A NEW STC STRUCTURE TO ACHIEVE GENERALIZED OPTIMAL DIVERSITY WITH A REDUCED DESIGN COMPLEXITY

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Abstract - We propose a new space-time code (STC) structure that can achieve generalized optimal diversity (GOD) with a reduced design complexity, and also provide an increased coding gain. As a class of GOD STC structure, the new structure achieves the full diversity and an optimized coding gain with minimum delay irrespective of the number of transmit antennas (Nt) and the rate (R), if the rate is less than or equal to the rank of the channel matrix. In the new structure, only R transmit antennas at a time slot are engaged in signal transmission, and a different set of R transmit antennas at a different time slot is used. Nt data symbols are combined together using complex weights for each antenna at a time slot. The same set of data symbols is repeated at all the remaining time slots in a code block, but is transmitted through different transmit antennas using different sets of combining weights at different time slots. In particular, the above structure eliminates cross-code product terms in the determinant of codeword difference matrices when R < Nt, thus increasing the coding gain. In addition, we propose a simplified code structure to facilitate the code design, in which a higher-rate GOD STC can be obtained by coupling R rate-one codes using different rotations. We apply two design constraints such as the power and orthogonality constraints, and provide some guidelines to reduce further the design complexity. In conclusion, the proposed structure provides a higher coding gain as compared with the original GOD structure [1] and the existing linear dispersion STCs (LD-STCs), when R < Nt and can reduce significantly the design complexity when the rate becomes higher and/or the number of transmit antennas gets larger.

Keywords - Space-time codes, linear dispersion codes, generalized optimal diversity, minimum delay

I. INTRODUCTION

Recently, several linear dispersion STCs [1]-[7] that achieve the optimal diversity-multiplexing tradeoff [8] in a multiple-input multiple-output (MIMO) context have been proposed. Such codes are constructed through linear combination of dispersion matrices. However, some of LD-STCs such as the tilted-QAM code [3] and the Golden code [4] are designed only for two transmit antennas and rate 2. Sethuraman et al. [5] and Damen et al. [6] have proposed their own generalized STC structures to construct full-diversity and full-rate (FDFR) codes with minimum delay by invoking algebraic number theories. Those structures combine data symbols using only real weights or equal gain complex weights, so that the channel resource could not be utilized fully. Heath et al. [7] have used a fully generalized STC structure that combines together all the data symbols transmitted in a code block for each transmit antenna at a time slot. The structure also uses unconstrained complex combining weights. However, it requires an unaffordable design complexity to obtain an optimized set of complex weights, even in the case of two transmit antennas. Therefore, a simplified code structure that can achieve GOD with a feasible design complexity should be developed.

In our previous work [1], we have presented a GOD STC structure. When R = Nt, it can provide the capacity-lossless design. When R < Nt, however, the original GOD structure provides only a limited coding gain since it generates a lot of cross-code product terms in the determinant of codeword difference matrices. These cross-code product terms might decrease greatly the minimum product distance, and in turn reduce the coding gain. In addition, the original GOD structure still has an unaffordable design complexity when the number of complex weights becomes larger.

In this paper, we propose a new GOD STC structure which has a lower design complexity, and also can increase the coding gain. It uses only R of Nt transmit antennas for signal transmission at a time slot, and at a different time slot, a different set of R transmit antennas is used. The structure combines together Nt data symbols using complex weights for each transmit antenna at a time slot. The same set of symbols is repeated at all the remaining time slots in a code block, but is transmitted through different transmit antennas using different sets of combining weights at different time slots. In particular, the proposed structure eliminates cross-code product terms in the determinant of codeword difference matrices if R < Nt. When R = Nt, however, the new STCs are same as their original GOD counterparts [1]. In addition, we propose a simplified code structure, in which a higher-rate code can be designed by only coupling rate-one codes using rotational parameters. To reduce further the design complexity, we apply two design constraints such as the power and orthogonality constraints, and provide some guidelines. We use the well-known rank and determinant criteria [9] to obtain an optimized sample code. We give some design examples for two and three transmit antennas. Computer simulations are performed to demonstrate the performance of the proposed code structure. As proper performance measures, we use the minimum product distance of codeword difference matrices and the bit error rate (BER).

