Constraint Multiproduct Joint-Replenishment Inventory Control Problem Using Uncertain Programming

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Abstract
An uncertain economic order quantity (UEOQ) model with payment in advance is developed to purchase high-price raw materials. A joint policy of replenishments and pre-payments is employed to supply the materials. The rate of demand is considered LR-fuzzy variables, lead-time is taken to be constant, and it is assumed that shortage does not occur in the cycles. The cycle is divided into three parts; the first part is the time between the previous replenishment-time to the next order-time ($t_0$), the second part is the period between $t_0$ to a payment-time ($t_k$), and the third part is the period between $t_k$ to the next replenishment-time. At the start of the second part ($t_0$), $\alpha$% of the purchasing cost is paid. The $(1-\alpha)$% remaining purchasing cost is paid at the start of the third part ($t_k$). The cost of the model is purchasing under incremental discount for each order with rough cost per unit, clearance cost, fixed-order cost, transportation cost, holding, and capital cost. Holding cost is for on-hand inventory and capital cost is for the capital that is paid for the next order. The constraints of the problem are space, budget, and the number of orders per year. Further, lead-time is considered less than a cycle time. We show that the model of this problem is a fuzzy integer-nonlinear-programming type and in order to solve it, a hybrid method of harmony search, fuzzy simulation, and rough simulation is proposed. In order to validate the results and examine the performance of the proposed method, a genetic algorithm, as well as a particle swarm optimization method is also employed. The results of a numerical example show that the proposed procedure has the best performance in terms of the mean of the objective function in different simulation runs. At the end, a case study along with a sensitivity analysis is given to demonstrate the applicability of the proposed methodology in real world inventory control problems and to provide some managerial insights.

Keywords: Inventory Control; Multi-product Multi-constraint; Joint Replenishment; Raw Materials; Soft Computing Techniques; Harmony Search Algorithm; Uncertain Programming

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1. Introduction
Since its formulation in 1915, the square-root-formula for the economic order quantity (EOQ) has been used in the inventory literature for a long time. This formula is based on the assumption of a constant demand. The discrete case of the dynamic version of EOQ was first discussed by Wagner and Whitin (1958). Regarding the continuous-time dynamic EOQ models, Silver and Meal (1969) were the first to suggest a simple modification of the classical square-root-formula in the case of time-varying demand. Later, Silver and Meal (1973) developed an approximate solution procedure, known as the Silver–Meal-heuristic, for general case of a deterministic, time-dependent demand pattern. However, Donaldson (1977) discussed, for the first time, the classical no-shortage inventory policy for the case of a linear, time-dependent demand. His treatment was fully analytical and needed extensive computational effort to obtain the optimal solution.

The question of inventory shortages and backlogging were not considered at all by the aforementioned researchers. Deb and Chaudhuri (1987) were the first to incorporate shortages into the inventory lot-sizing problem with a linearly increasing time-varying demand. EOQ models for deteriorating items with a trended demand were considered by Goswami and Chaudhuri (1991, 1992), Xu and Wang (1990), Chung and Ting (1993, 1995), Kim (1995), Hariga (1996a, b), Benkherouf (1995), Jalan et al. (1996), Jalan and Chaudhuri (1999), Giri and Chaudhuri (1997), Lin et al. (2000), etc. In the model of Deb and Chaudhuri (1987), shortages are allowed in all cycles except the last one. Each of the cycles in which shortages are permitted starts with replenishment and ends with a shortage which is backlogged in the next cycle. Numerous studies have been carried out to address the problems of imperfect quality EMQ model with rework (see, for example, Hayek and Salameh 2001; Chiu 2003, 2007; Chiu et al. 2004; Jamal et al. 2004). Chiu and Chiu (2006) studied optimal replenishment policy for an imperfect quality EMQ model with backlogging and failure in repair using conventional approach. That is to derive the optimal lot size by utilizing differential calculus on the expected cost function with the need to prove optimality first. Grubbstrom and Erdem (1999) first introduced an algebraic method to solve the classic EOQ and economic production quantity (EPQ) model without the use of derivatives. A few researches have then been carried out using the same method (see, for example, Cardenas-Barron 2001; Wee and Chung 2007). In these researches that extend the algebraic approach to an imperfect quality EMQ model examined by Chiu and Chiu (2006), the use of differential calculus is replaced with an algebraic method, and the optimal lot size solution is obtained under the expected cost minimization. Taleizadeh et al. (2008a) introduced a joint replenishment EOQ model to import the raw material with certain demand.

In this paper, to extend the model of Taleizadeh et al. (2008a), the EOQ model is extended to purchase expensive imported raw material in a joint replenishment policy. In addition, holding costs (including capital, storage and insurance cost for on hand inventory), purchasing cost under incremental discount, clearance and fixed order costs, transportation cost and capital costs for the next order are employed to the problem. All parameter of the problem are crisp and the quantity of the orders must be integer multiples of packets, each containing more than one product. Also we assume that the lead time is less than a cycle length.

The rest of the paper is organized as follows: In section 2, the problem along with its assumptions is defined. Some definitions in Fuzzy and rough theory are given in sections 3 and 4, respectively. In section 5, we model the problem. Section 6 contains
the proposed solution algorithm. Finally, a numerical example is solved in section 7 to demonstrate the applicability of the proposed methodology.

2. Problem Definition

Inventory holding as a tactical-level decision against non-secure situations to increase confidence level of responding to customer demands and also to resist against all kind of non-deterministic situations in receiving raw materials made inventory control and management an important concept in supply chain management. The model that is proposed in this research is an applied model developed based on the real constraints and environments of manufacturing companies. In these companies, the annual demands of different products are first estimated at the start of the year. Then, based on these estimates, the inventory and planning departments proceed to material planning. In some cases, improper material planning and control policies and loss of sufficient raw materials in proper times and quantities, result to customer complaints and loss of customer and market share. In some instances gaining the lost-market-shares needs more promotion and advertising costs and causes increased production cost. To confront with these instances and to minimize raw-material shortages, in this research a model for material planning and control is developed.

