Magneto-Thermo-Viscoelastic Interactions in an Unbounded Non-homogeneous Body with a Spherical Cavity Subjected to a Periodic Loading

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Abstract

The paper presents the analytical solution for magneto-thermo-elastic non-homogeneous medium with a spherical cavity subjected to periodic loading is presented. The effect of thermal relaxation times on the wave propagation in magneto-thermo-visco-elastic using the GL theory is discussed. We take the material of the spherical cavity to be of Kelvin-Voigt type. The displacement, temperature and stress components have been obtained in analytical form. The numerical calculations for the displacement, temperature and the components of stresses, and explain the special cases from this study. The results are displayed graphically to illustrate the effect of relaxation times, magnetic field, mechanical relaxation time, non-homogeneity, frequency are very pronounced and illustrated graphically.

Keywords: Viscoelastic, Periodic loading, Non-homogeneity, Wave propagation, Magneto-thermo.

1- Introduction

Increased interest in magneto-thermo-elasticity during recent years can be attributed to the fact that the study of magneto-thermo-mechanical coupled behavior in smart structures.

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The dynamical problem of magneto thermo viscoelasticity has received much attention in the literature during the past decade. In recent years the theory of magneto-thermo-viscoelasticity which deals the interactions among strain, temperature and electromagnetic fields has drawn the attention of many researchers because of its extensive uses in diverse fields, such as geophysics for understanding the effect of the Earth's magnetic field on seismic waves, damping of acoustic waves in a magnetic field, emission of electromagnetic radiations from nuclear devices, development of highly sensitive superconducting magnetometer, electrical power engineering optics, supersonic airplanes, rockets and missiles etc. Abd-Alla and Mahmoud [1,2] solved magneto-thermo elastic problem in rotating non-homogeneous orthotropic hollow cylindrical under the hyperbolic heat conduction model, and investigated effect of the rotation on propagation of thermo elastic waves in a non-homogeneous infinite cylinder of isotropic material. Abd-Alla, et al.[3-8] investigated some problems as the propagation of S-wave in a non-homogeneous anisotropic incompressible and initially stressed medium under influence of gravity field, the generalized magneto-thermo elastic Rayleigh waves in a granular medium under influence of gravity field and initial stress, also investigated the problem of transient coupled thermo elasticity of an annular fin, effect of the rotation on plane vibrations in a transversely isotropic infinite hollow cylinder, the magneto-thermo-viscoelastic interactions in an unbounded body with a spherical cavity subjected to a periodic loading, effect of the rotation on a non-homogeneous infinite cylinder of orthotropic material. Some problems of wave propagation in cylindrical poroelastic dry bones and in the non-homogeneity orthotropic elastic media are investigated by Mahmoud [9,10] respectively. Mukhopadhyay [11] investigated the effects of thermal relaxations on thermo-visco-elastic interactions in an unbounded body with a spherical cavity subjected to a periodic loading on the boundary. Effects of thermal relaxations on thermo elastic interactions in an unbounded body with a spherical cavity or cylindrical hole subjected to a periodic loading on the boundary respectively is investigated by Roychoudhuri and Banerjee [12]. The thermally induced vibrations in a generalized thermo elastic solid with a cavity have been investigated by Erbay, et al.[13] and Rehbinder [14]. Abd-Alla, and Abo-Dahab [15] investigated the time-harmonic sources in a generalized magneto-thermo-viscoelastic continuum with and without energy dissipation.

In this paper, the magneto-thermo-elastic equation of spherical cavity is decomposed into non-homogeneous equation with boundary conditions. The effect of thermal relaxation times on the wave propagation in magneto thermo viscoelastic using the GL theory will be discussed. We took the material of the spherical cavity to be of Kelvin – Voigt type. Thus, the exact expressions for the transient response of displacement, stresses and temperature in spherical cavity are obtained. The numerical calculations will be investigated for the displacement, temperature and the components of stresses, and explain the special case from this study when the magnetic field and non-homogeneity are neglected. Finally, numerical examples are calculated and discussed.

