Control Structure Selection for Optimal Disturbance Rejection

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Abstract— Control structure selection is an important part of control system design which has a strong influence on the resulting plant performance. This paper presents a systematic procedure for choosing control structure by extending previous work based on steady-state optimization to include the dynamic performance in a consistent manner. To address the problem that the dynamic performance depends on the type and the parameterization of the controllers used when comparing control structures, NMPC controllers are assumed. For each structure, the performance weights of these controllers are optimized with respect to the economic performance to yield an optimal tuning. This gives the best performance that is attainable for each structure and the structures are compared on equal grounds. The approach is demonstrated for a distillation control problem.

I. INTRODUCTION

In a processing plant, on different levels of the control hierarchy, there are usually a considerable number of degrees of freedom and measurements. Choosing manipulated and controlled variables among these variables is not a trivial task. This is the main objective of control structure selection.

Control structure selection consists of two steps [13]:

• Choice of sets of manipulated and regulated variables used in continuous and quasi-continuous automatic feedback control,
• Choice of controller structure (single-loop controllers, single loop controllers with compensation...)

The result of control structure selection often has a stronger influence on the performance of the resulting closed-loop system than the choice of control algorithms. Quantitative controllability analysis and control structure selection have been covered by many papers from the control community. However most of this work focuses on the dynamic tracking and regulation performance rather than the resulting economic performance of plant in the presence of disturbances and parameter variations.

An early contribution on the issue of control structure selection from the point of view of plant performance was made by Morari et al. [9] where several important aspects of the control of the chemical processes are discussed.

Yi and Luyben [1] used steady-state disturbance sensitivity to evaluate different control structures. They evaluated the ability of control system to respond to the effect of steady-state disturbances limited to step changes in single processes variables. McAvoy [21] selected a set of manipulated variables which require the smallest changes that are needed to compensate the effects of a set of disturbances. The approach was applied to a linear process model leading to a mixed-integer linear problem. Perkins and colleagues [6], [10] included the choice of control structures and the computation of controller parameters in the optimization of the plant design, leading to a large mixed-integer dynamic optimization problem. The structure of the controllers was fixed a priori, e.g. PI controllers. However, because of the convergence problem and immense computation times, this approach could only be applied to simple processes. Seferlis [12] used economic and static controllability criteria for screening control structures and applied this to a flow sheet with multiple reaction and separation steps with strong couplings due to recycles. The dynamic behaviour was however not investigated.

Skogestad [20], [21] followed the same line of thinking as [9], introducing the idea of “self-optimizing” control which means that a close-to-optimal steady state performance of the process in the presence of disturbances can be established by a well chosen control structure. The basic idea is to choose the controlled variables such that a cost function is minimized by keeping these variables close to the set-points in the presence of disturbances. A stepwise approach is proposed where first the promising control structures are determined based on a stationary analysis. A similar approach was proposed by [16], [23] with some refinements replacing the informed judgement by objective criteria and optimization, and in particular considering the effect of measurement errors. The promising structures were assessed using dynamic performance indicators.

The work mentioned above deals with stationary models only, the disturbances are assumed to be slowly time-varying or constant. The effect of dynamic disturbances has not yet been addressed which may lead to a wrong judgement of the performance of the controlled plant. In this paper the dynamic analysis is included seamlessly extending the analysis of [16], [23] based upon full process models.
II. PLANT PERFORMANCE AND CONTROL STRUCTURE SELECTION

From the process engineering point of view, the purpose of automatic feedback control is not to track set-point changes or to keep some variables as close to their set-points as well as possible but to keep the process operation close to the optimal one in the presence of disturbances and differences between the plant model used for the design and the real plant. This means that a “good” controller that can perfectly follow a suboptimal set-point in the presence of strong disturbances may yield a worse plant profit than a “bad” controller that cannot manage to nicely track a well chosen set-point. A way to avoid the problem of choosing pairs of controlled variables and manipulated variables and their set-points is to use online optimization, which exploits all available degree of freedom based upon a full nonlinear model in order to maximize the profit over a time horizon and is nowadays practically possible for slow processes [15], [17]. However in industry, feedback control of selected variables is preferred because of the reliability and simplicity in dealing with the effect of disturbances and plant-model mismatch.