In a manufacturing company, the steps involved in the ordering process of the materials are (Taleizadeh et al. 2008a):

1. Forecast the number of finished products.
2. Forecast the required raw-materials.
3. Order the required materials that consist of:
   a. Releasing orders of the materials to a supplier.
   b. Paying a percentage $(\alpha\%)$ of the purchasing cost at $t_0$.
   c. Paying the remaining payment $(1-\alpha)\%$ at $t_k$.
   d. Transporting materials and receiving them at $T$.

Figure (1) depicts the inventory control cycle of a material.

There are two payment methods in a material ordering process; (1) the credit transaction with a specific lead time and (2) cash method that has smaller lead time. In the cash method the purchasing cost of the total ordered material is paid to the supplier at the ordering time. However, in the credit transaction method $\alpha\%$ of the material purchasing cost is paid at the order release time $(t_0)$ and the remaining $(1-\alpha)\%$ is paid at the starting time of the material transportation $(t_k)$. The ordered materials are received at $T$ (Taleizadeh et al. 2008a).

Let us assume that the credit transaction method is used for payments in which the lead-time is deterministic and constant. Materials ordering and their transportation are performed in batch form and the amount of raw materials in each batch and the number of batches in each vehicle is deterministic. Based on the product groups' consistency, the ordered materials can be carried together by finite-capacity vehicles with maximum capacity of $F$ (in volume).

In this paper, an inventory problem is considered in which the demand follows a LR fuzzy number and based on Liu (2008), the concepts of the credibility, possibility and necessity theory, as well as credibility of fuzzy event and the expected value of a fuzzy variable have been adopted. In this problem, service rates, warehouse capacity and budget are considered constraints and the incremental discount policies are used to purchase the items. Furthermore, the quantity of the orders must be integer multiples of packets, each containing more than one product. The objective is to determine the optimal order quantity of each material in a joint replenishment policy.
such that the total cost is minimized. The costs are: (1) purchasing cost under incremental discount for each order, (2) holding cost for on hand inventory (including capital, warehouse, and insurance costs), (3) capital cost of the next order, (4) transportation cost, (5) clearance cost, and (6) fixed-order costs. Furthermore, we assume that the lead times are less than the cycle times of the materials.

In the following section, some brief backgrounds on rough theory are given. The parameters and the variables required to model the problem are introduced in section 4.

![Inventory Control Cycle](image)

Figure (1): Inventory Control Cycle

3. Rough Theory

In this part of the paper, some concepts in rough environment as well as rough space and rough variable are introduced. The descriptions of these concepts are presented in the following based on Liu (2008).

**Definition 7:** Let $\Lambda$ be a nonempty set, $A$ be a $\sigma$-algebra of subset of $\Lambda$, $\Delta$ be an element in $A$, and $\pi$ be a nonnegative, real valued, additive set function. Then $(\Lambda, \Delta, A, \pi)$ is called rough space.

Also, a rough variable is defined as a measurable function from a rough space to the real line.

**Definition 8:** A rough variable $\delta$ on the rough space $(\Lambda, \Delta, A, \pi)$ is a function from $\Lambda$ to the real line $\mathbb{R}$ such that for every Borel set $O$ of $\mathbb{R}$, we have $\{\theta \in \Lambda | \delta(\theta) \in O\} \in A$. The lower and upper approximation of the rough variable $\delta$ are then respectively defined as follows,
\( \delta = \{ \delta(\varnothing) | \varnothing \in \Delta \} \), \( \bar{\delta} = \{ \delta(\varnothing) | \varnothing \in \Lambda \} \) \hspace{1cm} (9)

For example a rough variable \( (\llbracket a, b \rrbracket, \llbracket c, d \rrbracket) \) with \( c \leq a \leq b \leq d \) is a measurable function from a rough space \( (\Lambda, \Delta, A, \pi) \) to the real line, where, \( \Lambda = \{ \chi | c \leq \chi \leq d \} \), \( \Delta = \{ \chi | a \leq \chi \leq b \} \) and \( \delta(\chi) = \chi \) for all \( \chi \in \Lambda \).

4. Problem Modeling

To model the problem at hand, we first introduce the parameters and the variables of the problem in section 4.1. Then, the model of the problem is developed in section 4.2.

4.1. The Parameters and the Variables

For \( i = 1, 2, \ldots, n \) and \( j = 1, 2, \ldots, P \) the parameters and the variables of the model are:

- \( P \): Is the number of products
- \( D_j \): Is the annual constant demand of the \( j^{th} \) product
- \( \bar{D}_j \): Is the fuzzy annual constant demand of the \( j^{th} \) product
- \( n_j \): Is the number of items in the packets of the \( j^{th} \) product
- \( h_{j1} \): Is the holding cost per unit of on hand inventory of the \( j^{th} \) product during a period
- \( h_{j2} \): Is the capital cost per unit of the \( j^{th} \) ordered product during the period before the payment of the remaining purchasing cost (10\% of the total purchasing cost)
- \( h_{j3} \): Is the capital cost per unit of the \( j^{th} \) ordered product during the period after the payment of the remaining purchasing cost
- \( C_j^c \): Is the clearance cost for each unit of the \( j^{th} \) product
- \( C_j^p \): Is the constant purchasing cost of the \( j^{th} \) product in the \( i^{th} \) discount break point
- \( \bar{C}_j^p \): Is the rough purchasing cost of the \( j^{th} \) product in the \( i^{th} \) discount break point
- \( q_{j} \): Is the \( i^{th} \) discount break point of the \( j^{th} \) product
- \( ROP_j \): Is the reorder point of the \( j^{th} \) product
- \( f_j \): Is the space required for each packet of the \( j^{th} \) product
- \( L \): Is the constant joint lead-time for each order
- \( A \): Is the fixed order cost per each order
- \( A_f \): Is the fixed transportation cost per each shipment
- \( \hat{f} \): Is the total available space in each truck
- \( Q \): Is a decision variable representing the order quantity of the \( j^{th} \) product
- \( m_j \): Is a decision variable representing the number of packets that have been ordered for the \( j^{th} \) product
- \( TB \): Is total available budget
- \( T \): Is the decision variable representing the joint cycle length
\( N \): Is the annual number of orders \((N = 1/T)\)

\( N_T \): Is the upper limit for number of orders

\( C_H \): Is the annual total holding cost of the products

\( C_c \): Is the annual total clearance cost

\( C_p \): Is the annual total purchasing cost of the products

\( C_T \): Is the annual total transportation cost of the products.