II. SYSTEM AND CHANNEL MODELS

Consider an MIMO system with Nt transmit and Nr receive antennas, in which respective channel coefficients are independent identically-distributed (i.i.d.) zero-mean complex Gaussian random variables with unit variance. We as-
sume perfect channel state information only at the receiver. Then, a received signal matrix \( \mathbf{Y} \in \mathbb{C}^{N_t \times N_r} \) corresponding to an input code block over \( N_t \) time slots can be expressed as

\[
\mathbf{Y} = \mathbf{H} \mathbf{X} + \mathbf{N},
\]

where \( \mathbf{H} \in \mathbb{C}^{N_r \times N_t} \) denotes a channel matrix with \( \mathbb{C} \) denoting the field of complex numbers; \( \mathbf{N} \in \mathbb{C}^{N_r \times N_t} \) denotes a noise matrix whose elements have i.i.d. zero-mean complex white Gaussian distribution, \( \mathbf{N}(0, I_{N_r}) \); and \( \mathbf{X} \in \mathbb{C}^{N_r \times N_t} \) is a codeword matrix. We can rewrite (1) in a vectorized form by stacking all the columns of \( \mathbf{Y} \), that is, \( \mathbf{y} = \text{vec}(\mathbf{Y}) \), (See [1]-[7] for details.) as

\[
y = \mathbf{H}_{\text{stack}} \Phi \mathbf{s} + \mathbf{n},
\]

where \( \mathbf{H}_{\text{stack}} = \text{diag}(\mathbf{H}, \mathbf{H}, \ldots, \mathbf{H}) \) is a stacked channel matrix; \( \mathbf{s} \in \mathbb{C}^{N_r \times 1} \) and \( \mathbf{n} \in \mathbb{C}^{N_r \times N_t} \) denote a stacked data symbol vector and a stacked noise vector, respectively; and \( \Phi \in \mathbb{C}^{N_t \times N_t \times N_r} \) denotes a dispersion matrix. In addition, \( \mathbf{x} \in \mathbb{C}^{N_r \times 1} \) denotes a stacked codeword vector generated by combining \( N_r \) data symbols using the dispersion matrix as

\[
\mathbf{x} = \left( x_1^1 \mid x_2^1 \mid \cdots \mid x_{N_r}^1 \right) = \left( w_{1,1}^1 \mid w_{1,2}^1 \mid \cdots \mid w_{N_r,1}^1 \right) \Theta \mathbf{s},
\]

where \( w_{i,j}^1 \) and \( s_1 \), respectively, denote a complex weight for the \( j \)-th data symbol transmitted through the \( i \)-th transmit antenna at the \( t \)-th time slot and the \( k \)-th data symbol in a code block; and \( x_t^k \) denotes a signal transmitted through the \( i \)-th transmit antenna at the \( t \)-th time slot.

III. ORIGINAL GOD CODE STRUCTURE [1]

The original GOD code structure is characterized as

\[
\mathbf{x}_{\text{GOD}} = \left( \begin{array}{c}
\sum_{j=0}^{N-1} w_{1,j}^1 s_j \\
\sum_{j=0}^{N-1} w_{2,j}^2 s_j \\
\vdots \\
\sum_{j=0}^{N-1} w_{N_r,j}^{N_r} s_j \\
\end{array} \right),
\]

where row and column indices indicate transmit antennas and time slots, respectively. Then, GOD STCs are obtained by finding a set of the complex combining weights \( \{w_{i,j}^1\} \) in (4), so that an optimized coding gain as well as the full diversity at the given rate \( R \) could be achieved. As you can see in (4), it has \( N_r! \) number of product terms in the determinant of codeword difference matrices even in rate-one codes, thus resulting in huge cross-code product terms. Here, we define the cross-code product as the product of two or more linear data polynomials each consisting of different data symbols. Such cross-code product terms keep the coding gain from being further increased. In addition, \( N_r^2 \) complex weights should be simultaneously optimized, thus the design complexity being unaffordable even using the above-mentioned two design constraints as \( R \) becomes higher and/or \( N_t \) gets larger.

