Novel Low-Complexity SLM Schemes for PAPR Reduction in OFDM Systems

Chih-Peng Li1,2, Sen-Hung Wang2, Kun-Sheng Lee1, and Chin-Liang Wang3

Abstract - The selected mapping (SLM) is a major scheme for peak-to-average power ratio (PAPR) reduction in orthogonal frequency division multiplexing (OFDM) systems. It has been shown that the complexity of the traditional SLM scheme can be substantially reduced by adopting the conversion vectors to replace the inverse fast Fourier transform (IFFT) operations [4]. Each conversion vector is obtained by taking the IFFT of the phase rotation vector. Unfortunately, the corresponding phase rotation vectors of the conversion vectors in [4] do not have equal magnitude, leading to significant degradation in bit error rate (BER) performance. This drawback can be remedied by adopting the perfect sequences as the conversion vectors. This paper presents two novel classes of perfect sequences, which are shown to be compositions of certain base vectors and their cyclic-shift versions. Then, two novel low-complexity SLM schemes are proposed by utilizing the special structures of the perfect sequences. The BER performances of both the proposed schemes are exactly the same as the traditional SLM scheme.

Index Terms - Orthogonal frequency division multiplexing (OFDM), peak-to-average power ratio (PAPR), selected mapping (SLM), perfect sequence.

I. INTRODUCTION

Orthogonal frequency division multiplexing (OFDM) is a promising technique for high data rate transmission because of its high spectral efficiency and immunity to interferences caused by the multi-path channels. However, one major drawback of OFDM is the high peak-to-average power ratio (PAPR) of the transmitted signal. Due to the large number of subcarriers, the amplitude of the transmitted signal has a large dynamic range, leading to inter-modulation distortion and out-of-band radiation when it is passed through the power amplifier. There are a number of methods proposed for PAPR reduction in OFDM systems, e.g. clipping [1], coding [2], selected mapping (SLM) [3-4], and partial transmit sequences (PTS) [5].

As opposed to the clipping method, the SLM scheme does not affect the signal spectrum and bit error rate (BER), but require a bank of IFFTs to produce candidate signals, resulting in dramatic increase in computational complexity. To overcome the drawback of traditional SLM schemes, two low-complexity SLM schemes have been proposed by Wang and Ouyang [4], where conversion vectors are adopted to replace the IFFT blocks and the candidate signals are generated by performing circular convolution of the IFFT of the original data sequence with the conversion vectors. Each conversion vector can be obtained by taking the IFFT of the phase rotation vector. Unfortunately, for most of the conversion vectors proposed in [4], the elements of the phase rotation vectors have different magnitudes. Therefore, signals of different sub-carriers may have different gains and the signal power of some sub-carriers may be attenuated, leading to serious degradation in BER performance.

This paper shows that the above-mentioned drawback can be remedied by using perfect sequences as conversion vectors. In this paper, two novel classes of perfect sequences are introduced and demonstrated to be compositions of certain base vectors and their cyclic-shift versions. Based on these findings, two low-complexity SLM schemes are proposed, where one of them require only one IFFT operation. Both methods have exactly the same BER performance as the traditional SLM scheme.

The rest of this paper is organized as follows. Section II describes the system models. Section III presents the structures of two novel classes of perfect sequences. The proposed SLM schemes are demonstrated in Section IV. The computational complexity is analyzed in Section V. Section VI evaluates the PAPR performance. Some concluding remarks are provided in Section VII.

II. SYSTEM MODEL

As proposed by Wang and Ouyang [4], the conversion vectors can be adopted to replace the IFFT operations in traditional SLM schemes. The proposed architecture is shown in Fig. 1, where N is the number of sub-carriers and M is the number of candidate signals. In this paper, this scheme is termed as Wang’s Scheme I. In order to
have a better approximation of the true PAPR in discrete-time case, oversampling the candidate signals is required. An oversampling rate of $L$ for an OFDM system with $N$ sub-carriers can be achieved by inserting $(L-1)\cdot N$ zeros in the middle of the modulated symbol vector to form an $1 \times LN$ data vector $X$ as shown in (1),