2- Formulation of the problem

Considering spherical cavity in an a magnetic field $\mathbf{H}(0,0,H_{r})$, letting the spherical coordinates of any represents point be $(r, \theta, \phi)$ and assuming that spherical cavity is subjected to a rapid change in temperature $T(r,t)$. For the axisymmetric plane
strain problem, the components of displacement and magnetic field in spherical coordinates 
\((r, \theta, \phi)\) system are expressed as \(u_\theta = u_\phi = 0, and u_r = u_r(r, t)\) respectively. Let us consider the medium is a perfect electric conductor and the linearized Maxwell equations governing the electromagnetic field, in the absence of the displacement current (SI) in the form as in Roychoudhuri and Mukhopadhyay [12] for a perfectly conducting, elastic body are given by:

\[
\mathcal{J} = \text{curl } \mathbf{h}, \quad -\mu_e \frac{\partial \mathbf{h}}{\partial t} = \text{curl } \mathbf{E},
\]

\[
\text{div } \mathbf{h} = 0, \quad \text{div } \mathbf{E} = 0, \quad \mathbf{E} = -\mu_e \left( \frac{\partial \mathbf{U}}{\partial t} \times \mathbf{H} \right),
\]

where \(\mathbf{h}\) is the perturbed magnetic field over the primary magnetic field, \(\mathbf{E}\) is the electric intensity, \(\mathcal{J}\) is the electric current density, \(\mu_e\) is the magnetic permeability, \(\mathbf{H}\) is the constant primary magnetic field and \(\mathbf{U}\) is the displacement vector.

Applying an initial magnetic field vector \(\mathbf{H}(0, 0, H_\phi)\) in spherical coordinate \((r, \theta, \phi)\) to Eq.(2.1) we have

\[
\begin{align*}
\mathbf{H} &= \mathbf{H}(0, 0, H_\phi), \quad \mathbf{E} = \mu(0, -H_\phi \frac{\partial u}{\partial t}, 0), \\
\mathbf{h} &= \text{curl}(\mathbf{U} \times \mathbf{H}) = (0, 0, -H_\phi \left( \frac{1}{r} \frac{\partial (ru)}{\partial r} \right)) = (0, 0, -H_\phi \left( \frac{\partial u}{\partial r} + \frac{u}{r} \right)), \\
\mathbf{J} &= (0, 0, h_\phi), \quad \mathcal{J} = (0, \frac{\partial h_\phi}{\partial r}, 0), \\
\mathbf{E} &= (0, E_z, 0).
\end{align*}
\]

Let us consider an infinite isotropic non-homogeneous viscoelastic solid and the viscoelastic nature of the material is described by the Voigt type of linear viscoelasticity. The medium is assumed to have a spherical cavity of radius \(a\). The equations governing the displacement field \(\mathbf{U}\), the stresses \(\sigma_{ij}\) and the small temperature derivation \(T\) from a constant initial temperature \(T_0\) are related by the following fundamental equations of the generalized thermoelasticity for a Kelvin-Voigt type, the magneto-elastic dynamic equation of the non-homogeneous sphere becomes

\[
\sigma_{ij} + \tau_{ij} = \rho \dddot{u} , \quad i, j = 1, 2, 3
\]

\[
(2.3)
\]
\[ KT_j = \rho c_v (T + \tau_1 \dot{T}) + \gamma T_0 u_{j,j}, \quad j = 1, 2, 3 \]  
\[ \sigma_{ij} = (\lambda \tau_m e_k - \gamma (T + \tau_2 \dot{T})) \delta_{ij} + 2 \mu \tau_m e_{ij}, \quad i, j = 1, 2, 3 \]

where \( \rho \) is density of the material, \( K \) is thermal conductivity, \( c_v \) is specific heat of the material per unit mass, \( \tau_1, \tau_2 \) are thermal relaxation parameters, \( \alpha \) is coefficient of linear thermal expansion, \( \lambda, \mu \) are Lame elastic constants, \( \theta \) is the absolute temperature, \( \gamma = \alpha (3 \lambda + 2 \mu) \), \( T_0 \) is reference temperature solid, \( T \) is temperature difference \( (\theta - T_0) \), \( \tau_0 \) is the mechanical relaxation time due to the viscosity, \( \tau_m = (1 + \tau_0 \frac{\partial}{\partial t}) \) and \( \sigma_{ij} \) is the mechanical stress.

Maxwell’s electro-magnetic stress tensor \( \tau_{ij} \) is given by:
\[ \tau_{ij} = \mu_e [H_i h_j + H_j h_i - (H \cdot H) \delta_{ij}], i, j = 1, 2, 3, \]
and
\[ e_{ij} = \frac{1}{2} (u_{i,j} + u_{j,i}), \quad e_{ij} = \Delta, \quad i, j = 1, 2, 3. \]

The non-vanishing displacement component is \( u_r = u(r,t) \), so that
\[ e_r = \frac{\partial u}{\partial r}, \quad e_{\theta \theta} = e_{\phi \phi} = \frac{u}{r}. \]