\( Z \): Is the annual total costs

In the next section, the inventory model of the problem is developed.

### 4.2. The Objective Function

The objective function of the model is to minimize the total cost of the joint replenishment problem given in equation (10).

\[ Z = C_T + C_p + C_c + C_d + C_H \]  \( \quad \text{(10)} \)

The terms in the right hand side of equation (10) are derived as follows.

#### 4.2.1. Transportation Cost \((C_T)\)

The transportation cost is calculated based on equation (11), in which \( f_jm_j \) is the required space to ship the order of \( j^{th} \) product from the supplier.

\[
C_T = \begin{cases} 
  A_f & ; \quad 0 < \sum_{j=1}^{p} f_j m_j \leq \hat{f} \\
  2A_f & ; \quad \hat{f} < \sum_{j=1}^{p} f_j m_j \leq 2\hat{f} \\
  \vdots & \\
  K\hat{f} & ; \quad (K-1)\hat{f} < \sum_{j=1}^{p} f_j m_j \leq K\hat{f} 
\end{cases} \quad \text{(11)}
\]

By introducing the binary variables \( Y_k; k = 1, 2, \ldots, K \), the annual total transportation cost (occurring \( N \) times per year) can be incorporated with the mathematical model of the problem as:

\[
C_T = N\sum_{k=1}^{K} kA_f Y_k = \frac{1}{T}\sum_{k=1}^{K} kA_f Y_k \\
0 < \sum_{j=1}^{p} f_j m_j \leq \hat{f}Y_1 \\
\hat{f}Y_2 < \sum_{j=1}^{p} f_j m_j \leq 2\hat{f}Y_2 \\
\vdots \\
(K-1)\hat{f}Y_k < \sum_{j=1}^{p} f_j m_j \leq K\hat{f}Y_k \\
Y_1 + Y_2 + \cdots + Y_k = 1 \\
Y_k = 0, 1 \quad \forall k = 1, 2, \ldots, K \quad \text{(12)}
\]
4.2.2. Purchasing Cost under Incremental Discount ($C_p$)

The purchasing cost of the company for the $j^{th}$ product at each period and each order can be calculated using the incremental discount policy. It is necessary to indicate that, discount from supplier is considered for each order and not during a year. Let the incremental discount policy be:

$$
C_j^p = \begin{cases} 
C_{1j}Q_j & ; \quad 0 < Q_j \leq q_{1j} \\
C_{1j}q_{1j} + C_{2j}(Q_j - q_{2j}) & ; \quad q_{1j} < Q_j \leq q_{2j} \\
\vdots & \\
C_{1j}q_{1j} + C_{2j}(q_{2j} - q_{1j}) + \ldots + C_{nj}(Q_j - q_{n-1,j}) & ; \quad Q_j \geq q_{nj}
\end{cases}
$$

(13)

Where $q_{ij}$ and $C_{ij} ; i = 1, 2, \ldots, n$ are the discount points and the purchasing costs for each unit of the $j^{th}$ product that corresponds to its $i^{th}$ discount break point, respectively. In order to include the incremental discount policy in the inventory model, we use equation (14) to model the incremental discount policy.

$$
C_j^p = C_{1j}Q_j + C_{2j}Q_{2j} + \ldots + C_{nj}Q_{nj} \\
Q_j = Q_{1j} + Q_{2j} + \ldots + Q_{nj} \\
q_{1j} \lambda_{2j} \leq Q_{1j} \leq q_{1j} \lambda_{1j} \\
(q_{2j} - q_{1j}) \lambda_{3j} \leq Q_{2j} \leq (q_{2j} - q_{1j}) \lambda_{2j} \\
\vdots \\
0 \leq Q_{nj} \leq M \lambda_{nj} \\
\lambda_{1j} \geq \lambda_{2j} \geq \ldots \geq \lambda_{nj} \\
\lambda_{ij} = 0,1 \quad \forall i, \quad i = 1, 2, \ldots, n, \quad \text{and} \quad M \quad \text{is a very big number.}
$$

(14)

Finally the total annual purchasing cost will be (occurring $N$ times per year):

$$
C_p = \sum_{j=1}^{P} \sum_{i=1}^{n} C_{ij}Q_{ij} = \frac{1}{T} \sum_{j=1}^{P} \sum_{i=1}^{n} C_{ij}Q_{ij} \\
Q_j = Q_{1j} + Q_{2j} + \ldots + Q_{nj} \\
q_{1j} \lambda_{2j} \leq Q_{1j} \leq q_{1j} \lambda_{1j} \\
(q_{2j} - q_{1j}) \lambda_{3j} \leq Q_{2j} \leq (q_{2j} - q_{1j}) \lambda_{2j} \\
\vdots \\
0 \leq Q_{nj} \leq M \lambda_{nj} \\
\lambda_{1j} \geq \lambda_{2j} \geq \ldots \geq \lambda_{nj} \\
\lambda_{ij} = 0,1 \quad \forall j, \quad j = 1, 2, \ldots, P , \forall i, \quad i = 1, 2, \ldots, m
$$

(15)

4.2.3. Clearance Cost ($C_c$)

The clearance cost of the company for the $j^{th}$ product at each period is $C_j^c$, the order quantity is $Q_j$, and the number of order per year is $N$. Hence, the total annual clearance cost of the company will be:
\[ C_c = N \sum_{j=1}^{p} C_j Q_j = \sum_{j=1}^{p} C_j D_j \]  

(16)

