Designing a Petri net Model for Resource Allocation in an Automated Assembling Shop

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Abstract

In today's competitive manufacturing systems it is critical to be responding quickly to the demands of customers and decreasing total cost of production. Long cycle times are associated with higher costs of work in process (WIP), increased demand forecasting errors, and lower yields due to slower control feedback. Automated Assembling Shop (AAS) is a class of systems exhibiting concurrency and synchronicity. Achieving high performance AAS, it's necessary to use methods to minimization makespan (production cycle time) and minimization WIP in buffers and maximization machine utilization. This paper intends to focus resource/machine allocation for decreasing WIP and reduction in cycle times using timed Petri net (TPN). A case study is discussed to show the effectiveness of the model.

Keywords

Petri net; automated assembling shop; production cycle time; WIP

1. Introduction

Scheduling problems have been extensively studied in many manufacturing systems. Research has mainly concentrated on the important class of systems called flow shops, in which components are moved linearly through the system, and manufacturing stations are totally dedicated. In these years automatic tools such as CNC machines and different type of robots have been used in assembly line that called automated assembling shop. Many modeling approaches have been considered to solve scheduling problems in AAS, including the finite automata decomposition approach, fuzzy control, and genetic algorithm [3]. These have all been successfully applied to obtain optimal or near-optimal schedules. Due to the discrete nature of manufacturing system, Petri nets (PN) are widely used for modeling manufacturing system [6, 10, and 11]. The scheduling of manufacturing system is usually an NP-hard problem. This means that only heuristic algorithms can be used to provide near-optimal schedules.

Petri net is a graphical and mathematical modeling tool for describing and studying systems. In the early development of Petri nets [7, 8], it was particularly concerned with the description of the causal relationships between events. Much of the early theory, notation, and representation of Petri nets developed for discrete event systems. This work showed how Petri nets could be applied to the modeling and analysis of systems of concurrent components. In today’s competition among the global major industrial nations has renewed interest in the issues of increasing productivity through manufacturing technology [9]. Automated assembling shop consists of several types of CNC (computer numerical control) machines, Robots, and automated guided vehicle and are designed to produce a great variety of products in multiple lines. However, this facility poses complex problems for their planning, designing, scheduling, controlling and monitoring. Many products can be manufactured and assembled in automated assembling shop. The parts to be assembled are transferred by conveyor and robots. Robots transfer the parts from the conveyor to buffer. The AAS features multiple machines that work together in workspaces concurrently. The main problem of designing of...
AAS is to reach a compromise between the makespan and WIP of the system, i.e. to obtain minimization makespan of the system ensuring at the same time its minimum WIP.

In this paper the researchers developed a model of production that receives an order from customer. According to the order they design an initial layout and machine allocation of production line. Then they simulate our initial layout using Petri net model to obtain production cycle time and WIP. The solutions could be obtained by testing the possible resource allocation in available layout. The machines would be allocated to different locations as they can perform same operation with varying time process.

2. Petri net modeling

Petri nets are one of the most rigorous and powerful modeling tools for event-driven systems. Various extensions have greatly enhanced their applicability to different types of discrete event dynamic systems. A timed PN enables a time dependent system to be described. Two methods exist to model timing: either timing associated with places (the PN is said to be place-timed Petri net, or P-timed PN), or timing associated with transitions (the PN is said to be transition-timed Petri net, or T-timed PN). One can show that P-timed PNs and T-timed PNs are equivalent, and it is possible to move from one model to another.

2.1. Basic notations of timed Petri nets [1, 2 and 5]

Let \( F = (P, T, I, O, M_0, \mu) \) be a timed Petri net, where
- \( P \) the set of places (of size \( m \))
- \( T \) the set of transitions( of size \( n \))
- \( I \) the input function
- \( O \) the output function
- \( M: P \rightarrow \{0, 1, 2, \ldots\} \) is a \(|P|\) Dimensional vector with \( M(P) \) being the token count of place \( P \). \( M_0 \) is an initial marking.
- \( \mu: P \rightarrow \mathbb{IN} \) is a time function which associates to each place of \( P \) which may a deterministic rational or probability, \( \mathbb{IN} \) is the set of positive integers.

The number of occurrences of an input place \( p_i \) in a transition \( t_j \) is given by \( \#(p_i, I(t_j)) \), and the number of occurrences of an output place \( p_i \) in a transition \( t_j \) is given by \( \#(p_i, O(t_j)) \).

The matrix of the PN structure, \( X \) is \( X = (P, T, N^-, N^+) \), where \( P, T \) are the finite sets of places and transitions, respectively. \( N^- \) and \( N^+ \) are matrices by \( m \) rows (one for each place) by \( n \) columns (one for each transition):
- \( N^- = [I, j] = \#(p_i, I(t_j)) \), matrix of input incidence matrix, which maps the necessary placement of the tokens for a transition to fire.
- \( N^+ = [O, j] = \#(p_i, O(t_j)) \), matrix of output incidence matrix, indicating the placement of the tokens as a result of the transition firing.

And the incidence matrix \( N \) is given by \( N = N^+ - N^- \).

A Petri net from stage \( k \) to stage \( k+1 \) can be expressed by the following state equation:
\[
M_{k+1} = M_k + e_t N^T
\]

Where \( M_k \) is the current marking state vector and \( e_t \) is the characteristic vector of \( t \), where \( e_t(x) = 1 \) if \( x=t \) is a firing transition, else \( =0 \).