On the other side if the third term is large then online optimizing control should be used instead of feedback control. The above equation gives the loss of profit only for one disturbance and a chosen control structure. The overall performance of a control structure should be evaluated with all the disturbances and their probabilities of occurrence.

There are two kinds of disturbances which affect the process: slow and fast varying disturbances. The effect of the former ones is taken into account by analyzing the steady state plant performance. In order to treat both kinds of disturbances, first the steady state performance of plant is considered since most plants work mostly in the vicinity of a steady state, then the dynamic part is addressed in order to incorporate dynamic disturbances.

The slowly-varying disturbances are incorporated into the static optimization where the measurement errors of the controlled variables must be taken into account because their presence may make the resulting inputs $u_{con}$ differ considerably from the desired values although they may be a very good approximation of the optimal inputs in the nominal case, as illustrated by Fig. 1. This is taken into account by considering the worst case performance of the regulatory control loops trying to keep the controlled variables in a range around the set-points defined by the measurement errors. A control structure that yields a comparatively small profit in the presence of measurement errors should be excluded.

The above analysis is extended here based upon a full dynamic process model and dynamic controllers to include the fast-varying disturbances in a consistent manner. The type and the parameterization of the controllers used can have a strong influence on the dynamic performance of the system which poses problems for the comparison of control structures. In this approach, an optimization-based scheme using dynamic models is used. The weights used in the cost functions of the NMPC-controllers are optimized for each structure to yield an optimal economic performance for the disturbance scenarios considered. The result is the best performance which each structure can achieve, and the best structure is found by comparing all the candidates.

III. CONTROL STRUCTURE SELECTION PROCEDURE

Our control structure selection procedure consists of 6 steps:

1. Define the optimization problem:

Determine the degrees of freedom available for optimization and choose a set of manipulated variables (u) that represent the degrees of freedom. Formulate a scalar profit function $J$ to be maximized for the optimal operation and specify the constraints that have to be satisfied during process operation.
defined in \( k \), the following manipulated variables on the measurements is denoted by:
\[
\Delta = \left( \arg \max_{\Delta u} \text{J} \right) - \left( \arg \min_{\Delta u} \text{J} \right).
\]

Small singular values of \( S \) mean small influences of the sensor errors therefore structures with large maximal singular values are excluded.

Other useful criterion for pre-screening are RHP zeros and conventional RGA analysis which excludes structures with negative diagonal elements. The generalized non-square relative gain (NRG) given by \cite{19} can also be applied. For the case of many candidate measured outputs, one may consider not using those outputs corresponding to the rows in NRG where the sum of elements is much smaller than 1. This can be explained by the fact that the row sums of the NRG are equal to the square of the output and should not be too small.

4. Selection of the set-points for regulatory control

To fully exploit the potential of feedback control, the set-points are determined by optimization over the set of disturbance scenarios instead of optimizing for only nominal conditions (no plant-model mismatch, no disturbances). The optimal set-points are found by solving:

\[
\begin{align*}
\max \sum_{i=1}^{n} \text{J}(x, u_i, d_i) \\
\text{s.t.: } \forall d_i : \\
\hat{x} = f(x, u_i, d_i) = 0 \\
h(x, u) \leq 0 \\
y_{\text{opt}} = m(x) \\
\Delta \leq u \leq u_{\text{max}} \\
x_{\text{min}} \leq x \leq x_{\text{max}}
\end{align*}
\]

The above optimization problem can be infeasible indicating that there is no common set-point which can be attained for all disturbances and the given constraints on the process inputs and process states.

5. Quantitative evaluation of the benefits of the control structures with static disturbances:

For all scenarios of disturbances \( d_i \), the following optimization problem is solved to get the worst case control performance for regulation of the controlled variables to values in the range around the nominal set-point \( y_{\text{opt}} \) defined by the measurement error \( \varepsilon_{\text{sensor}} \):

\[
\min \text{J}(x, u, d_i) \\
\text{s.t.: } \forall d_i : \\
\hat{x} = f(x, u, d_i) = 0 \\
h(x, u) \leq 0 \\
y = m(x) \\
\sum_{i} \varepsilon_{\text{sensor}} \leq y \leq \sum_{i} \varepsilon_{\text{sensor}} + \varepsilon_{\text{sensor}}
\]