### 4.2.4. Fixed Order Cost \((C_A)\)

The fixed order cost of each order per period is \(A\) and the number of orders per year is \(N\). Hence, and the total annual fixed order cost in a disjoint ordering policy will be \(NA\). However, since we are using the joint replenishment policy, the fixed order cost will change to:

\[ C_A = \frac{A}{T} \]  

(17)

### 4.2.5. Holding Cost \((C_H)\)

According to Figure (1), the holding cost during a cycle is:

\[ \sum_{j=1}^{p} \frac{h_j^1 Q_j T}{2} \]  

(18)

The first part of the capital cost occurs between the \(\alpha\)% and \((1-\alpha)\)% - payment times and is derived as:

\[ \sum_{j=1}^{p} h_j^2 Q_j (t_k - t_0) \]  

(19)

The other part of the capital cost occurs between\([t_k , T]\) and is calculated as:

\[ \sum_{j=1}^{p} h_j^3 Q_j \left[ L -(t_k - t_0) \right] \]  

(20)

So the total holding cost during a year will be:

\[ C_H = N \left[ \sum_{j=1}^{p} \frac{h_j^1 Q_j T}{2} + \sum_{j=1}^{p} h_j^2 Q_j (t_k - t_0) + \sum_{j=1}^{p} h_j^3 Q_j \left[ L -(t_k - t_0) \right] \right] \]  

(21)

Also, according to (Figure 1) we have

\[ Q_j = D_j T_j \]  

(22)

Knowing that

\[ N = \frac{1}{T} \rightarrow NT = 1 \]  

(23)

Then, equation (21) can be written as (24) for a joint order case.

\[ C_H = \sum_{j=1}^{p} \frac{h_j^1 D_j T}{2} + \sum_{j=1}^{p} h_j^2 D_j (t_k - t_0) + \sum_{j=1}^{p} h_j^3 Q_j \left[ L -(t_k - t_0) \right] \]  

(24)

### 4.3. The Constraints

As the total available warehouse space is \(F\), the space required for each unit of the \(j^{th}\) product is \(f_j\), and the order quantity of the \(j^{th}\) product is \(Q_j\), the space constraint will be:

\[ \sum_{j=1}^{p} f_j Q_j \leq F \]  

(25)

Furthermore, since the total available budget is \(TB\), and the purchasing cost that is calculated in section (3.2.2) is \(C_p = \frac{1}{T} \sum_{j=1}^{p} \sum_{i=1}^{m} C_{ij} Q_{ij}\), the budget constraint will be:
\[
\frac{1}{T} \sum_{j=1}^{P} \sum_{i=1}^{n} C_{ij}Q_{ij} \leq TB
\]  

(26)

For the sake of convenience in ordering, clearance, transportation and some other activities, we assume an upper limit for the number of orders per year. In other words

\[
N \leq N_T \rightarrow \frac{1}{T} \leq N_T \rightarrow T \geq \frac{1}{N_T}
\]  

(27)

Moreover, the quantities of the orders must be integer multiples of packets, each containing more than one product. That is

\[
Q_j = n_j m_j
\]  

(28)

Knowing that

\[
Q_j = D_j T
\]  

(29)

We have

\[
TD_j = n_j m_j
\]  

(30)

Finally, the model of the problem becomes:

\[\begin{align*}
\text{Min:} & \quad Z = C_T + C_p + C_C + C_A + C_H \\
& = \frac{1}{T} \sum_{k=1}^{K} kA_T Y_k + \frac{1}{T} \sum_{j=1}^{P} \sum_{i=1}^{n} C_{ij}Q_{ij} + \frac{1}{T} \sum_{j=1}^{P} C_j D_j + \frac{A}{T} + \sum_{j=1}^{P} \frac{h_j^2 D_j T}{2} \\
& \quad + \sum_{j=1}^{P} h_j^2 D_j (t_k - t_0) + \sum_{j=1}^{P} h_j^2 D_j [L - (t_k - t_0)] \\
& = \frac{1}{T} \left[ A + \sum_{k=1}^{K} kA_T Y_k + \sum_{j=1}^{P} \sum_{i=1}^{n} C_{ij}Q_{ij} \right] + \left[ \sum_{j=1}^{P} \frac{h_j^2 D_j}{2} \right] \left[ T \right] \\
& \quad + \left[ \sum_{j=1}^{P} C_j D_j + \sum_{j=1}^{P} h_j^2 D_j (t_k - t_0) + \sum_{j=1}^{P} h_j^2 D_j [L - (t_k - t_0)] \right] \\
\end{align*}\]

s.t.:
\[\begin{align*}
\frac{1}{T} \sum_{j=1}^{P} \sum_{i=1}^{n} C_{ij}Q_{ij} & \leq TB \\
\sum_{j=1}^{P} f_j Q_j & \leq F \\
T & \geq \frac{1}{N_T} \\
0 & < \sum_{j=1}^{P} f_j m_j \leq \hat{f} Y_1 \\
\hat{f} Y_2 & < \sum_{j=1}^{P} f_j m_j \leq 2 \hat{f} Y_2 \\
& \vdots \\
(K-1) \hat{f} Y_k & < \sum_{j=1}^{P} f_j m_j \leq K \hat{f} Y_k
\end{align*}\]
\[ TD_j = n_j m_j \]
\[ Q_j = Q_{ij} + Q_{2j} + \ldots + Q_{nj} \]
\[ q_{ij} \lambda_{2j} \leq Q_{ij} \leq q_{ij} \lambda_j \]
\[ (q_{2j} - q_{ij}) \lambda_{2j} \leq Q_{2j} \leq (q_{2j} - q_{ij}) \lambda_j \]
\[ \vdots \]
\[ 0 \leq Q_{nj} \leq M \lambda_{nj} \]
\[ Y_1 + Y_2 + \ldots + Y_k = 1 \]
\[ \lambda_{ij} \geq \lambda_{2j} \geq \ldots \geq \lambda_{nj} \]
\[ \lambda_{ij} = 0,1; Y_k = 0,1 \quad \forall k = 1,2,\ldots,K, \quad \forall j = 1,2,\ldots,P, \forall i, i = 1,2,\ldots,m \]
\[ T \geq 0 \]
\[ m_j, Q_j \geq 0 \text{ integer} \]

