance cancellation does not help with the reference tracking. It is under this background that we investigate the ADRC design for the NMP systems, to seek a better solution in a unified ADRC-feedforward design.

The rest of the paper is organized as follows. The ADRC design for NMP systems is presented first in Section 2, to address the disturbance rejection problem. To further improve the tracking performance, the feedforward design for NMP systems is discussed in Section 3. Section 4 gives some design examples, and is followed by the concluding remarks in Section 5.

## 2 ADRC Design for NMP Systems

In this section, the regular ADRC design is briefly reviewed, followed by the modifications to the ADRC design specifically for different NMP systems.

### 2.1 Regular ADRC Design

Consider a general second order system

\[
\dot{y} = bu + f(y, \dot{y}, w, t)
\]

where \( y \) is the system output and \( u \) is the system input, \( b \) is a system related coefficient, and \( f(\cdot) \) represents the total disturbance [6], which includes both external disturbance \( w \) and unknown internal dynamics. Define the state as \( x_1 = y \), \( x_2 = \dot{y} \), and the extended state as \( x_3 = f \), the state space representation of (4) is then

\[
\dot{x} = Ax + Bu + Ef
\]

\[
y = Cx
\]

where \( x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \), \( A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \), \( B = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \), \( E = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \), and \( C = [1 \ 0 \ 0] \).

The extended state observer (ESO) for (5) is

\[
\dot{z} = Az + Bu + L(x_1 - z_1)
\]

where \( z = \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} \) is the estimation of \( x \), \( b_0 \) is a design parameter, which should be as close to \( b \) as possible in most cases, and \( L = \begin{bmatrix} l_1 \\ l_2 \\ l_3 \end{bmatrix} \) is the observer gain vector. The controller law is given as

\[
u = K(r - z)/b_0
\]

where \( r = \begin{bmatrix} r \\ \dot{r} \end{bmatrix} \) is the reference signal, and \( K = [k_1 \ k_2 \ k_3] \) is the controller gain vector. In practice, \( \dot{r} \) and \( \ddot{r} \) are set to zero if they are either not available or unbounded.

To simplify the tuning process, it is convenient to place all the poles of the observer at \( -\omega_c \), hence \( l_1 = 3\omega_c \), \( l_2 = 3\omega_c^2 \), \( l_3 = \omega_c^3 \). Similarly, \( k_1 = \omega_c^2 \), \( k_2 = 2\omega_c \) will place the poles of the controller at \( -\omega_c \). Here \( \omega_c \) and \( \omega_e \) are referred to as observer and controller bandwidth, respectively [9].

The regular ADRC design works very well for MP systems [10, 11], with theoretical analysis provided [12, 13]. But for NMP systems, some difficulties are encountered because the observer bandwidth is found to be limited, which prohibits the ESO to convert the MP part of the system to the ideal form of cascaded integrators. To solve the problem, individual remedies are explored to improve the ADRC design for the NMP systems.

### 2.2 ADRC Design for Systems with RHP Zeros

Considering the ADRC design for system (2), two different formulations were studied in [14]. One uses the relative order of one, results in a negative \( b_0 \) as follows.

\[
\dot{y} = b_0 u + f
\]

\[
= -u + \left(p_1 + p_2\right)y - p_1 p_2 \int y dt + z_2 \int ud t
\]

The other formulation ignores the RHP zero and considers the system as a second order system.

\[
\dot{y} = b_0 u + f
\]

\[
= z u + \left(p_1 + p_2\right)y - p_1 p_2 y - u
\]

Through simulations, the first formulation was never made stable regardless of the choice of the observer bandwidth; the second formulation is also found having a limited observer bandwidth, hence oscillatory response as a result (see the green solid curve in Fig. 3).

