AN ARTIFICIAL NEURAL NETWORKS BASED MODEL TO IDENTIFY FAULTS IN MULTIVARIATE QUALITY CONTROL CHARTS

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Abstract: Most multivariate quality control procedures evaluate the in-control or out-of-control condition based upon an overall statistic, like Hotelling’s T\(^2\). Although T\(^2\) is optimal for finding a general shift in mean vectors, it is not optimal for shifts that occur for some subset of variables. This introduces a persistent problem in multivariate control charts, namely the interpretation of a signal, which often discourages practitioners in applying them. In this paper, we propose an artificial neural network based model to diagnose faults in out-of-control conditions and to help identify aberrant variables when Shewhart type multivariate control charts based on Hotelling’s T\(^2\) is used. The results of the model implementation on a couple of numerical examples are encouraging.

Key Words: Statistical Process Control, Multivariate Control Charts, Artificial Neural Networks

1. INTRODUCTION

In many quality control settings, the product (process) under examination may have two or more correlated quality characteristics and we need an appropriate approach to monitor all these characteristics simultaneously. This has formed the basis of extensive research performed in the field of multivariate control chart since the 1940’s, when Hotelling (1947) recognized that the quality of a product might depend on several correlated characteristics. However, because of computational complexity, researchers and practitioners did not pursue the multivariate quality control at that time. Now that the development of high-speed computers and technological advances in industrial control procedures has alleviated this problem, many researchers have proposed several multivariate control charts, where each has some advantages as well as disadvantages. Jackson (1957) mentioned that multivariate control charts should possess three important properties, namely, they should produce signal answers to (1) whether the process is in-control, (2) whether the specified type I error has been maintained, and (3) whether they have taken the relationships between the variables into account?

One can classify the multivariate quality control procedures in two broad categories. First, when quality test includes several parameters; second, when a production line includes several serial stages as a sequential system. In the sequential case, like chemical industry, production lines have several stages and the quality of the product in each stage depends not only on the process of the current stage, but also on the quality of the input, which is the output of the previous stage, to the current stage. In the non-sequential case, where there are several correlated characteristics, the purpose of each test is to detect where the process has gone “out-of-control” and which of the variable(s) is (are) the cause of deterioration. In this research, we will focus on the non-sequential case.

2. LITERATURE REVIEW

Early research on multivariate Shewhart charts goes back to Hotelling (1947) where he introduced the problem of correlation between the quality characteristics of a process and came up with the well-known $T^2$ statistic to identify whether the whole process is out-of-control. A major advantage of Hotelling’s $T^2$ statistic is that it is the optimal test statistic for detecting a general shift in the process mean vector for an individual multivariate observation (Hawkins 1991). However, the technique has several practical drawbacks. One of the most important ones is that when the $T^2$ statistic indicates that a process is out of control, it does not provide information on which variable or set of variables is out of control. Moreover, it is difficult to distinguish location shifts from scale shifts since the $T^2$ statistic is sensitive to both types of process changes.

Murphy (1987) proposed a method to identify the “out-of-control” variables based on discriminant analysis. We can view this quality control method as trying to discriminate between the process of being “in control” or “out-of-control”. He divided the complete set of variables into two subsets and then tried to determine which one caused the “out-of-control” signal. Extensions of the Murphy’s work are Niaki and Moeinzadeh (1997) and Niaki et. al. (2001). They developed a statistic and an algorithm for the cause-selecting problem in which the population parameters are not known and are to be estimated.

The principal component analysis is a way of explaining the variance-covariance structure in a multivariate environment by the use of a few linear combinations of the original variables. Jackson (1980) gave a detailed description of principal components and its possible use as a multivariate quality control tool. The problem with principal component is that they are not easily interpretable in many cases, and they do not have a one-to-one relation with the original variables. Nevertheless, in some cases they can be very useful, depending on the context.

Doganoskoy et. al. (1991) proposed the use of the univariate t-statistic for ranking the variables most likely to have changed. Then, to further strengthen the belief that a certain variable has changed, they applied the Bonferroni type interval. The obvious drawback of this method is that it only tells you which variable is most likely to have shifted, which is not conclusive. Moreover, this method does not allow the user to study the trends.

Mason et. al. (1995) proposed a cause-selecting procedure using the decomposition of $T^2$ statistic. By decomposing $T^2$ statistic, the user can see the contribution of each variable. This decomposition also allows the user to detect which variable(s) with significant contribution is (are) the cause of deviation. The drawback of this method is extensive computation and sensitivity to the number of variables.