IV. PROPOSED NEW GOD CODE STRUCTURE

A. Code Structure

The new GOD code structure is characterized by using the following \( N_t \times N_r \) codeword matrix \( \mathbf{X}_{N_t \times N_r} \) as

\[
\mathbf{x}_{\text{new}} = \left( \begin{array}{c}
\sum_{j=0}^{N-1} w_{1,j}^1 s_j \\
\sum_{j=0}^{N-1} w_{2,j}^2 s_j \\
\vdots \\
\sum_{j=0}^{N-1} w_{N_r,j}^{N_r} s_j \\
\end{array} \right),
\]

where \( \Theta^m = e^{j m \pi / 6} \), \( m = 1, \ldots, (R-1) \) denote the set of rotational parameters to couple rate-one codes. From (5), we can note that signals are transmitted through only \( R \) transmit antennas at a time slot in a code block. Hence, the proposed structure has only \( R \) product terms in the determinant of codeword difference matrices when \( R < N_t \), thus no cross-code product terms. Such elimination of cross-code product terms might increase the coding gain. Now, STC design can be done by finding only \( N_r^2 \) complex combining weights \( \{w_{i,j}^1\} \) and the \( (R-1) \) rotational parameters.

B. Existence of Cross-Code Products

We examine the existence of the cross-code product terms in the determinant of codeword difference matrices. We consider the original GOD code structure and the new GOD code structure for 3x2 codes as examples.

First, we consider the 3x2 original GOD code \( \mathbf{x}_{\text{GOD}}^{3 \times 2} \) as

\[
\mathbf{x}_{\text{GOD}}^{3 \times 2} = \left( \begin{array}{c}
w_{1,0}^3 s_0 + w_{1,1}^3 s_1 \\
w_{2,0}^3 s_0 + w_{2,1}^3 s_1 \\
w_{3,0}^3 s_0 + w_{3,1}^3 s_1 \\
w_{1,0}^3 s_1 + w_{1,1}^3 s_0 \\
w_{2,0}^3 s_1 + w_{2,1}^3 s_0 \\
w_{3,0}^3 s_1 + w_{3,1}^3 s_0 \\
w_{1,1}^3 s_0 + w_{2,1}^3 s_0 \\
w_{2,1}^3 s_0 + w_{3,1}^3 s_0 \\
w_{3,1}^3 s_0 + w_{3,1}^3 s_0 \\
\end{array} \right),
\]

\[
\mathbf{x}_{\text{GOD}}^{3 \times 2} \text{ code has three cross-code product terms in the determinant of the codeword matrix as follows:}
\]

\[
\begin{align*}
&(w_{1,0}^3 s_0 + w_{1,1}^3 s_1) \cdot (w_{2,0}^3 s_4 + w_{2,1}^3 s_5) \cdot (w_{3,0}^3 s_3 + w_{3,1}^3 s_4), \\
&(w_{2,0}^3 s_0 + w_{2,1}^3 s_1) \cdot (w_{3,0}^3 s_4 + w_{3,1}^3 s_5) \cdot (w_{1,0}^3 s_3 + w_{1,1}^3 s_4), \\
&(w_{3,0}^3 s_0 + w_{3,1}^3 s_1) \cdot (w_{1,0}^3 s_4 + w_{1,1}^3 s_5) \cdot (w_{2,0}^3 s_3 + w_{2,1}^3 s_4). 
\end{align*}
\]
Therefore, the determinant of codeword difference matrices also has corresponding cross-code product terms. Next, we consider the 3x2 new GOD code $X_{3x2}$ as

$$X_{3x2} = \begin{bmatrix}
    w_{1,0} + w_{1,1} + w_{1,2} & u_{1,0} + u_{1,1} + u_{1,2} \\
    w_{2,0} + w_{2,1} + w_{2,2} & u_{2,0} + u_{2,1} + u_{2,2} \\
    0 & 0
\end{bmatrix}. \quad (8)$$

In (8), note that $X_{3x2}$ has no cross-code product terms in the determinant, thus having the following determinant as

$$\det(X_{3x2}) = \prod_{k=1}^{N_{t}} \sum_{h=0}^{1} w_{k,h}^t s_h - e^{j \theta R}. \quad (9)$$

V. DESIGN CONSTRAINTS AND CRITERIA

In this section, we present two design constraints to reduce the design complexity. We also present design criteria to obtain an optimized code under a given rate and antenna configuration.