Based on the frequency domain analysis, it is shown that: 1) if the DC gain of the system is positive, a negative \( b_0 \) in the ADRC design will destabilize the system; 2) with the second formulation, the system will become unstable if the observer bandwidth is too high; 3) an increased \( b_0 \) may allow higher achievable observer bandwidth. The last point, in fact, provides a practical solution to this problem. By increasing the value of \( b_0 \), better response was achieved (see the red dotted curve in Fig. 3). Thus the simulation study suggests that the original ADRC design for systems with RHP zeros should adopt the second formulation with \( b_0 \) chosen to be 20 to 100 times of its nominal value.
2.3 Further Analysis

By increasing $b_0$, we can get a satisfactory performance at the cost of high observer bandwidth, which, however, may make the system more sensitive to the measurement noise. To design an improved control system with lower bandwidth, consider a general second order NMP process,

$$G_2(s) = \frac{-b_1 s + b}{s^2 + a_1 s + a_2}$$  \hspace{1cm} (10)

where, $a_1$, $a_2$, $b_1$, $b$ are positive values. Dividing the estimation $\hat{f}$ of the total disturbance into two parts,

$$\hat{f} = \frac{\omega_o}{(s + \omega_o)^3} f$$

$$= \frac{\omega_o^3}{(s + \omega_o)^3} [-a_2 y - a_2 y + (b - b_0) u + d - b_3 u]$$

$$= \frac{\omega_o^3 f_1}{(s + \omega_o)^3} u$$

with $f_1 = -a_2 y - a_2 y + (b - b_0) u + d$, the structure of ESO can be represented as below.

As shown in Fig. 4, the equivalent transfer function in the red box can be easily obtained.

$$G_1(s) = \frac{1}{b_0 s - b_3 s - b_1 s + b} \frac{1}{(1 + s/\omega_o)^3}$$  \hspace{1cm} (12)

It is obvious that $G_1$ tends to be a positive feedback system when $\omega_o$ approaches infinity. Rewrite $G_1$ as follows,

$$G_1(s) = \frac{(s + \omega_o)^3}{b_0 s^3 + (3b_0 \omega_o)^2 s^2 + (3b_0 \omega_o^2 - b_1 \omega_o) s + b_0 \omega_o^3}$$  \hspace{1cm} (13)

Based on the Routh stability criterion, the necessary condition of the ESO for the process (10) is

$$\omega_o \leq \frac{8 b_0}{3 b_1}$$  \hspace{1cm} (14)

From (14), it can be concluded that the bandwidth of ESO is strictly limited associated with the ratio of $b_0/ b_1$. The larger is $b_1$, the smaller is the RHP zero and the narrower is the allowable bandwidth of ESO. As a tuning guidance, a larger bandwidth can be obtained with an increased $b_0$.

2.4 Two Alternative Model-assisted Modifications

Shown in (12) is the intrinsic dilemma. In such case, the total disturbance consists of two parts: 1) the regular one of the states and external disturbances; 2) the additional one from the RHP zero. In general, a high observer bandwidth is preferred for good disturbance rejection, subject to the constraint of (14). In this section, an alternative to this strategy is explored by incorporating the model information into the ESO so that good disturbance rejection can be achieved even with a small $b_0$ and low bandwidth.

The main idea is that, with the bandwidth limited by the RHP zero, the general purpose ADRC alone may be inadequate in obtaining the desired results. In this case, additional model information, representing the known dynamics, could be embedded into ESO so that it only needs to estimate the unknown dynamics. This reduces the load and makes the ESO more efficient, as will be shown below. To this end, a practical solution in the form of the modified ESO, denoted as MESO, is designed for the plant in (10), assuming the approximated model is obtained:

$$\begin{align*}
\dot{x}_e &= A_e x_e + B_e v \\
z &= C_e x_e
\end{align*}$$  \hspace{1cm} (15)

where $x_e$ and $z$ are the state and output vector respectively.

Furthermore, with the approximated value of $b_1$ obtained, a modified ADRC (MADRC) structure is proposed as shown in Fig. 5. Basically, to avoid positive feedback, a practical derivative term of control variable is added to balance the positive feedback effects caused by the RHP zero.
Here, $c$ is the filter parameter and $\hat{b}_1$ is a tunable parameter which can be set a little larger than $b_1$. Under such a structure, the estimation of the internal uncertainties and external disturbances will be of higher accuracy by choosing a more suitable $b_0$ and $\omega_o$.

Applied in the same example in [14],

$$G(s) = \frac{-s + 1}{s^2 + 7s + 10},$$

the simulation results for original ESO ($b_0=100$, $\omega_o=20$), MESO ($b_0=3$, $\omega_o=3.9$) and MADRC ($b_0=1$, $\omega_o=4.3$, $\hat{b}_1 = 1$, $c = 10^{-5}$) are shown in Fig. 6.