The MEWMA control charts use all the observations since the detection of the last special event rather than only the last observation vector as in the Shewhart type charts. Their advantages over the latter charts are that their average run length is smaller for small shifts in the process mean. In MEWMA category several researchers proposed different procedures, to name a few see Alwan (1986), Lowry et. al. (1992), and Prabhu and Runger (1997). Lawry et. al. (1992) presented a multivariate extension of the exponentially weighted moving average (EWMA) control chart. They compared their chart to a multivariate cumulative sum (MCUSUM) control chart based on the average run length (ARL) performance. They concluded that their chart was similar to the CUSUM chart in detecting a shift in the mean vector of a multivariate normal distribution, and that the ARL performance of the MEWMA chart, as well as the Hotelling’s and MCUSUM charts depended on the underlying mean vector and covariance matrix only through the value of the non-centrality parameter. They stated that in order to avoid the potential inertia problems, one should always use the MEWMA and MCUSUM charts in conjunction with the Hotelling’s chart. In order to improve the detection of small shifts in multivariate statistical process control, Prabhu and Runger (1997) provided some recommendations to select the parameters of a multivariate exponentially weighted moving average control chart.

The properties of MCUSUM control charts are quite similar to those of the MEWMA charts. In this category, Woodall and Neube (1985) proposed methods to approximate parameters of the distribution of the minimum of the run length of the univariate CUSUM charts. For the bivariate normal distributions, they show that their MCUSUM method works better than the Hotelling’s $T^2$ procedure. Healy (1987) discussed the natural applications of CUSUM procedures to the multivariate normal distribution. Crosier (1988) presented the design procedures and the average run length for two MCUSUM quality control procedures. The first MCUSUM procedures reduced each multivariate observation to a scalar, and then formed a CUSUM of the scalars. The second MCUSUM method formed a CUSUM vector directly from the observations. They compared these two procedures to a multivariate Shewhart chart and discussed the robustness of the procedures. Pignatiello and Runger (1990) considered several approaches for controlling the mean of a multivariate normal process. They compared the performance of these approaches, as well as the performance of their two newly proposed charts, based on the estimated ARL and reported the results.
For the first time, Noorossana et al. (2003) presented an Artificial Neural Network (ANN) model to detect and classify non-random disturbances in auto-correlated processes. The preliminary results of their method implications indicated that the ANN modeling is an effective method for cause-selecting problems.

In section 3, we briefly introduce Neural Network Modeling and their applications in non-sequential and multivariate quality control environments. Section 4 contains the proposed Neural Network methodology in cause-selecting situations. The implementation of the proposed methodology comes in section 5. At the end, we state the conclusions and recommend some areas for future research.

3. NEURAL NETWORK MODELING in MULTIVARIATE QUALITY CONTROL ENVIRONMENTS

Artificial Neural Networks (ANN) modeling is an optimizations tool for the output process (responses). They mimic biological neural networks to model and solve a variety of problems arising in prediction or forecasting, function approximation, pattern classification, clustering, and categorization. A neural network, which consists of a number of interconnected nodes called neurons, plays like a computational algorithm for information processing (Guh 1999 and Noorossana 2003).

In pattern classification application of the ANN, we usually assign an observation to a pre-identified pattern. There are two stages in pattern classification, namely, characteristics identification stage and classification stage. In the first stage, we select the basic characteristics of a pattern that distinguishes it from the others. We will later use them as a criterion for decision-making process. In the second stage, we design a classification machine such that it takes the characteristics as input and produces patterns as output.

There are many topologies of the ANN in pattern classification as many researchers have proposed several classification machines so far. However, studies show that the Multilayer Perceptron (MLP) network with error back propagation has better performance than that of traditional statistical classification methods (Guh 1999). MLP is a neural network that does nonlinear classification and has three layers: input layer, output layer and hidden layer. Input layer distributes input to all neuron in the hidden layer, which contains the sigmoid transfer function. The task of the output layer is to determine the pattern.

The network training process is an important task before network implementation. There are two general types of training process, namely, the supervised and the non-supervised process. In the supervised training process, while the network does not know the environment, the trainer (user) knows it completely and plays an important role as the network learns. However, in a non-supervised process the user lets the network train itself. In order to train a MLP network with error back propagation, in the first step one needs to generate sufficient data containing all of the classified patterns. In the second step, he uses the data as input to the network and compares the output of the network with a pre-specified target. Then, in the next step, based upon some performance criteria, such as the mean squared error, where the error is defined as the difference between the target value and the output, the error back propagation algorithm modifies the weights (W’s) and the threshold values (b’s) and the training process goes to the next step and so on. He continues the training process until either one of two situations occurs: either the mean squared error gets sufficiently close to zero or a pre-specified number of cycles (epochs) is reached. Figure 1 shows the structure of a MLP neural network in which learning is supervised and error back propagation algorithm is used (James 1987).