The overall performance of a control structure with all disturbances is evaluated by the following expected value:

\[
\Delta J = \int_{-d_{\text{max}}}^{d_{\text{max}}} \int_{-d_{\text{max}}}^{d_{\text{max}}} \omega(d) \text{J}(u_{\text{max}}, d) - \text{J}(u_{\text{min}}, d) \text{d}d_1 \ldots \text{d}d_n
\]

As \( \omega(d) \) is usually not known, the above formulation is approximated by a weighted sum over a set of disturbance scenarios. A comparatively large value of the maximum loss means that the corresponding control structure is not able to avoid a poor performance in the presence of the measurement errors and should be excluded.
6. Quantitative evaluation of the benefits of the control structures with dynamic disturbances:

The main idea here is to find out what performance can be achieved if disturbances occur and the controlled variables are kept as close to the set-points as possible. In order to accomplish this, a simulation of an online optimization scheme that tracks the set-points is implemented. The objective function is defined by:

\[
P(t_k) = \min_{u} \left\{ \int_{t_k}^{t_k+H_p} \left( \|y(t) - y_{set}\|_P + \|u(t) - u_{set}\|_Q \right) dt \right\}
\]

s.t.: \( \dot{x}(t) = f(x(t), u(t), d(t)) \)

\[
h(x, u) \leq 0
\]

\[
y(t) = m(x(t))
\]

where \( \|x\|_X \) denotes the norm which is defined by:

\[
\|x\|_X = u^T X u, \quad X \text{ is a positive semi-definite matrix.}
\]

\( P \) and \( Q \) are degrees of freedom and should be chosen such that the economic profit function \( J \) is maximized. The prediction horizon \( H_p \) is chosen long enough to capture all effects of the disturbances. An upper layer optimization is used to compute \( P \) and \( Q \):

\[
\max_{P, Q} \int_{t_k}^{t_k+H_p} \sum_{i=0}^{N_c} J(x, u, d) dt
\]

s.t.: \( P, Q \succ 0 \)

(P1)

The results for all remaining control structures are compared as in step 5. The structure which yields the best performance will be chosen.

IV. CASE STUDY

The above methodology is applied to the continuous distillation column shown in Fig. 2.

The distillation column is used for the separation of a binary mixture of Methanol and n-Propanol. The overhead vapor is totally condensed in a water-cooled condenser which is open to atmosphere. The reboiler is heated electrically. The preheated feed stream enters the column at the feed tray as saturated liquid. The model of the process is based on the following assumptions: total condenser, negligible vapour holdup, variable liquid holdup, liquid outflow determined by Francis weir formula, constant pressure losses, perfect mixing, the mixture is at equilibrium temperature. Murphee efficiency is applied for each tray. With 40 trays, this results in a large and stiff model of 82 differential and 122 algebraic variables [7].

The reflux rate and the heat supply are the two operating degrees of freedom. It is assumed that a composition measurement is too expensive and unreliable (it also introduces a time delay into the system). The controlled variables are the temperatures on two trays. Hence there are:

\[
C_{40}^2 = \binom{2}{40} = \frac{40!}{(40-2)!2!} = 780
\]

possible control structures. The profit function is chosen to be:

\[
J = c_{\text{Methanol}} H \cdot (x_{\text{Methanol}} - \text{desiredvalue}) n_{\text{Methanol}} - c_{\text{heat input}} H
\]

\( H \) is the Heaviside step function which is approximated by a tangent hyperbolic function to avoid the problem of a discontinuous function. The income is available only when the purity of distillate product satisfies the requirement, which is 0.99 in our case. The disturbances are chosen to be: a change in the feed flow rate: ±4 l/h, a change in the feed concentration: ±0.1, a change in the feed temperature: ±5 K for both steady state and dynamic cases. For the steady state case, the additional following disturbances are considered: a change in the condenser temperature: ±5 K, a change in the heat loss: ±0.2 kW. In the dynamic scenarios, the disturbances are assumed to be uniformly distributed within the lower and upper bounds. The sensor error is assumed to be 0.33 K.