A. Power Constraints [1]

We have two types of power constraints such as (i) an equal average transmit power level during a code block interval is allocated to all the transmit antennas, and (ii) all the average transmit power levels allocated for respective data symbols in a code block are equal. These can be expressed as

$$|w_{1,0}^t + \ldots + w_{N_t-1}^t|^2 = \ldots = |w_{N_t,0}^t + \ldots + w_{N_t,N_t-1}^t|^2 = P_t / R, \quad (10)$$

$$|w_{1,0}^t + \ldots + w_{N_t,0}^t|^2 = \ldots = |w_{N_t,N_t-1}^t + \ldots + w_{N_t,N_t-1}^t|^2 = P_t / R. \quad (11)$$

Here, $P_t$ denotes an average total transmit power over all transmit antennas at each time slot.

B. Orthogonality Constraints [1]

For design simplicity, we set that the columns of the dispersion matrix are mutually orthogonal. From (3) and (5), we obtain a reduced set of the constraint equations as follows:

$$w_{1,0}^t w_{1,0}^* + \ldots + w_{N_t,0}^t w_{N_t,0}^* = 0,$$

$$w_{1,0}^t w_{1,1}^* + \ldots + w_{N_t,0}^t w_{N_t,1}^* = 0,$$

$$\vdots$$

$$w_{1,0}^t w_{1,N_t-1}^* + \ldots + w_{N_t,0}^t w_{N_t,N_t-1}^* = 0,$$

$$w_{1,1}^t w_{1,1}^* + \ldots + w_{N_t,1}^t w_{N_t,1}^* = 0,$$

$$\vdots$$

$$w_{1,N_t-1}^t w_{1,N_t-1}^* + \ldots + w_{N_t,N_t-1}^t w_{N_t,N_t-1}^* = 0. \quad (12)$$

C. Determinant and Rank Criteria [9]

We use the determinant criterion as a proper metric to measure the coding gain as follows:

$$\delta^2 = \min_{X \neq X'} \left| \det(X - X') \right|^2. \quad (13)$$

From (13), we see that the full rank criterion can also be guaranteed from the determinant criterion by choosing a set of combining weights, which satisfies $\delta^2 \neq 0$. Hence, an optimized coding gain and the full diversity can be achieved simultaneously by choosing a set of combining weights, which maximizes $\delta^2$.

VI. DESIGN EXAMPLES

A. STC with Rate 1 for Two Transmit Antennas

The proposed STC for $N_t = 2$ and $R = 1$, $X_{2x4}$ can be expressed as

$$X_{2x4} = \begin{bmatrix}
    w_{1,0}^t s_0 + w_{1,1}^t s_1 & 0 \\
    0 & w_{2,0}^t s_0 + w_{2,1}^t s_1
\end{bmatrix}. \quad (14)$$

From (14), the dispersion matrix can be written as

$$\Phi_{2x4} = \begin{bmatrix}
    w_{1,0}^t & 0 & 0 & w_{2,0}^t \end{bmatrix}^t, \quad (15)$$

where ($\bullet^T$) denotes transpose of the matrix.

From the power constraints, we obtain

$$|w_{1,0}^t|^2 + |w_{1,1}^t|^2 = P_1, \quad |w_{2,0}^t|^2 + |w_{2,1}^t|^2 = P_1,$$

$$|w_{1,0}^t|^2 + |w_{2,0}^t|^2 = P_1, \quad |w_{1,1}^t|^2 + |w_{2,1}^t|^2 = P_1. \quad (16)$$