Now, in multivariate quality control environment, when an out-of-control signal is detected by Hotelling’s $T^2$ statistic or other multivariate procedures, we may model the cause-selecting problem as a pattern classification problem. In this case, we classify the signal in several classes of patterns. As an example in bivariate case there are three distinguished classes of patterns as 1) $1^{st}$ variable being out-of-control, 2) $2^{nd}$ variable being out-of-control, and 3) $1^{st}$ and $2^{nd}$ variable both are out-of-control. Note that the signal must belong to one of them.

We will implement the MLP neural network for pattern classification in fault diagnosis process. The number of neurons in input layer is the number of variables in the multivariate environment, and the number of neurons in output layer is the number of classes ($2^p - 1$, where $p$ is number of variables). The number of neurons in the hidden layer is a designer choice and usually we select it based on some performance criteria obtained from examining different numbers of hidden nodes. In the design process, one must generate sufficient data to train the ANN adequately. Each one of the multivariate control charts is sensitive for a special range of shifts. Therefore, to create data for the training process, one must consider the multivariate control chart method that signals the overall out-of-control situation. In Multivariate Shewhart Chart (MSCH), we usually consider a range of shift in the mean vector to be equal or greater than 2.50 times the standard deviation. In this research, we apply the Matlab computer software to generate data for the training process. As an example, in a bivariate case, where we apply MSCH for the overall out-of-control signal detection with the known mean vector

740
$(\mu_1, \mu_2)$ and known covariance matrix $\Sigma$, as an input to the network, we shift the mean vector as

$$(\mu_1 + 2.5\sigma_{11} + \mu_2, \mu_1 + 2.5\sigma_{22} + \mu_2)$$

and

$$(\mu_1 + 2.5\sigma_{11}, \mu_2 + 2.5\sigma_{22})$$.

In this case the target values for the output should be $(1,0)$, $(0,1)$, and $(1,1)$, respectively. When sufficient data for training process is generated and the ANN is trained, we can use the multivariate Shewhart procedure for fault detection along with the ANN for the diagnosis process. We note that, in the cases where the output values of the trained network are close to either zeroes or ones, we round them down or up respectively to get the exact zero and ones as the output values. Figure 2 shows a schematic for the detection and diagnostic process.

**Figure 1. The Structure of a MLP Neural Network**

**Figure 2. Detection and Diagnostic by Multivariate Control Chart and Neural Networks**
3.1 Numerical Examples

Through the following numerical examples, we will demonstrate how one can implement the ANN modeling along with a multivariate Shewhart chart to both detect and classify shifts in mean vectors in multivariate quality control settings.

3.1.1 Example one

Consider the diagnostic of a signal when we have only two process variables. We use this simple case to illustrate the way the proposed model works. The data represent measurements of stiffness $X$ and bending strength $Y$ in unit of lbs/sq.inch for a particular grade of lumber. The standard values either known or derived from a large amount of past data are:

$$\mu_{0x} = 265, \mu_{0y} = 470, \sigma_{0x} = 10, \sigma_{0y} = 11, \rho = 0.6$$

In matrix notation we have

$$\mu = \begin{pmatrix} 265 \\ 470 \end{pmatrix}, V = \begin{pmatrix} 100 & 66 \\ 66 & 121 \end{pmatrix}, R = \begin{pmatrix} 1 & 0.6 \\ 0.6 & 1 \end{pmatrix}$$

In order to detect an out-of-control condition we use the MSCH method with $\alpha$ being equal to 0.05. There are two characters and $2^2 - 1$ patterns. Therefore, we design a MLP network that has two neurons in input layer and two neurons in output layer. For the number of neurons in the hidden layer, we tried different numbers and based on the mean squared error we came up with nine neurons. Figure 3 shows the neural network for example 1.

![Neural Network of Example 1](image)

We trained the neural network with mean vector shifted $2.5 \sigma$. To do so, first we created 100 set of vector means for each of the three cases of $(265+2.5*10, 470)$, $(265, 470+2.5*11)$ and $(265+2.5*10, 470+2.5*11)$ with covariance matrix being $V$ in each case. If we denote the out-of-control variable as 1 and the in-control variable as 0, then the target vectors for these cases are $(1 \ 0)^T$, $(0 \ 1)^T$, and $(1 \ 1)^T$ respectively. Then, applying the Matlab computer software, we detected the overall out-of-control condition using Hotelling $T^2$ for each set of the shifted vectors and computed the mean squared error for several combinations of the weights and the thresholds in each case. We ended the training process when the mean squared error reached a value of $10^{-13}$. The number of required cycles (epochs) for this process was 1902.