Since there are nearly 800 possibilities of choosing control structures, pre-screening should be performed to reduce their number. First the RGA criterion is used. Those outputs corresponding to the rows in the RGA, where the sum of the elements is much smaller than 1, will not be considered further. 429 structures with a sum less than 0.1 are discarded. Next, a sensitivity analysis is performed, 78 structures are left and no structure of these has RHP zeros.

In step 4, all structures have a common set-point and continue to step 5. Only those structures which result in a profit in the worst case greater than 85% of the nominal operating profit are considered for the final step. 12 structures that satisfy this criterion are compared in the final step using the NMPC simulation.

The NMPC simulation is done by using MUSCOD II which stands for "MUltiple Shooting COde for Direct
Optimal Control” MUSCOD is an optimization software package designed by IWR Heidelberg to efficiently and reliably solve optimal control problems for systems described by ordinary differential equations (ODE) or by differential-algebraic equations (DAE) of index one, [2].

To make the problem simpler, $P$ and $Q$ are assumed to be diagonal matrices; as usually used in practice. The diagonal elements of $P_{ii} Q_{jj}$ of $P$, $Q$, can be subsumed to the vector $r$ with $r=(p_{11} \ldots p_{nn} q_{11} \ldots q_{mm})^T$; $n$ and $m$ are sizes of $P$ and $Q$. The above problem is formulated in a simpler form:

$$\max_{r}\int_{\tau_0}^{\tau_{\infty}} \sum_{i=1}^{s} J(x,u,d)dt$$

s.t.: $r \geq 0$

$$P(t) = \min_{u} \left\{ \int_{t_{i}}^{t_{f}} \left[ \left\| y(t) - y_{ref}\right\|_P + \left\| u(t) - u_{ref}\right\|_Q \right] dt \right\}$$

s.t.: $\dot{x}(t) = f(x,u,d)$

$$y(t) = m(x(t))$$

The upper layer is an optimization problem with only simple bounds on variables. However conventional optimization algorithms using derivatives can not be used for solving this problem since the lower layer is a complex numerical procedure which introduces noise if a finite difference method is used. The disturbances in the plant model also pose the same problem. To address this, derivative free optimization (DFO) is implemented. This family of optimization algorithms is especially suited for optimization problem which the objective function is computationally expensive to evaluate or is corrupted by noise. The software package used is CONDOR [4].

Due to the excessive computation time if NMPC is used, first a linear MPC controller is used to screen the structures. The system is linearized around the operating point $x_0$, $u_0$ found in step 4.

$$\max_{P,Q} \int_{\tau_0}^{\tau_{\infty}} \sum_{i=1}^{s} J(x,u,d)dt$$

s.t.: $P$, $Q > 0$

$$P(t) = \min_{u} \left\{ \int_{t_{i}}^{t_{f}} \left[ \left\| \Delta y(t) \right\|_P + \left\| \Delta u(t) \right\|_Q \right] dt \right\}$$

s.t.: $\Delta \dot{x}(t) = A \Delta x(t) + B \Delta u(t) + C \Delta d(t)$

$$\Delta y = D \Delta x(t)$$

with

$$A = \frac{\partial f}{\partial x} \bigg|_{x=x_0, \, u=0} , \quad B = \frac{\partial f}{\partial u} \bigg|_{x=x_0, \, d=0} , \quad C = \frac{\partial f}{\partial d} \bigg|_{x=x_0, \, u=0}. $$

$$x = x_0 + \Delta x, u = u_0 + \Delta u$$

The task of linear MPC is to drive the controlled variables to the origin. From the linear MPC simulation result, the 3 most promising structures are found: (10,28), (10,31), (17,27) (the numbers indicate the number of the tray
A methodology for control structure selection was presented with the aim of optimizing the plant performance taking into account the presence of both steady-state and dynamic disturbances. The method was applied successfully to the example of a distillation column. The future work will employ the real-time iterative procedure proposed by Diehl [7], which will significantly reduce the computation time. Parameter uncertainties of the plant model will also be considered.

Step 1 – Step 6 give a set of control structures that lead to a good performance with respect to the profit for stationary, slowly-varying and fast-varying disturbances. It may happen that some of these structures are not suitable for dynamic operation (e.g. in the presence of plant-model mismatch). To check this, a dynamic controllability analysis using linear technique can be performed [14], [18].

V. CONCLUSION AND FUTURE WORK

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REFERENCES