Based on definition (6) since \( h_j^\top T/2, h_j^\top (t_k - t_0) \), and \( h_j^\top [L - (t_k - t_0)] \) are positive constants, the model of the problem in uncertain state with triangular fuzzy demand and rough purchasing price under discount becomes:

\[ \text{Min: } Z = C_r + C_p + C_c + C_A + C_H \]
\[ = \frac{1}{T} \sum_{k=1}^{K} k A_T Y_k + \frac{1}{T} \sum_{j=1}^{P} \sum_{i=1}^{n} \hat{C}_{ij} Q_{ij} + \sum_{j=1}^{P} C_j^c \bar{D}_j + \frac{A}{T} + \sum_{j=1}^{P} h_j^\top \bar{D}_j \frac{T}{2} \]
\[ + \sum_{j=1}^{P} h_j^\top \bar{D}_j (t_k - t_0) + \sum_{j=1}^{P} h_j^\top \bar{D}_j [L - (t_k - t_0)] \]
\[ = \frac{1}{T} \left[ A + \sum_{k=1}^{K} k A_T Y_k + \sum_{j=1}^{P} \sum_{i=1}^{n} \hat{C}_{ij} Q_{ij} \right] + \left[ \sum_{j=1}^{P} h_j^\top \bar{D}_j \right] \frac{T}{2} \]
\[ + \left[ \sum_{j=1}^{P} C_j^c \bar{D}_j + \sum_{j=1}^{P} h_j^\top \bar{D}_j (t_k - t_0) + \sum_{j=1}^{P} h_j^\top \bar{D}_j [L - (t_k - t_0)] \right] \]

s.t.:
\[ \frac{1}{T} \sum_{j=1}^{P} \sum_{i=1}^{n} \hat{C}_{ij} Q_{ij} \leq TB \]
\[ \sum_{j=1}^{P} f_j Q_j \leq F \]
\[ T \geq \frac{1}{N_f} \]
\[ 0 < \sum_{j=1}^{P} f_j m_j \leq \hat{f}Y_1 \]
\[ \hat{f}Y_2 < \sum_{j=1}^{P} f_j m_j \leq 2 \hat{f}Y_2 \]
\[ \vdots \]
\[(K - 1)\bar{Y}_k < \sum_{j=1}^{p} f_j m_j \leq K\bar{Y}_k\]

\[TD_j = n_j m_j\]

\[Q_j = Q_{ij} + Q_{2j} + \ldots + Q_{nj}\]

\[q_{ij} \lambda_{2j} \leq Q_{ij} \leq q_{ij} \lambda_{ij}\]

\[\left(q_{2j} - q_{ij}\right) \lambda_{2j} \leq Q_{ij} \leq \left(q_{2j} - q_{ij}\right) \lambda_{2j}\]

\[0 \leq Q_{ij} \leq M \lambda_{ij}\]

\[Y_t + Y_2 + \ldots + Y_k = 1\]

\[\lambda_{ij} \geq \lambda_{2j} \geq \ldots \geq \lambda_{ij}\]

\[\lambda_{ij} = 0, 1; Y_k = 0, 1 \quad \forall k = 1, 2, \ldots, K, \quad \forall j, \quad j = 1, 2, \ldots, P, \forall i, \quad i = 1, 2, \ldots, m\]

\[T \geq 0\]

\[m_j, Q_j \geq 0 \quad \text{integer}\]

5. **A Solution Algorithm**

Since the model in (32) is mixed integer-nonlinear in nature, reaching an analytical solution (if any) to the problem is difficult (Gen 1997). Furthermore, as efficient treatment of mixed integer nonlinear optimization is one of the most difficult problems in practical optimization (El-Sharkawi 2008), we need to employ a meta-heuristic search algorithm to solve it. This algorithm is a hybrid method of harmony search algorithm, rough simulation, and fuzzy simulation, described as follows.

5.1. **Rough Simulation**

In order to estimate uncertain demands of rough variable, we employ a simulation technique. Rough simulation plays an important role in rough system. If \(Z(\hat{C}, \hat{Q})\) is a real valued continues function and \(\hat{C}_{ij}\) is a rough vector defined on rough space \((\Lambda, \Delta, A, \pi)\), in order to estimate the expected value \(E[Z(\hat{C}, \hat{Q})]\), rough simulation approach may be employed. In this approach, by denoting \(\delta_i\) by \(\delta_i = (\delta_{ij}, \delta_{2j}, \ldots, \delta_{lj})\) we generate \(\hat{\delta}_i = (\hat{\delta}_{ij}, \hat{\delta}_{2j}, \ldots, \hat{\delta}_{lj})\) from \(\Lambda\), and \(\bar{\delta}_i = (\bar{\delta}_{ij}, \bar{\delta}_{2j}, \ldots, \bar{\delta}_{lj})\) from \(\lambda\) according to the measure \(\pi\). So the objective function will be a function of \(\hat{C}(\hat{\delta}_i)\) and \(\hat{C}(\bar{\delta}_i)\). However, to evaluate the objective function we need to estimate \(\hat{C}(\hat{\delta}_i)\) and \(\hat{C}(\bar{\delta}_i)\).