$$X = \begin{bmatrix} X[0], \ldots, X\left[\frac{N}{2} - 1\right], 0, \ldots, 0, X\left[\frac{N}{2}\right], \ldots, X[N-1] \end{bmatrix}$$  \tag{1}

where $X[i]$ is the modulated symbol of the $i$th subcarrier. After this, an $LN$-point IFFT is performed to produce the oversampled time-domain signal vector $x$, where the $n$th element of $x$ is

$$x[n] = \frac{1}{\sqrt{LN}} \sum_{k=0}^{LN-1} X[k] \cdot \exp\left(\frac{j2\pi nk}{LN}\right), \quad 0 \leq n \leq LN - 1.  \tag{2}$$

It is shown in [6] that $L=4$ is sufficient to capture the peak information of the continue-time signals $x(t)$. Then, each candidate signal is generated by performing an $LN$-point circular convolution of the time-domain signal vector $x$ with the conversion vector $G_m$, where $G_m$ is a $1 \times LN$ vector, $m = 1, 2, \ldots, M - 1$. Therefore, the $m$th candidate signal vector $y_m$ can be written as:

$$y_m = x \otimes_{LN} G_m,  \tag{3}$$

where $\otimes_{LN}$ denotes the $LN$-point circular convolution.

Finally, the candidate signal vector which has the lowest PAPR is selected for transmission.

III. STRUCTURES OF THE ADOPTED PERFECT SEQUENCES/CONVERSION VECTORS

For most of the conversion vectors proposed by Wang and Ouyang, the elements of the corresponding phase rotation vectors do not have the same magnitude, leading to significant degradation in BER performance. It is worthy of note that, for all the elements of the phase rotation vectors to have the same magnitude, it is easy to show that the periodic auto-correlation function (PACF) of the conversion vectors must be a delta function, i.e.

$$\sum_{m=0}^{N-1} g[m] \cdot g^*[(m-n)_N] = E \cdot \delta[n], \quad 0 \leq n \leq N-1, \tag{4}$$

where $g[m]$ is the $m$th element of the conversion vector, $*$ is the complex conjugate operation, $(\cdot)_N$ denotes the modulo $N$ operation, $E$ is a constant, and $\delta[n]$ is the delta function.

The sequences that satisfy (4) are well-known as perfect sequences [7-8]. Extensive computer searches were conducted for finding the perfect sequences that are appropriate for our applications. As demonstrated in the following, the adopted perfect sequences are compositions of certain base vectors. In addition, our proposed architectures take the $LN$-point circular convolution of the time domain signal vector $x$ with the base vectors. In order to keep the conversion process to have a low computational load, the following constraints are imposed in searching for the suitable perfect sequences:

1. The number of nonzero elements in the base vectors is limited to 4.

2. The nonzero elements in the base vectors must belong to the set $\{\pm 1, \pm j, \pm 1 \pm j\}$.  

In the following, two classes of perfect sequences of length $LN$ are presented, where $LN$ is any positive integer multiple of 2 for Class I sequences and any positive integer multiple of 4 for Class II sequences.  

1) Class I perfect sequence/conversion vectors

The Class I conversion vector $G_a$ is a $1 \times LN$ vector expressed as:

$$G_a = [g_a[0], g_a[1], \ldots, g_a[LN-1]].  \tag{5}$$

In particular, $G_a$ is a composite of two $1 \times LN$ vectors, $G_{a1}$ and $G_{a2}^0$, which have the following forms:

$$G_{a1} = [ -1, 0, \ldots, 0, 1, 0, \ldots, 0],  \tag{6}$$

and

$$G_{a2}^0 = [1, 0, \ldots, 0, 1, 0, \ldots, 0],  \tag{7}$$

where $LN/2 \geq 1$. The Class I conversion vectors are given by:

$$G_a(c, w, m) = c \cdot \left\{ G_{a1} + w \cdot G_{a2}^m \right\},  \tag{8}$$

where $c$ is any complex constant, $w \in \{\pm 1, \pm j\}$, and $G_{a2}^m$ denotes the $m$th right cyclic shift of $G_{a2}^0$, $m = 1, 2, \ldots, LN/2 - 1$. Therefore, the $n$th element of $G_{a2}^m$ is given by:

$$g_{a2}^m[n] = g_{a2}^0[(n-m)_LN].  \tag{9}$$
For given $c$ and $w$, there are a total of $LN/2$ conversion vectors. Since the constant $c$ has no effect on PAPR, $c = 1$ is adopted to save the computational complexity. It is worthy of note that some of the conversion vectors presented in [4] have equal-magnitude elements of the phase rotation vector. These conversion vectors are special cases of our Class I conversion vectors.