The magneto elastodynamic equation of the non-homogeneity spherical if \( u = u(r,t) \), becomes
\[ \frac{\partial \sigma_{r r}}{\partial r} + \frac{2}{r} (\sigma_{r r} - \sigma_{\theta \theta}) + f_r = \rho \frac{\partial^2 u_r}{\partial t^2}, \]
where \( f_r \) is defined as Lorentz’s force [13], which may be written as
\[ f_r = \mu_e (\vec{J} \times \vec{H}) = \mu_e H_z \frac{\partial}{\partial r} \left( \frac{\partial u}{\partial r} + \frac{u}{r} \right). \]

For a spherical symmetric system the non-vanishing stresses components of a non-homogeneous sphere subjected to a rapid change in temperature, is expressed as
\[ \sigma_{r r} = \tau_m (\lambda + 2 \mu) \frac{\partial u}{\partial r} + 2 \lambda \tau_m \frac{u}{r} - \gamma (T + \tau_2 \dot{T}), \]
\[ \sigma_{\theta \theta} = 2 \tau_m (\lambda + \mu) \frac{u}{r} + \lambda \tau_m \frac{\partial u}{\partial r} - \gamma (T + \tau_2 \dot{T}), \]
where \( \sigma_{r r} \) and \( \sigma_{\theta \theta} \) are radial and hoop stresses respectively.

The heat conduction equation is
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\[ K \left( \frac{\partial^2 T}{\partial r^2} + \frac{2}{r} \frac{\partial T}{\partial r} \right) = \rho c_v \left( \frac{\partial T}{\partial t} + \tau_m \frac{\partial^2 T}{\partial t^2} \right) + \gamma T \left[ \frac{\partial}{\partial r} \left( \frac{1}{r} \frac{\partial}{\partial r} \right) + \frac{2}{r} \right] u . \]  

(2.13)

where \( K \) is the thermal conductivity, \( \gamma = \alpha_c \left( 3\lambda + 2\mu \right) \).

3- Boundary conditions

(i) Omitting the Maxwell tensor the surface of the non-homogeneous sphere the corresponding boundary conditions are

\[ u(1,t) = 0 , \]  

(3.1a)

\[ \sigma_{rr} = [\tau_m (\lambda + 2\mu) \frac{\partial u}{\partial r} + 2\lambda \tau_m \frac{u}{r} - \gamma (T + \tau_2 T)]_{r=a} + p e^{ext} = 0 , \]  

(3.1b)

where \( T = T(r,t) \).

(ii) The initial conditions are

\[ u(r,0) = 0 , \quad \frac{\partial u(r,0)}{\partial t} = 0 . \]  

(3.1c)

In the above formula, the non-homogeneity of material is characterized by a special law as follows:

\[ \lambda = \lambda_0 r^{2m} , \quad \mu = \mu_0 r^{2m} , \quad \gamma = \gamma_0 r^{2m} , \quad \rho = \rho_0 r^{2m} , \quad \mu_\epsilon = \mu_\epsilon^0 r^{2m} , \]  

(3.2)

where \( \lambda_0 , \mu_0, \gamma_0, \rho_0 \) and \( \mu_\epsilon^0 \) are Lame's constant, shear modulus, thermal modulus, mass density and magnetic permeability coefficient of homogeneous material, respectively, and \( m \) expresses a non-homogeneous exponent of material which is an arbitrary real number.

Substituting (3.2) into Eq.s (2.10), (2.11) and (2.12) yield

\[ \sigma_{rr} = r^{2m} [\tau_m (\lambda_0 + 2\mu_0) \frac{\partial u}{\partial r} + 2\lambda_0 \tau_m \frac{u}{r} - \gamma_0 (T + \tau_2 T)], \]  

(3.3)

\[ \sigma_{\theta \theta} = r^{2m} [2\tau_m (\lambda_0 + \mu_0) \frac{u}{r} + \lambda_0 \tau_m \frac{\partial u}{\partial r} - \gamma_0 (T + \tau_2 T)], \]  

(3.4)

and

\[ f_r = \mu_\epsilon^0 H \frac{\partial}{\partial r} \left( \frac{\partial u}{\partial r} + \frac{u}{r} \right) , \]  

(3.5)