For simplicity, without loss of generality, we set $P_1 = 1$. From (16), we obtain the following equalities

$$|w_{1,0}^t| = |w_{2,1}^t|; \quad |w_{1,1}^t| = |w_{2,0}^t|. \quad (17)$$

To consider an unequal gain combining of data symbols, we can parameterize (16) using the equalities in (17) as follows:

$$w_{1,0}^t = e^{j\delta t_1} \sqrt{(1 + r^2)}, \quad w_{1,1}^t = r e^{j\delta t_0} w_{1,0}^t,$$

$$w_{2,0}^t = e^{j\delta t_0} w_{2,1}^t, \quad w_{2,1}^t = e^{j\delta t_0} w_{2,0}^t. \quad (18)$$

where $r$ is a positive real number. Therefore, we can rewrite the complex combining weights as

$$w_{1,0}^t = e^{j\delta t_1} \sqrt{(1 + r^2)}, \quad w_{1,1}^t = r e^{j\delta t_0} w_{1,0}^t,$$

$$w_{2,0}^t = e^{j\delta t_0} w_{2,1}^t, \quad w_{2,1}^t = e^{j\delta t_0} w_{2,0}^t. \quad (19)$$

From the orthogonality constraints, we obtain

$$w_{1,0}^t w_{1,1}^* + w_{2,0}^t w_{2,1}^* = 0. \quad (20)$$

From (19) and (20), we derive the phase relation as follows:

$$\theta_1 + \theta_2 = \theta_3 + \pi. \quad (21)$$

For simplicity, without loss of generality, we can set $\theta_1 = 0$. Then, 5 independent parameters in (19) are reduced to 3 independent parameters as $\delta t_0, \delta t_1$ and $r$.

By maximizing $\delta^2$ in (13) as a function of $(r, \delta t_0, \delta t_1)$, we can obtain a set of optimum $\delta t_0, \delta t_1$ and $r$ as follows:

$$r = \frac{1}{2}, \delta t_0 = \frac{\pi}{4}, \delta t_1 = \frac{3\pi}{4}. \quad (22)$$

Therefore, an optimized set of complex weights for 2x1 code $X_{2x4}$ can be obtained if the entries in (19) satisfy (21) and (22).
B. STC with Rate 2 for Two Transmit Antennas

The proposed STC for $N_t=2$ and $R=2$, $X_{2x2}$ can be easily obtained by coupling two $X_{2x2}$ codes in (14) with rotation as

$$X_{2x2} = \left[ \begin{array}{c}
  w_{1,0}x_0 + w_{1,1}x_1 \\
  w_{1,0}x_0 + w_{1,1}x_1 \\
  w_{2,0}x_0 + w_{2,1}x_1 \\
  w_{2,0}x_0 + w_{2,1}x_1 \\
\end{array} \right]
\left[ \begin{array}{c}
  e^{j\theta_1} \\
  e^{j\theta_1} \\
  e^{j\theta_2} \\
  e^{j\theta_2} \\
\end{array} \right],$$

(23)

where $\theta_1$ denotes a rotation angle for coupling two rate-one codes.

From (14) and (23), we see that $X_{2x2}$ can be easily designed by using the intermediate results in the subsection IV.A, if only one additional parameter $\theta_1$ is incorporated. In this case, by maximizing $\delta^2$ in (13) as a function of $(r, \theta_1, \theta_2, \theta_3)$ instead of $(r, \theta_1, \theta_2)$, we obtain a set of optimum $\theta_2$, $\theta_3$, and $r$ as follows:

$$r = \frac{1 + 1.5}{2}, \quad \theta_1 = 0, \quad \theta_2 = 3\pi/2, \quad \theta_1 = \pi/4.$$  

(24)

Therefore, an optimized set of complex weights for 2x2 code $X_{2x2}$ can be obtained if the entries in (19) and the rotation angle $\theta_1$ satisfy (21) and (24).