5.2. **Fuzzy Simulation**

In order to estimate the uncertain demands of the fuzzy model, we employ a simulation technique. Denoting \(\hat{D}_j\) by \(\hat{D}_j = (\hat{D}_1, \hat{D}_2, \ldots, \hat{D}_p)\), \(\mu\) as the membership function of \(\hat{D}\), and \(\mu_j\) are the membership functions of \(\hat{D}_j\), we randomly generate \(D_{jz}\) from the \(\alpha\)-level sets of fuzzy variables \(\hat{D}_j\), \(j = 1, 2, \ldots, P\) and \(z = 1, 2, \ldots, N\) as \(\xi_z = (\xi_{1z}, \xi_{2z}, \ldots, \xi_{pz})\) and
\[ \mu(\xi_z) = \mu_1(\xi_{z1}) \land \mu_2(\xi_{z2}) \land \cdots \land \mu_p(\xi_{zp}) , \quad \text{where } \alpha \text{ is a sufficiently small positive number.} \]

Based on the definition in equation (11), the expected value of the fuzzy variable is

\[ E\left[ Z(\tilde{D}, Q) \right] = \int_{-\infty}^{+\infty} Cr \left\{ Z(\tilde{D}, Q) \geq r \right\} dr - \int_{-\infty}^{0} Cr \left\{ Z(\tilde{D}, Q) \leq r \right\} dr \tag{33} \]

Then, provided \( N \) is sufficiently large, for any number \( r \geq 0 \), \( Cr \{ Z(\tilde{D}, Q) \geq r \} \) can be estimated by:

\[ Cr \{ Z(\xi_z, Q) \geq r \} = \frac{1}{2} \left( \text{Max}_{z=1,2,\ldots,Z} \{ \mu_z [ Z(D_z, Q) \geq r ] \} + 1 - \text{Max}_{z=1,2,\ldots,Z} \{ \mu_z [ Z(\xi_z, Q) < r ] \} \right) \tag{34} \]

And for any number \( r < 0 \), \( Cr \{ Z(\xi_z, Q) \leq r \} \) can be estimated by:

\[ Cr \{ Z(\xi_z, Q) \leq r \} = \frac{1}{2} \left( \text{Max}_{z=1,2,\ldots,Z} \{ \mu_z [ Z(\xi_z, Q) \leq r ] \} + 1 - \text{Max}_{z=1,2,\ldots,Z} \{ \mu_z [ Z(\xi_z, Q) > r ] \} \right) \tag{35} \]

Algorithm (1) shows the procedure of estimating \( E\left[ Z(\tilde{D}, Q, \hat{C}) \right] \) by hybrid method of rough simulation and fuzzy simulation.

**Step1.** Set \( E=0 \)

**Step2.** Randomly generate \( D_{\tilde{z}} \) from \( \alpha \)-level sets of fuzzy variables \( \tilde{D}_j \), and set

\[ \hat{\xi}_z = (\xi_{z1}, \xi_{z2}, \ldots, \xi_{zp}) \]

**Step3.** Set

\[ a = Z(D_1, Q) \land Z(D_2, Q) \land \cdots \land Z(D_Z, Q). \]
\[ b = Z(D_1, Q) \lor Z(D_2, Q) \lor \cdots \lor Z(D_Z, Q). \]

**Step4.** Randomly generate \( r \) from Uniform \([a, b]\).

**Step5.** Generate \( C_{ijl} \) from \( \Delta \) according to measure \( \pi \).

**Step6.** Generate \( C_{ijkl} \) from \( \Delta \) according to measure \( \pi \).

**Step7.** If \( r \geq 0 \), then \( E \leftarrow E + Cr \left\{ \frac{Z(D, Q, \hat{C}(\tilde{D})) + Z(D, Q, \hat{C}(\tilde{D}))}{2L} \geq r \right\} \).

**Step8.** If \( r < 0 \), then \( E \leftarrow E - Cr \left\{ \frac{Z(D, Q, \hat{C}(\tilde{D})) + Z(D, Q, \hat{C}(\tilde{D}))}{2L} \leq r \right\} \).

**Step9.** Repeat the fourth to eight steps for \( N \) times.

**Step10.** \( E\left[ Z(\tilde{D}, Q, \hat{C}) \right] = a \lor 0 + b \land 0 + E \frac{b-a}{N} \).

**5.3. Harmony Search Algorithm**

Many researchers have successfully used meta-heuristic methods to solve complicated optimization problems in different fields of scientific and engineering disciplines. Some of these meta-heuristic algorithms are simulating annealing (Aarts and Korst 1989, Kirkpatrick et al. 1994, Taleizadeh et al. 2008b), threshold accepting (Dueck and Scheuer 1990), Tabu search (Joo and Bong 1996), genetic algorithms (Al-Tabtabai and Alex 1999), neural networks (Gaidock et al. 2002), ant colony optimization (Dorigo and Stutzle 2004), fuzzy simulation (Taleizadeh et al. 2009),
evolutionary algorithm (Laumanns et al. 2002), and harmony search (Lee and Geem 2004, Geem et al. 2001 and Taleizadeh et al. 2008a, Fesanghari et al. 2008).

Lee and Geem (2004) and Vasebi et al. (2007) showed that the harmony search (HS) algorithm outperforms genetic algorithm (GA) (Goldberg 1989) because of its multi-vector consideration and fast computation. The HS optimization method has been applied successfully to various engineering problems such as satellite heat pipe design (Geem and Hwangbo 2006), vehicle routing (Geem et al. 2005), water network design (Geem et al. 2002 and 2005) and structural design (Lee and Geem 2004). Mahdavi et al. (2007) described an improved harmony search (IHS) algorithm for solving optimization problems. IHS employs a novel method for generating new solution vectors that enhances accuracy and convergence rate of HS algorithm.

The selection of a proper algorithm to solve any optimization problem is not an easy task. That is why in addition to a HS algorithm, a GA, as well as a particle swarm optimization algorithm is developed to solve the problem at hand. A comparison study of the methods' performances can assure one of the validity of the results obtained. The HS optimization algorithm applied in this paper is performed by the following steps.

5.3.1. Initialization

The initialization process has two parts; parameter initialization and harmony memory (HM) initialization as described below.

5.3.2. Parameter Initialization

The constant parameters of the HS algorithm include harmony memory size (HMS), harmony memory considering rate (HMCR), pitch adjusting rate (PAR), number of decision variables (N), and the maximum number of improvisations (NI) (Taleizadeh et al. 2008a).