From (3.3), (3.4) and (3.5) into (2.9) we have
\[
[\tau_m + \mu_0 H^2 \frac{\partial^2 u}{\partial r^2} + [2(m + 1)\tau_m + \mu_0 H^2 \frac{1}{r^2} \frac{\partial u}{\partial r}] + \frac{4m \lambda_0 \tau_m}{(\lambda_0 + 2\mu_0)} - 2\tau_m - \frac{\mu_0 H^2}{(\lambda_0 + 2\mu_0)} \frac{u}{r^2} - \frac{2m + \frac{\partial}{\partial r}}{r} \frac{\gamma_0}{(\lambda_0 + 2\mu_0)} (T + \tau_2 T^*)
\]
\[
= \frac{\rho_0}{(\lambda_0 + 2\mu_0)} \frac{\partial^2 u}{\partial t^2}.
\]

Let \( c_1 = \frac{\lambda_0}{(\lambda_0 + 2\mu_0)} \), \( c_2 = \frac{\gamma_0}{(\lambda_0 + 2\mu_0)} \),
\[c_3 = \frac{\mu_0 H^2}{(\lambda_0 + 2\mu_0)} \), \( c_\nu = \sqrt{\frac{(\lambda_0 + 2\mu_0)}{\rho_0}} \).

Then Eq.(3.6) becomes
\[
(\tau_m + c_1) \frac{\partial^2 u}{\partial r^2} + [2(m + 1)\tau_m + c_3] \frac{1}{r^2} \frac{\partial u}{\partial r} + [4mc_1\tau_m - 2\tau_m - c_3] \frac{u}{r^2}
\]
\[
- c_2 \left[ \frac{2m + \frac{\partial}{\partial r}}{r} \right] (T + \tau_2 T^*) = \frac{1}{c_\nu} \frac{\partial^2 u}{\partial t^2}.
\]

Also the heat conduction equation is
\[
K \left( \frac{\partial^2 T}{\partial r^2} + \frac{2}{r} \frac{\partial T}{\partial r} \right) = \rho_0 r^2 c_\nu \left( \frac{\partial T}{\partial t} + \tau_1 \frac{\partial^2 T}{\partial t^2} \right) + \gamma_0 r^2 T_0 \left[ \frac{\partial}{\partial r} + \frac{2}{r} \right] u.
\]

4- Dimensionless quantities

We now use the following dimensionless quantities are taken as:
\[
U = \frac{u}{a} \), \( k = \frac{K}{\rho \ c_\nu} \), \( \eta = \frac{k c_\nu}{a} \),
\[Z = \frac{T}{T_0} \), \( \tau'_0 = \frac{c_\nu}{a} \tau_0 \), \( \tau'_1 = \frac{c_\nu}{a} \tau_1 \), \( \tau'_2 = \frac{k c_\nu}{a} \tau_2 \), \( w = \frac{a}{k c_\nu} \omega \), \( R = \frac{r}{a} \),
\[
\sigma_{\nu\nu} = \frac{\sigma_{\nu\nu}}{(\lambda_0 + 2\mu_0)} \), \( \sigma_{\nu\theta} = \frac{\sigma_{\nu\theta}}{(\lambda_0 + 2\mu_0)} \).
\]

Substituting of Eq.(4.1) into Eq.(3.8) and (3.9) gives the displacement equilibrium equation of the non-homogeneous spherical as follows:
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\[ [(1 + \tau'_0 \frac{\partial}{\partial \eta}) + c_5] \frac{\partial^2 U}{\partial R^2} + [2(m+1)(1 + \tau'_0 \frac{\partial}{\partial \eta}) + c_3] \frac{1}{R} \frac{\partial U}{\partial R} \]
\[ + [(4mc_1 - 2)(1 + \tau'_0 \frac{\partial}{\partial \eta}) - c_3] \frac{U}{R^2} - c_2 T_0[1 + \tau'_0 \frac{\partial}{\partial \eta}](\frac{2L}{R} + \frac{1}{c_z^2} \frac{\partial}{\partial R})Z = k^2 \frac{\partial^2 U}{\partial \eta^2}, \]

and the heat conduction equation is

\[ \left( \frac{\partial^2 Z}{\partial R^2} + \frac{2 \partial Z}{R \partial R} \right) = l_1 (1 + \tau'_1 \frac{\partial}{\partial \eta}) \frac{\partial Z}{\partial \eta} + l_2 \left[ \frac{\partial^2 U}{\partial \eta^2} + \frac{2}{R} \frac{\partial U}{\partial \eta} \right], \]

where

\[ l_1 = \frac{ac_v}{k}, \quad l_2 = \frac{a\gamma_0}{\rho_0} \]