C. STC with Rate 1 for Three Transmit Antennas

The proposed STC for $N_t=3$ and $R=1$, $X_{3x1}$ can be expressed as

$$X_{3x1} = \left[ \begin{array}{c}
  w_{1,0}x_0 + w_{1,1}x_1 + w_{1,2}x_2 \\
  0 \\
  0 \\
  0 \\
\end{array} \right].$$

(25)

From (25), the dispersion matrix can be written as

$$\Phi_{3x1} = \left[ \begin{array}{cc}
  w_{1,0} & 0 \\
  0 & w_{1,0} \\
  0 & 0 \\
  0 & 0 \\
  w_{2,0} & 0 \\
  0 & w_{2,0} \\
  0 & 0 \\
  0 & 0 \\
  w_{3,0} & 0 \\
  0 & w_{3,0} \\
  0 & 0 \\
  0 & 0 \\
\end{array} \right].$$

(26)

Invoking the power and orthogonal constraints, the combining weights in (25) (26) should satisfy the following equalities. From the power constraints, we obtain

$$w_{1,0}^2 + w_{1,1}^2 + w_{1,2}^2 = P_1, \quad w_{2,0}^2 + w_{2,1}^2 + w_{2,2}^2 = P_2, \quad w_{3,0}^2 + w_{3,1}^2 + w_{3,2}^2 = P_3.$$  

(27)

From the orthogonality constraints, we obtain

$$w_{1,0}w_{1,1} + w_{1,0}w_{1,2} + w_{1,0}w_{1,3} = 0, \quad w_{2,0}w_{2,1} + w_{2,0}w_{2,2} + w_{2,0}w_{2,3} = 0, \quad w_{3,0}w_{3,1} + w_{3,0}w_{3,2} + w_{3,0}w_{3,3} = 0.$$  

(28)

To consider an unequal gain combining of data symbols, we can parameterize (27) as follows

$$w_{1,0} = r_1 e^{j\theta_1}/r_n, \quad w_{1,1} = r_2 e^{j\theta_1}/r_n, \quad w_{1,2} = r_3 e^{j\theta_1}/r_n, \quad w_{2,0} = r_1 e^{j\theta_2}/r_n, \quad w_{2,1} = r_2 e^{j\theta_2}/r_n, \quad w_{2,2} = r_3 e^{j\theta_2}/r_n, \quad w_{3,0} = r_1 e^{j\theta_3}/r_n, \quad w_{3,1} = r_2 e^{j\theta_3}/r_n, \quad w_{3,2} = r_3 e^{j\theta_3}/r_n.$$  

(29)

where $\{r_1, r_2, r_3\}, i=1,2,3$ are positive real numbers and $r_n$ denotes a normalization factor. From (27), we can derive the following equations

$$r_1^2 + r_2^2 + r_3^2 = r_1^2 + r_2^2 + r_3^2 = r_1^2 + r_2^2 + r_3^2 = r_2^2.$$  

(30)

Even using the two types of design constraints, the code has still an unaffordable design complexity. Hence, we apply a new set of constraints (i.e., the symmetry constraints [1]), which can simplify further the code design. One possible set of symmetry constraints is given by

$$r_1 = r_2^*, \quad r_2 = r_3^*, \quad r_3 = r_3^*.$$  

(31)

Substituting the relations of (31) into (30), we can derive the following equalities

$$r_1 = r_2^*, \quad r_2 = r_3^*, \quad r_3 = r_3^*.$$  

(32)

Then, we can express the complex combining weights as follows:

$$w_{1,0} = r_1 e^{j\theta_1}/r_n, \quad w_{1,1} = r_1 e^{j\theta_1}/r_n, \quad w_{1,2} = r_1 e^{j\theta_1}/r_n, \quad w_{2,0} = r_2 e^{j\theta_2}/r_n, \quad w_{2,1} = r_2 e^{j\theta_2}/r_n, \quad w_{2,2} = r_2 e^{j\theta_2}/r_n, \quad w_{3,0} = r_3 e^{j\theta_3}/r_n, \quad w_{3,1} = r_3 e^{j\theta_3}/r_n, \quad w_{3,2} = r_3 e^{j\theta_3}/r_n.$$  

(33)

Substituting the relations of (31) into (30), we obtain the following equalities

$$r_1 r_2 e^{j(\theta_1-\theta_2)} + r_2 r_3 e^{j(\theta_2-\theta_3)} + r_3 r_1 e^{j(\theta_3-\theta_1)} = 0,$$

$$r_1 r_2 e^{j(\theta_1-\theta_2)} + r_2 r_3 e^{j(\theta_2-\theta_3)} + r_3 r_1 e^{j(\theta_3-\theta_1)} = 0,$$