To test the trained neural network, applying the Matlab computer software, we generated 100 samples vector that shifted $2 \sigma$, 100 samples vector that shifted $2.5 \sigma$, and 100 samples vector that shifted $3 \sigma$. We applied the MSCH procedure to detect the overall out-of-control in each case. Table 1 shows the results. As the results in Table 1 show, the MLP modeling can classify the out-of-control signals in almost every case.
Table 1. Results of the MLP Modeling on Example 1

<table>
<thead>
<tr>
<th>Shift</th>
<th>The Results of the MLP model</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1,0)</td>
</tr>
<tr>
<td>(2σ,0)</td>
<td>82</td>
</tr>
<tr>
<td>(0,2σ)</td>
<td>0</td>
</tr>
<tr>
<td>(2σ,2σ)</td>
<td>3</td>
</tr>
<tr>
<td>(2.5σ,0)</td>
<td>95</td>
</tr>
<tr>
<td>(0,2.5σ)</td>
<td>0</td>
</tr>
<tr>
<td>(2.5σ,2.5σ)</td>
<td>3</td>
</tr>
<tr>
<td>(3σ,0)</td>
<td>94</td>
</tr>
<tr>
<td>(0,3σ)</td>
<td>0</td>
</tr>
<tr>
<td>(3σ,3σ)</td>
<td>1</td>
</tr>
</tbody>
</table>

3.1.2 Example two

As another example of a higher dimensional problem, consider the example discussed in Doganaksay et. al. (1991), which was taken from Jackson (1980), which concerns the testing of ballistic missiles. In this example $p=4$ and the mean vector and the covariance matrix are given by:

$$
\mu_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad \Sigma = \begin{bmatrix} 102.74 & 88.34 & 67.03 & 54.06 \\ 88.34 & 142.74 & 86.55 & 80.02 \\ 67.03 & 86.55 & 84.57 & 69.42 \\ 54.06 & 80.02 & 69.64 & 99.06 \end{bmatrix}
$$

The goal is to detect shifts in the mean vector by $2.5\sigma$.

We designed a MLP neural network with four neurons in input layer, four neurons in output layer, and came up with thirty-one neurons in hidden layer by trying different numbers of hidden nodes and comparing the final mean squared error for each case. Figure 4 shows the neural network of example 2.
Again, we detected the overall out-of-control condition with the Hotelling’s $T^2$ with $\alpha$ being equal to 0.05. We trained the network by a shift in the mean vector of two standard deviations. For example, one of the generated input vectors was $(0 \ 0 + 2(142.74) \ 0 \ 0)^T$ with a target vector of $(0 \ 1 \ 0 \ 0)^T$ in this case. The 15 patterns then were recognized using the Matlab software designed for the MLP modeling with 1802 epochs, where the mean squared error reached a value of $10^{-7}$ at the end. Then, in our simulation study, we generated 50 sample vectors of data that shifted only 2.5 $\sigma$ from the mean vector in the same way as of the example 1 and implemented them to the trained model as an input vectors. Table 2 shows the simulation results of the MLP modeling. One more time the results of Table 2 show that the MLP modeling is able to classify the signals in most of the time.

<table>
<thead>
<tr>
<th>Shift=2.5 $\sigma$</th>
<th>The Results of the MLP model</th>
</tr>
</thead>
<tbody>
<tr>
<td>The Number of the Shifted Variable(s)</td>
<td>Number of True Diagnosis</td>
</tr>
<tr>
<td>1</td>
<td>45</td>
</tr>
<tr>
<td>2</td>
<td>46</td>
</tr>
<tr>
<td>3</td>
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<tr>
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<td>1,2,3,4</td>
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4. CONCLUSIONS AND RECOMMENDATIONS FOR FUTURE RESEARCH

In this paper, we presented a new procedure to classify the signals of out-of-control condition in multivariate quality control environment by designing a MLP type artificial neural network model. We trained the ANN model for specific shifts in the input vector mean and implemented the model on two situations. The results of the model implementation are encouraging. We believe that an ANN modeling is a good and strong procedure to diagnose signals in multivariate quality control and could be used in other aspects of it. For example, it could be applied to determine the variable(s) with shifts in standard deviations, to evaluate the value of a shift in either mean or standard deviation, to determine the out-of-control stage(s) in sequential processes, and to use it along with fuzzy modeling to change the output of the ANN to exact zero and one values.

5. REFERENCES