2) Class II perfect sequence/conversion vectors

The Class II conversion vector, $G_b$, is a composite of two $1 \times LN$ vectors, $G_{b_1}$ and $G_{b_2}^0$, which have the following forms:

$$
G_{b_1} = \begin{bmatrix} g_{b_1}[0], 0, \ldots, 0, g_{b_1}[LN/4], 0, \ldots, 0 \end{bmatrix},
$$

$$
G_{b_2}^0 = \begin{bmatrix} g_{b_2}^0[0], 0, \ldots, 0, g_{b_2}^0[LN/4], 0, \ldots, 0 \end{bmatrix},
$$

where $LN/4 \geq 1$, $g_{b_1}[0] \in \{1,-1\}$ and $g_{b_1}[LN/4]$, $g_{b_2}^0[LN/4] \in \{1+j,1-j,1-j-1,1+j-1\}$. It is worthy of note that $g_{b_1}[LN/4] = v \cdot g_{b_2}^0[LN/4]$, where $v \in \{1,-1,j,-j\}$. In addition, $g_{b_2}^0[0]$, $g_{b_1}[LN/2]$, and $g_{b_2}^0[LN/2]$ are given by:

$$
g_{b_2}^0[0] = -g_{b_1}[0]/v,
$$

$$
g_{b_1}[LN/2] = \frac{j \cdot g_{b_1}[0]}{\text{Re}\{g_{b_1}[LN/4]\} \cdot \text{Im}\{g_{b_1}[LN/4]\}},
$$

$$
g_{b_2}^0[LN/2] = \frac{-j \cdot g_{b_1}[0]}{v \cdot \text{Re}\{g_{b_2}^0[LN/4]\} \cdot \text{Im}\{g_{b_2}^0[LN/4]\}},
$$

where $\text{Re}\{g\}$ and $\text{Im}\{g\}$ are the real part and the image part of $g$, respectively. The Class II conversion vectors are given by:

$$
G_{b_1}(c,m) = c \cdot \left(G_{b_1} + G_{b_2}^m\right),
$$

where $c$ is any complex constant, and $G_{b_2}^m$ denotes the $m$th right cyclic shift of $G_{b_2}^0$, $m = 0, 1, \ldots, LN/2-1$.

Therefore, the $n$th element of $G_{b_2}^m$ is given by:

$$
ge_{b_2}^m[n] = g_{b_2}^m[(n-m)LN].
$$

For given $g_{b_1}[0]$, $g_{b_1}[LN/4]$, $g_{b_2}^0[LN/4]$ and $c$, there are a total of $LN/2$ conversion vectors. Again, $c=1$ is adopted to save the computational complexity.

IV. THE PROPOSED LOW-COMPLEXITY SLM SCHEMES

Base on the two classes of conversion vectors and their structures, two low-complexity SLM schemes are proposed in this section.

1) Proposed Scheme I

Because the Class I conversion vectors, $G_a$, are composites of two different base vectors and the circular convolution is a linear operation, the candidate signals generated by the Class I conversion vectors with $c=1$ can be written as:

$$
y = x \otimes_{LN} G_a = x \otimes_{LN} G_{a_1} + w \cdot x \otimes_{LN} G_{a_2}^m
$$

$$
\equiv A_{a_1} + w \cdot A_{a_2}^m,
$$

where $A_{a_1} \equiv x \otimes_{LN} G_{a_1}$ and $A_{a_2}^m \equiv x \otimes_{LN} G_{a_2}^m$. The $n$th element of $A_{a_2}^m$ can be written as:

$$
A_{a_2}^m[n] = \sum_{q=0}^{LN-1} g_{a_2}^m(q) \cdot x[(n-q)LN] = \sum_{q=0}^{LN-1} g_{a_2}^m(q) \cdot x[((n-m)-q)LN]
$$