(34)

$$r_2 r_3 e^{j(\theta_2-\theta_3)} + r_3 r_1 e^{j(\theta_3-\theta_1)} + r_1 r_2 e^{j(\theta_1-\theta_2)} = 0.$$  

(35)

We would choose a possible relation $r_1 r_2 = r_1 r_3 + r_2 r_3$ from (34). Then, invoking again the orthogonal constraints, we can derive the following phase relations as

$$\theta_1 - \theta_2 = \theta_2 - \theta_3 = \theta_3 - \theta_1 + \pi, \quad \theta_1 - \theta_3 = \theta_2 - \theta_3 = \theta_3 - \theta_1 + \pi, \quad \theta_1 - \theta_2 = \theta_1 - \theta_3 = \theta_1 - \theta_2 + \pi.$$  

(36)

We would choose a possible relation $r_1 r_2 = r_1 r_3 + r_2 r_3$ from (34). Then, invoking again the orthogonal constraints, we can derive the following phase relations as

$$\theta_1 - \theta_2 = \theta_2 - \theta_3 = \theta_3 - \theta_1 + \pi, \quad \theta_1 - \theta_3 = \theta_2 - \theta_3 = \theta_3 - \theta_1 + \pi, \quad \theta_1 - \theta_2 = \theta_1 - \theta_3 = \theta_1 - \theta_2 + \pi.$$  

(35)

Note that all the phase angles $\theta_i = 1, \ldots, 9$ can be determined from (35). Hence, an optimized set of combining weights in (25) is obtained by maximizing $\delta^2$ in (13) as a function of only $r_1$ and $r_2$ as

$$r_1 = 0.974, \quad r_2 = r_3/(r_1 + r_2), \quad r_3 = 0.381.$$  

(36)

Therefore, an optimal set of combining weights for 3x1 code $X_{3x1}$ can be obtained if the entries in (29) satisfy (35) and (36).

D. STC with Rate 2 for Three Transmit Antennas

The proposed STC for $N_t=3$ and $R=2$, $X_{3x2}$ can be expressed as
where $\theta_i$ denotes a rotational phase angle for coupling two $X_{3\times3}$ codes. Hence, we can obtain $X_{3\times3}$ by maximizing $\delta_e^2$ in (13) as a function of $(r_1, r_2, \theta_1)$. Then, we can obtain a set of optimum $r_1$, $r_2$, and $\theta_1$ as follows:

$$r_1 = 0.68, \quad r_2 = r_3/(r_1 + r_2), \quad r_3 = 0.52, \quad \theta_1 = \pi/4.$$ (38)

Therefore, an optimized set of combining weights for 3x2 code $X_{3x2}$ can be obtained if the entries in (29) and the rotation angle $\theta_1$ satisfy (35) and (38).

### E. STC with Rate 3 for Three Transmit Antennas

The proposed STC for $N_t = 3$ and $R = 3$, $X_{3x3}$ can be expressed as

$$X_{3x3} = \begin{pmatrix}
  w_{1,0,1} + w_{2,0,2} + w_{3,0,3}
  & 0 & w_{1,1,1} + w_{2,1,2} + w_{3,1,3} e^{j\theta_1} \\
  w_{1,1,1} + w_{2,1,2} + w_{3,1,3} e^{j\theta_1}
  & 0 & w_{1,2,1} + w_{2,2,2} + w_{3,2,3} e^{j\theta_2} \\
  w_{1,2,1} + w_{2,2,2} + w_{3,2,3} e^{j\theta_2}
  & 0 & w_{1,3,1} + w_{2,3,2} + w_{3,3,3} e^{j\theta_3}
\end{pmatrix},$$ (39)

where $\theta_1$ and $\theta_2$ denote rotational phase angles for coupling three $X_{3\times3}$ codes. Hence, we can obtain $X_{3\times3}$ by maximizing $\delta_e^2$ in (13) as a function of $(r_1, r_2, \theta_1, \theta_2)$. Then, we can obtain a set of optimum $r_1$, $r_2$, $\theta_1$, and $\theta_2$ as follows:

$$r_1 = 1, \quad r_2 = \frac{r_1 r_2}{(r_1 + r_2)}, \quad r_3 = 0.8, \quad \theta_1 = \frac{\pi}{36}, \quad \theta_2 = \frac{2\pi}{9}.$$ (40)

Therefore, an optimized set of complex weights for 3x3 code $X_{3x3}$ can be obtained if the entries in (29) and two rotation angles $\theta_1$ and $\theta_2$ satisfy (35) and (40).