The HMS is the number of simultaneous solution vectors in HM. Based on the frequently used HMS values in other HS applications available in the literature (Geem 2006, Geem and Hwangbo 2006, Geem et al. 2001, 2002, and 2005) it seems that using a small HMS is a good and logical choice with the added advantage of reducing space requirements. In this paper, the numbers 10, 20, and 30 are chosen as different values of HMS (Taleizadeh et al. 2008a).

The HMCR is the probability of choosing HM. Choosing a very small HMCR decreases the algorithm efficiency and the HS algorithm behaves like a pure random search, with less assistance from the HM. Hence, it is generally better to use a large value for the HMCR (i.e. ≥ 0.9). In this research 0.93, 0.95, and 0.99 have been used for HMCR (Taleizadeh et al. 2008a).

The pitch adjustment is similar to the adjustment of each musical instrument in a jazz so that pleasing harmony can be achieved. The efficiency of the algorithm lies within this pitch adjustment because of the fact that once a feasible design is determined, it searches new solution vectors around this design vector rather than generating arbitrary design vectors (Taleizadeh et al. 2008a). The PAR is the probability of pitch adjustment where its typical value ranges from 0.3 to 0.99. In this research, 0.3, 0.7 and 0.9 have been utilized for PAR.

The value of N, the number of variables for optimization, is fully depended on the characteristics of the problem. For the proposed HS algorithm, since the main variable is T and the other variables depend on T, the value of N has been chosen one (Taleizadeh et al. 2008a).
Finally, $NI$ is the maximum number of iterations of the objective function evaluations. In this research, 100, 500, and 1000 are chosen as different iteration numbers.

5.3.3. Harmony Memory Initialization

In the proposed HS algorithm, the HM is a two-dimensional matrix with $HMS$ rows and 2 columns. The first column of HM is specified by the value of the objective function for each solution vector.

The HM is initialized with randomly generated solutions in a specific range limited by upper and lower bounds determined by the problem at hand. However, because of the constraints described in section 3.3, only those solution vectors that satisfy the constraints are included in HM.

5.3.4 New Harmony Generation

New harmony improvisation is based on three rules: (i) random selection (ii) HM consideration, and (iii) pitch adjustment. In random selection rule, the new value of each decision variable $x'_i$ is randomly chosen within the allowable range of the vector solution $X_j$. Then, $X = [x'_1, x'_2, ..., x'_N]$ will represent the new vector solution.

In HM algorithm, the random choosing from HM occurs with probability $HMCR$ and the random selection is performed with probability $(1 - HMCR)$.

In pitch adjustment, every component obtained by the memory consideration is examined to determine whether it should be pitch adjusted or not. The value of the decision variable is changed by equation (36) with probability of $PAR$, and this value is kept without any change with probability $1 - PAR$. In equation (36) the $BW$ stands for bandwidth and denotes the amount of change for pitch adjustment. Also, $rand$ is a uniform random number between 0 and 1. In this equation, for each component of the vector the selection for increasing or decreasing are carried out with the same probability.

$$X = X \pm (rand)(BW) ; \quad rand \sim U[0,1]$$ (36)

5.3.5. Harmony Memory Update

The constraint handling part of the algorithm is performed before the HM update. There are three constraints in each of the proposed models. The constraint handling part checks whether these constraints are satisfied or not. If they are satisfied, then the HM update action occurs. In this stage, by the objective function evaluation, if the new fitness value is better than the worst case in the HM, the worst harmony vector is replaced by the new solution vector. The remaining steps of the HM algorithm are performed after the HM updates (Taleizadeh et al. 2008a).

In summary, the steps involved in the HS algorithm used in this research are:

1. Initialize both the parameters and the HM of the HS algorithm.
2. Make a new vector $X'$. For each component $x'_i$:
   - With probability $HMCR$ pick the component from memory,
   - With probability $1 - HMCR$ pick a new random value in the allowed range.
3. Pitch adjustment: For each component $x'_i$:
   - With probability $PAR$, a small change is made to $x'_i$. 

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• With probability $1 - PAR$ do nothing.

4. If $X'$ is better than the worst $X^j$ in the memory, then replace $X^j$ with $X'$ by evaluating the objective function (based on Algorithm 1, to estimate the uncertain parameters).

5. Go to step 2 until a maximum number of iterations has been reached.

The detailed flow-chart of the proposed HS algorithm is shown in Figure (2).

6. A Case Study

The proposed methodology of this research is applicable in a company that uses expensive imported raw materials ordered in batch sizes to make the final product. The demand and the purchasing cost of the final product are nondeterministic. The lead-time is positive justifying the use of the clearance cost. Drugs, industrial machines, chemical raw material, washing powder, alloys, etc. are the types of imported raw materials that are imported to un-developed countries and falls within the boundaries of this research.

In order to demonstrate the application of the proposed methodology a case study is considered in this section. The case involves a dairy product company in Iran that processes and bottles milk in paper packs (called Tetra Pack) in different tastes of simple, cacao, Nescafe, and strawberry in different capacities of 100, 250, 500, and 1000 cc (16 product combinations). The products due to their nature have short cycle-times. The paper-role that produces Tetra packs of different sizes is the expensive imported raw material. Depending on the pack-sizes, each role produces a certain number of packs. Furthermore, due to storage capacity, the budget limitation on purchasing the expensive raw material and integer number of roles in each order an inventory model needs to be developed to contain these constraints.

When an order is placed for the paper roles, it usually takes almost a fixed amount of time to receive. This time includes the time to place the order, the time the supplier needs to produce the order, the shipping time from the supplier to the custom office placed at the Iran's border, the time in custom department to release the order, and the shipping time from the custom department to the company. That is why a fixed-cycle EOQ model is required to model the problem. The shipping costs are calculated per trucks and are excluded from the purchasing costs. Moreover, when the company places an order, an $\alpha\%$ of the purchasing price needs to be paid at the time of order and the remaining $(1 - \alpha)\%$ must be paid at the time the supplier starts shipping the roles. The supplier also offers an incremental discount policy on the price of the paper role.