$$
\equiv A_{a_2}^m[(n-m)LN].
$$

Therefore, $A_{a_2}^m$ is the $m$th right cyclic shift of $A_{a_2}^0$. A similar result can also be obtained for the Class II conversion vectors. Based on the above discussions, our first proposed SLM scheme (Proposed Scheme I) is depicted in Fig. 2, where candidate signals, $y(m_s)$ and $y(m_t)$, are generated using various combinations of...
cyclic shifts of $m_a$ and $m_b$. In addition, since our simulation results, which are omitted in this paper because of limited space, demonstrate that the usage of different $w$ does not improve the PAPR performance, $w$ is set to one in this paper.

2) Proposed Scheme II

To further improve the PAPR performance, Wang and Ouyang propose a modified scheme (Wang’s Scheme II) [4], which takes two parallel IFFTs and has a random phase rotation vector before the second IFFT operation. This modified scheme can also be adopted in our proposed architecture, as demonstrated in Fig. 3, and is termed as the Proposed Scheme II.

V. ANALYSIS OF COMPUTATIONAL COMPLEXITY

To evaluate the computational complexities of the Proposed Scheme I and II, let’s first note that the number of complex additions involved in performing the circular convolution of $x$ with $G_{a1}$, $G_{a2}$, $G_{b1}$, and $G_{b2}$ are $LN$, $LN$, $3LN$, and $3LN$, respectively. In addition, it takes $LN$ complex additions to combine the $A_{a1}$ or $A_{b1}$ with the cyclic shift versions of $A'_{a2}$ or $A'_{b2}$. To obtain better PAPR performance, both class I and class II conversion vectors are adopted in the Proposed Scheme I and II. Hence, the total number of complex additions for the Proposed Scheme I to generate $M$ candidate signals is $8LN + (M - 1) LN = (M + 7) LN$. Similarly, the total number of complex additions for the Proposed Scheme II to generate $M$ candidate signals is $(M + 14) LN$.

The computational complexities of various schemes are listed in Table I and depicted in Fig. 4 for $LN=256$, where an IFFT operation takes $(LN/2)\log_2(LN)$ complex multiplications and $(LN)\log_2(LN)$ complex additions. It can be seen that the number of complex additions of our Proposed Scheme I is smaller than that of Wang’s scheme I for $M > 5$. Moreover, the number of complex additions of our Proposed Scheme II is smaller than that of Wang’s scheme II for $M > 10$.

VI. SIMULATION RESULT

Simulation experiments were conducted to evaluate the PAPR and BER performances of our proposed schemes. We assume that the OFDM system has $N=64$ sub-carriers and data are 16-QAM modulated. To approximate the true PAPR, the OFDM signal is oversampled by a factor of $L=4$. The random phase rotation vectors adopted in traditional SLM scheme are randomly generated from the set $\{\pm 1, \pm j\}$. It is worthy of note that the traditional SLM scheme has the best PAPR performance since its phase rotation vectors are truly random. For the rest of the schemes investigated in this paper, since there are constraints on the selection of the conversion vectors, the randomness of the corresponding phase rotation vectors are limited, leading to certain degradation in PAPR performance.
Fig. 5. PAPR performances of various schemes. (Scheme I)

Fig. 6. PAPR performances of various schemes. (Scheme II)

VII. CONCLUSION

This paper presents two novel classes of perfect sequences, which are demonstrated to be compositions of certain base vectors and their cyclic-shift versions. Based on these findings, two low-complexity SLM schemes are proposed. Although the proposed architectures have certain PAPR performance losses when compared with the traditional SLM scheme, the proposed schemes have much lower complexities and their BER performances are exactly the same as the traditional SLM scheme. In particular, to generate 32 candidate signals for \( LN = 256 \), the number of complex additions for the proposed scheme I is 18.36% of the traditional SLM scheme and the number of complex multiplications is 3.12%, resulting in a PAPR performance loss of at most 0.64 dB. The proposed scheme II has a higher complexity, but achieves a better PAPR performance, where the PAPR performance loss is at most 0.22 dB.

REFERENCES