### VII. SIMULATION RESULTS AND DISCUSSIONS

We performed computer simulations to evaluate the performance of new GOD STCs generated by the proposed code structure except for the codes for $R = N_t$. We used $10^6$ channel blocks to obtain the BER performance at each SNR value in the case of 4-QAM. The elements of a MIMO channel are independent zero-mean circularly symmetric complex Gaussian random variables with unit variance and are constant over $N_t$ symbol periods. The ML decoder is used to decode all the STCs considered. We compared the proposed codes with the existing LD-STCs including the original GOD STC.

Table 1 shows $\delta_e$ (the positive square root of $\delta_e^2$ in (13)) for the proposed new GOD code and the original GOD code in the 2x1 code case. In the case of non-zero determinant, all the codes can achieve the full diversity. Hence, only the coding gain is a meaningful performance measure. From Table 1 and Fig. 1, the proposed code and the original GOD code have the same value of $\delta_e$ and the same BER performance.

![Fig. 1. BER performance of the original GOD code and the proposed code (2x1 code).](image1)

<table>
<thead>
<tr>
<th>m-QAM</th>
<th>Proposed</th>
<th>GOD [1]</th>
</tr>
</thead>
<tbody>
<tr>
<td>4-QAM</td>
<td>0.4039</td>
<td>0.1333</td>
</tr>
</tbody>
</table>

![Fig. 2. BER performance of the existing LD-STCs and the proposed code (2x1 code).](image2)

<table>
<thead>
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<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>4-QAM</td>
<td>0.4039</td>
<td>0.1333</td>
<td>0.1459</td>
<td>0.2123</td>
</tr>
</tbody>
</table>

From Table 2 and Fig. 2, we can see that among the four codes considered, the proposed code $X_{3\times3}$ has the maximum value of $\delta_e$, that is, the greatest coding gain, and show the best BER performance. We also see that among the three codes considered, the proposed code $X_{3\times2}$ obtained through coupling of two $X_{3\times3}$ codes with rotations has again the maximum value of $\delta_e$ and the best BER performance from Table 3 and Fig. 3.
Therefore, we see that the proposed structure can provide the full diversity with a better coding gain as compared with the existing FDFR LD-STCs including the original GOD codes by eliminating cross-code product terms in the determinant of codeword difference matrices when $R < N_t$.

Table 3. Maximum values of $\delta_c$ (3x2 code)

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>4-QAM</td>
<td>0.030998</td>
<td>0.00225</td>
<td>0.01147</td>
</tr>
</tbody>
</table>

Fig. 3. BER performance of the existing LD-STCs and the proposed code (3x2 code).

VIII. CONCLUSIONS

In this paper, we proposed a simplified STC structure that can achieve GOD with a reduced design complexity. It also provides an increased coding gain. The proposed structure uses only $R$ transmit antennas for signal transmission at a time slot, and a different set of $R$ transmit antennas is selected at a different time slot in a code block. $N_t$ data symbols are combined together using complex weights for each antenna at a time slot. The same set of data symbols is repeated at all the remaining time slots in a code block, but is transmitted through different transmit antennas using different sets of combining weights at different time slots. Hence, the structure eliminate the cross-code product terms in the determinant of codeword difference matrices, thus increasing greatly the coding gain. Using the proposed simplified structure, higher-rate codes can be designed by simply coupling rate-one codes using rotation parameters, thus reducing the design complexity drastically. We applied two design constraints such as the power and orthogonality constraints, and provided some guidelines to reduce further the design complexity. Our simulation results show that the proposed codes achieve the best BER performance when $R < N_t$ as compared with the original GOD codes and the existing FDFR LD-STCs.

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