There are different uncontrollable factors that affect the paper-role price. Some of these factors are the world-wide economic fluctuation, the variances in commodity values, and the sanction. These uncontrollable factors cause to use rough prices for the roles in the inventory model. Furthermore, a market investigation on the products shows that the demand does not follow a specific pattern such that it can neither be considered fixed nor to have a certain probability distribution, justifying the usage of triangular fuzzy demands.

The general data for the multi-product inventory control problem with sixteen products are given in Table (1). The company's total available warehouse space, the total budget and the maximum number of order are $F = 1000$, $TB = 400,000,000$ and $N_f = 6$, respectively. Moreover the lead-time is $L = 2.5$ months, the fixed ordering cost is $A_f = 2000$, the fixed transportation cost per shipment is $A_r = 300,000$, the total
available space in each truck is $f = 200$, the pre-payment time is assumed $(t_a) = 0$, and the remaining payment time is $(t_e) = 1.5$ Months.

The proposed methodology starts initializing the parameters of the HS algorithm. Then, feasible solution vectors are generated until the total memory is initialized. Next, for every randomly selected component of the solution vector the pitch adjustment is performed. Since the demand and the purchasing cost are uncertain (fuzzy and rough variables respectively), the hybrid rough and fuzzy simulation of Algorithm 1 is used to determine their deterministic values. The fourth step of the HS algorithm is to compare the solutions by evaluating the objective function, and finally the fifth step is to check whether the stopping criterion is met.

Table (2) shows different values of the HS parameters used to obtain the solution, where the maximum number of improvisations is set $NI = 100$. In this research, all of the 27 possible combinations of the HS parameters given in Table (2) are employed. Table (3) shows the objective function values based on the 27 parameter-combinations, each in 10 iterations. Since the Bartlett test cannot reject the equality of the variances of the 27 populations (the test statistic, 35.93, is less than the upper 0.05 percentile of the Chi-squared distribution with 26 degrees of freedom, 38.85), the mean responses are then compared using a one-way analysis of variance. The result of the analysis of variance shows the means are different (the $f$-test statistic, 8.57, is more than the upper 0.05 percentile of the F-distribution with 26 and 243 degrees of freedom, 1.52). Since the mean responses are statistically different, one can choose the one with the minimum value to select the best combination of the parameters. Table (4) shows the best combination of the HS algorithm. The best result of the hybrid harmony search is shown in Tables (5). The optimal common cycle time of the products is 0.3315 and the optimal total cost is 130,340,000. The optimal number of the ordered packets for each product and the optimal order quantities of the products are also given in this table. Furthermore, the convergence path of the best result of the objective function is shown in Figure (3).

In order to validate the results obtained and to examine the performance of the proposed method, the genetic algorithm of Taleizadeh et al. (2009) and the particle swarm optimization method of Taleizadeh et al. (2010) are employed to the numerical problem as well. Tables (6) and (7) show the best results of the genetic algorithm and the particle swarm optimization, respectively. Based on the results in Tables (5), (6), and (7), the proposed hybrid harmony search method performs better than the other two in terms of the objective function value.

Since the expensive imported paper-roles along with the capital and maintenance costs of the required facilities to transform them into Tetra packs, play important roles in the total costs, the lead-time and the pre-payment time are two of the factors that affect the value of the main decision variable (the cycle time). A further investigation on the effects of the percent changes on the lead-time and the pre-payment time, while the other factors are kept unchanged, on the cycle time and the objective function is summarized in Table (8). The results in Table (8) show the following:

- The cycle-time is sensitive to the pre-payment time and increases or decreases with an increase or decrease in the pre-payment time, respectively.
- On the one hand, the objective function value is sensitive to increases in the pre-payment time and is very sensitive to increases in the lead-time. In other words, an increase in the lead-time causes a drastic increase in the total cost and an increase in the pre-payment time causes the cost decrease moderately. On the
other hand, the objective function is sensitive to both decreases in the pre-payment and lead times. In other words, a decrease in the pre-payment time causes the total cost increase moderately and a decrease in the lead-time causes the objective function to decrease moderately.

7. Conclusion and Recommendations for Future Research
In this paper, a joint replenishment multi-product multi-constraint inventory control model to purchase high price raw materials was investigated. A mathematical model of the problem, in which incremental discount policy was used and transportation, clearance, fixed order, holding and shortage costs were considered, was developed and shown to be mixed integer-nonlinear programming. Then, a hybrid meta-heuristic algorithm (harmony search), rough and fuzzy simulation has been proposed to solve the fuzzy integer non-linear problem. A case study along with sensitivity analysis was given at the end to demonstrate the applicability of the proposed methodology and to provide some managerial insights.

The most important aspect of the proposed model of this research is to take advantage of fuzzy and rough variables to model uncertain demands when the purchaser is unable to determine its stochastic nature. This is usually the case in practice, where the purchaser usually has no prior knowledge of the probability distribution of the demand. Moreover, in many real-world inventory problems, some realistic situations such as joint replenishment, multiple constraints, discount policy, etc., exist for which the model of this research can help purchasers to find their optimal inventory policy.

Some recommendations for future works include inventory shortages and uncertain lead-times.

8. Acknowledgment
The authors are thankful for constructive comments of the reviewers that improved the presentation of the paper.
Figure (2): Flow-chart of the Proposed HS Algorithm

Start

- Initialize the parameters

- Generate a vector with specific boundaries for initialization

- Are constrains satisfied?

- Are the total memory initialized?

- Yes

- Rand ≥ HMCR

- HM selection

- Rand ≤ PAR

- Yes

- Pitch adjustment

- No

- Random selection

- Are constrains satisfied?

- Yes

- Update HM

- No

- Is stopping criterion reached?

- Yes

- Stop

- No
Table (1): General Data

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Table (2): The Parameters of the HS Algorithm

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Table (4): The Best Combination of the Hybrid HS Parameters

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Table (5): The Best Results of the Hybrid Harmony Search Method

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Table (6): The Best Results of the Particle Swarm Optimization Method

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Figure (3): The Convergence Graph
Table (8): The Results of a Sensitivity Analysis

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9. References