To demonstrate the above notations, we use the following example as shown in figure 1.

Example:

\[ P = \{p_1, p_2, p_3\} \]
\[ T = \{t_1, t_2, t_3\} \]
\[ I(t_1) = \{\}, I(t_2) = \{p_1, p_2\}, I(t_3) = \{p_1\} \]
\[ O(t_1) = \{p_1\}, O(t_2) = \{p_1\}, O(t_3) = \{p_2\} \]
Initial marking: \( M_0 = [1, 1, 0] \).

![Figure1. The PN example model](image-url)
\[
\begin{align*}
N^- &= P_1 \begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} P_2 \\
N^+ &= P_2 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} P_1 \\
N &= N^+ - N^- = P_2 \begin{bmatrix} 1 & -1 & 0 \\ 0 & -1 & 1 \\ 0 & 1 & -1 \end{bmatrix} P_1
\end{align*}
\]

\[M_1 = M_0 + e, \quad N_T = [1, 1, 0] + [0, 1, 0] \Rightarrow N_T = [0, 0, 1].\]

2.2. Petri net modeling of AAS

This paper addresses a typical AAS consisting of two conveyor robots, nine machines tool, five workspaces and fourteen processes. The operation process chart (OPC) has been drawn in figure 2. Also, the problem discussed in this paper is AAS configuration as shown in figure 3.

![Figure2. OPC](image-url)
3. The matrix description of P-timed PN

The PN model is used to determine the optimal location that is required to the AAS. That is, in order for a single or batch of products to be completed, the PN model must transition from an initial state, representing the system at the time the work is requested, to a final state, representing the system state that represents the work had been completed[4]. There are many means to transition to the required system state, and the PN model will aid in forming and selecting feasible transition sequences and facilitates the calculation of makespan and WIP. WIP is calculated according to maximum number of token in buffers (p4, p12, p18, p26, p39) and also the assigned to the places in model. For the AAS model as shown in Figure 4, the set of places is given by \( P = \{p_1, p_2, ..., p_{42}\} \). The location of the tokens in the set of system places \( P \), called a marking \( M \), where:

\[
M_0 = \begin{bmatrix} 2 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 1 & 1 & 0 & 1 & 0 & 0 & 1 & 0 \end{bmatrix}^T.
\]

There are twenty one possible transitions in the example. The connection between places and transitions is represented through an invariant flow relation matrix, denoted by \( N \), where \( N = N^+ - N^- \).

Place-timed Petri nets (p-timed PN) are used to model of system, in with transitions represent events and the places represent states, or conditions. The PN model has been given in figure 4.
The concept of places and transitions in the PN model are shown in Table 1.

Table 1. The concept of places and transitions

<table>
<thead>
<tr>
<th>Places</th>
<th>Transitions</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p_1$: work piece A available</td>
<td>$t_1$: Operation OP1 starts</td>
</tr>
<tr>
<td>$p_2$: Operation OP1</td>
<td>$t_2$: Operation OP1 finishes</td>
</tr>
<tr>
<td>$p_3$: Machine M1 available</td>
<td>$t_3$: Operation OP2 starts</td>
</tr>
<tr>
<td>$p_4$: work piece A ready for the operation OP2</td>
<td>$t_4$: Operation OP2 finishes</td>
</tr>
<tr>
<td>$p_5$: Buffer of work piece A available</td>
<td>$t_5$: Operation OP3 starts</td>
</tr>
<tr>
<td>$p_6$: Operation OP2</td>
<td>$t_6$: Operation OP2 finishes</td>
</tr>
<tr>
<td>$p_7$: Machine M2 available</td>
<td>$t_7$: Operation OP3 starts</td>
</tr>
</tbody>
</table>
4. Case Study and simulation results

The production planning of ten products is based on the above production line. The machines M2, M3, M8 are of the same type and non identical. For installation of machines M2, M3, M8, there exist six states (3!) of the layout. The simulation results have been given in Table2.

<table>
<thead>
<tr>
<th>state</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>OP</td>
<td>OP3</td>
<td>OP5</td>
<td>OP12</td>
<td>OP3</td>
<td>OP5</td>
<td>OP12</td>
</tr>
<tr>
<td>M</td>
<td>M3</td>
<td>M2</td>
<td>M8</td>
<td>M3</td>
<td>M2</td>
<td>M8</td>
</tr>
<tr>
<td>P</td>
<td>P4</td>
<td>P5</td>
<td>P7</td>
<td>P5</td>
<td>P7</td>
<td>P5</td>
</tr>
<tr>
<td>WIP</td>
<td>11</td>
<td>7</td>
<td>0</td>
<td>3</td>
<td>8</td>
<td>11</td>
</tr>
<tr>
<td>Make</td>
<td>155</td>
<td>155</td>
<td>116</td>
<td>116</td>
<td>116</td>
<td>116</td>
</tr>
<tr>
<td>Span</td>
<td>155</td>
<td>155</td>
<td>116</td>
<td>116</td>
<td>116</td>
<td>116</td>
</tr>
</tbody>
</table>

5. Conclusion

In this paper, the resource allocation of an AAS is discussed in detail. The system features different machines working same operation in the different work piece of product, and time variant manufacturing processes. A P-timed PN is applied to model the system. The MATLAB Petri net toolbox software has been used to calculate the WIP and product cycle time. The researchers obtain the optimal layout on the basis of well known approaches in multiple criteria decision making as they introduced WIP and product cycle time for minimizations.
References