![Comparison results for different methods](image)

Fig. 6 Comparison results for different methods

It can be seen from Fig. 6 that the MESO based method can gain almost the same tracking and disturbance rejection performance at a low bandwidth. But it leads to a steeper control action and correspondingly larger undershoot. It is also interesting to see that the MADRC method can obtain a smaller settling time with the same overshoot as the others. The MADRC method also has a better robustness in the presence of parameter uncertainty, as shown in Fig. 7.

![Results of robustness simulations](image)

Fig. 7 Results of robustness simulations

### 2.5 ADRC Design for Systems with Time Delay

Similar challenges exist in the ADRC design for system with time delay. Existing solutions include: 1) ignoring the time delay and designing the ADRC for the dynamics without time delay; 2) approximating the time delay with a first order dynamic using the relation $e^{-\tau s} \approx 1/(\tau s + 1)$ and adopting a higher order ADRC design; 3) predicting the system output or the control signal based on the Taylor expansion. But such techniques may not be accurate when the time delay $\tau$ is big.

A simple alternative is recently proposed in [15], where the information of the system delay is used to delay the controller output that is fed back to the ESO. Compared to the regular ADRC design, observer (6) is replaced by following new observer in the new design.

$$\dot{z}(t) = Az(t) + b_0Bu(t - \tau) + L(x(t) - z(t))$$

(16)

Since the system output is already delayed due to the system dynamic, the two inputs to the ESO are synchronized by (16), allowing higher observer bandwidth to be achieved, i.e. better disturbance cancellation. After the synchronization, the ESO provides more meaningful estimations of the system states and disturbances, all delayed by $\tau$.

To evaluate the solution of (16), consider a first order system with a large time delay as

$$G(s) = \frac{1.38 \times 10^{-3}}{s + 6.90 \times 10^{-5}} e^{-60ts}$$

(17)

Three different ADRC designs are carried out for the above system using methods 1) and 2), and the synchronization solution in (16). The parameters of the designs are chosen as follows: for the regular first order ADRC, $b_0 = 1.38 \times 10^{-3}$, $\omega_c = 0.015$; for the regular second order ADRC, $b_0 = 6.90 \times 10^{-5}$, $\omega_c = 0.02$ and $\omega_o = 0.04$; for the synchronized first ADRC, $b_0 = 1.38 \times 10^{-3}$, $\omega_c = 0.015$ and $\omega_o = 0.15$. A disturbance of magnitude 1 is added at 1000 seconds. The simulation results are shown in Fig. 8, where the advantage of the synchronization solution is clearly shown.

![Comparison between three ADRC designs](image)

Fig. 8: Three ADRC Designs for the system of (17).
3 Feedforward control for NMP systems

As can be seen from Fig. 2, the disturbance cancellation channel and the command tracking are independent to each other. To achieve a fast transient response after a set-point change, feedforward control, based on the inversion of the plant followed by a low pass filter, is an effective solution. But such solutions run into great difficulties for the NMP systems, such as those of (2) and (3). The common solution is to invert only the MP part of the plant.

Another important phenomenon in the systems with RHP zeros is the undershoot, which refers to the initial wrong way response when a change occurs in the control signal. And in general the faster the response, the larger the undershoot; thus even with the feedforward design, the task of control design for systems with RHP zeros is still complicated, because there is only so far the wrong way response can go before it endangers the physical integrity of the system. The question becomes that of finding the time optimal solution, given the maximum undershoot allowed; the solution is given in [16, 17] for systems with one or two RHP zeros. Here only the solution for one RHP zero case is presented, for the sake of conciseness. Consider the \( n \)th order system with one RHP zero,

\[
G(s) = \frac{(1-s/z)}{[(1+s/p)^n], \ z > 0, \ p > 0} \quad (18)
\]

the control law

\[
\begin{aligned}
    \begin{cases}
        u(t) = \left( e^{-t} - 1 \right) y_d r_{us}, & t \in [0, t_1) \\
        y_d, & t \in [t_1, \infty)
    \end{cases}
\end{aligned}
\quad (19)
\]

will achieve the minimum settling time as \( p \) goes to infinity, without violating the undershoot constraint

\[
- y(t)/y_d \leq r_{us}, \ t \in [0, \infty), \text{ where } y_d \text{ is the desired system output, and } r_{us} \text{ is the allowable relative undershoot.}
\]

Equation (19) can be used to construct the feedforward control law. The same model information can be used, as shown in Section 2.4, to improve the performance of ESO and ADRC. For the sake of conciseness, though, this proposed feedforward method is combined with the original ADRC design below to show improvement. Much work is still ahead to fully explore the potential of the various modifications of ADRC to address the NMP system problems.

4 Combined Feedforward-ADRC Design

Fig. 9 shows the combined feedforward and ADRC design. The reference signal passes through a feedforward signal generator and a compensator to generate the feedforward control, which goes through a system model to generate the reference system output for the feedback control loop. Here the error based ADRC is used to estimate and cancel the disturbance.

![Fig. 9: The feedforward and ADRC combined control design.](image)

\[
Q(s) = \frac{(1-s/1)(1+s/0.2)}{(1+s/6.2085)(1+s/0.2144)(1+s/(0.385 + 0.7373i))(1+s/(0.385 - 0.7373i))} \quad (20)
\]

\[
C(s) = \frac{(1+s/0.2)(1+s/4)^3}{(1+s/1.05)(1+s/0.22)} \quad (21)
\]

\[
Q_{real}(s) = \frac{(1+s/6.8285)(1+s/0.1944)(1+s/(0.385 + 0.7373i))(1+s/(0.385 - 0.7373i))}{(1+s/6.8285)(1+s/0.1944)(1+s/(0.385 + 0.7373i))(1+s/(0.385 - 0.7373i))} \quad (22)
\]

4.1 Example 1

In this example, a realistic hydraulic turbine control problem is used to demonstrate the practical significance of the proposed feedforward and ADRC combined solution. The overall transfer function of the hydraulic turbine generator is shown in (20). The output of the feedforward signal generator is calculated based on (19), with a 5% undershoot constraint enforced. The compensator is in the form of (21). The real system dynamic is given in (22). A third order ADRC is designed with the following parameters: \( b_0 = 460, \ \omega_d = 5 \) and \( \omega_s = 10 \), considering the results given in Section 2.2. The simulation result of the combined feedforward and feedback design is shown in Fig. 10 (green solid curve). The integral absolute error (IAE) for tracking reduces from 6.595 to 4.792 compared to the result of a pure ADRC design with the same parameters (see Fig. 10, red dash-dotted curve). The system response settles in 20 seconds after the disturbance is introduced.
4.2 Example 2

The same system as studied in Section 2.5 is used here again, but with the feedforward and ADRC combined solution. In this example, no feedforward signal generator is necessary and the compensator is in the following form.

\[ C(s) = \frac{(725s + 5)}{(10s + 1)} \]  (23)

The exact same ADRC parameters are used as in Section 2.5, i.e. \( b_0 = 1.38 \times 10^{-3} \), \( \alpha_r = 0.015 \) and \( \omega_n = 0.15 \). The simulation result is shown in Fig. 11. For comparison purpose, the pure ADRC design result obtained in Section 2.5 is repeated here. One can easily see that the two results have identical disturbance rejection responses, due to the identical ADRC design. The feedforward and ADRC combined results, however, has a faster transient response (98% settling time of 100 seconds) compared to pure ADRC solution (98% settling time of 165 seconds), and no overshoot compared to the 2% overshoot in the pure ADRC solution.

5 Concluding Remarks

In this paper, the control design for NMP systems is discussed. A particular feedforward control is introduced where the model information is incorporated into the ESO and ADRC, respectively, to improve disturbance rejection performance. Finally, as shown by the design examples, the feedforward design and ADRC are combined to form a cohesive solution for both tracking and disturbance rejection. Further theoretical analysis will be carried out to justify the proposed solutions, which have been tested in the simulation studies with promising results. Collectively, the methods proposed in this paper give practitioners a set of tools to deal with the unique problems of tracking and disturbance rejection for NMP systems.

References