The stress – strain relations can be right in the non-dimensional forms:

\[ \sigma_{\theta \theta} = (aR)^{2m} [(1 + \tau'_0 \frac{\partial}{\partial \eta}) \frac{\partial U}{\partial R} + 2\gamma_0 (1 + \tau'_0 \frac{\partial}{\partial \eta}) \frac{U}{R} - T_0 c_2 (1 + \tau'_0 \frac{\partial}{\partial \eta})Z], \]
\[ \sigma_{\phi \phi} = (aR)^{2m} [2(1 + \tau'_0 \frac{\partial}{\partial \eta}) \frac{U}{R} + c_4 (1 + \tau'_0 \frac{\partial}{\partial \eta}) \frac{U}{R} - T_0 c_5 (1 + \tau'_0 \frac{\partial}{\partial \eta})Z], \]

where

\[ c_4 = \frac{\gamma_0}{(\lambda_0 + 2\mu_0)}, \quad c_5 = \frac{\gamma_0}{(\lambda_0 + 2\mu_0)} \]

and

\[ f_r = \frac{\mu_0 H_0^2 \rho (aR)^{2m} \partial U}{a R} + \frac{U}{a R} = \frac{\mu_0 H_0^2 (aR)^{2m}}{a R} \left( \frac{\partial^2 U}{\partial R^2} + \frac{1}{R} \frac{\partial U}{\partial R} - \frac{U}{R^2} \right). \]

5- The solution method

Assuming that the general solution to the basic equation (4.2) of magneto-thermo-elastic motion is expressed in the form:

\[ U(R, \eta) = U^* (R) \ e^{i\omega \eta}, \]
\[ Z(R, \eta) = Z^* (R) \ e^{i\omega \eta}. \]

The equation of motion in Eq.(4.2) becomes

\[ [(1 + i\omega \tau'_0) + c_3] \frac{d^2 U^*}{dR^2} + [2(m+1)(1 + i\omega \tau'_0) + c_3] \frac{1}{R} \frac{dU^*}{dR} \]
\[ + [(4mc_1 - 2)(1 + i\omega \tau'_0) - c_3] \frac{U^*}{R^2} = -k^2 \omega^2 U^* + c_2 T_0 \gamma^*(\frac{2m}{R} + \frac{d}{dR})Z^*. \]
\[ \gamma' = (1 + i \tau'_1 \omega) \].

Or we can write in the form
\[
\frac{d^2 U^*}{dR^2} + \frac{1}{R} \frac{dU^*}{dR} + \frac{U^*}{R^2} = -m_1^2 U^* + \varepsilon \left( \frac{2m}{R} + \frac{d}{dR} \right) Z^*,
\]

where
\[
\eta_1 = \frac{(2m + 1)(1 + i \omega \tau'_0)}{(1 + i \omega \tau'_0) + c_3} + 1, \quad \eta_2 = \frac{(4mc_i - 1)(1 + i \omega \tau'_0)}{(1 + i \omega \tau'_0) + c_3} - 1,
\]
\[
\varepsilon = \frac{c_2 T_0}{(1 + i \omega \tau'_0) + c_3}, \quad m_1^2 = \frac{k^2 \omega^2}{(1 + i \omega \tau'_0) + c_3}.
\]

Also the heat conduction equation becomes
\[
(\nabla^2 + \beta_1) Z^* = \beta_2 \left[ \frac{d}{dR} + \frac{2}{R} \right] U^*,
\]

where
\[
\nabla^2 = \frac{d^2}{dr^2} + \frac{2}{r} \frac{d}{dr}, \quad \beta_1 = l_1 (\omega^2 \tau'_1 - i \omega), \quad \beta_2 = i \omega l_2,
\]

to solve equations, Eq.(5.4) and Eq.(5.6) , we let
\[
U^* (r) = \frac{d\phi(r)}{dr},
\]
\[
\frac{d}{dr} \left[ \frac{d^2 \phi(R)}{dR^2} + \frac{\eta_1}{R} \frac{d\phi(R)}{dR} + \frac{\eta_2}{R^2} \phi(R) \right] = -m_1^2 \frac{d\phi(R)}{dR} \varepsilon \left( \frac{2m}{R} + \frac{d}{dR} \right) Z^*,
\]

by comparing the coefficient of \( \frac{d}{dr} \) in Eq.(5.9) , we get
\[
\frac{d^2 \phi(R)}{dR^2} + \frac{\eta_1}{R} \frac{d\phi(R)}{dR} + \frac{\eta_2}{R^2} \phi(R) = \varepsilon Z^*,
\]

The heat conduction equation becomes
\[
(\nabla^2 + B_1) Z^* = B_2 \nabla^2 \phi(R).
\]

From Eqs.(5.10) and (5.11) , we have
\[
\frac{d^4 \phi(R)}{dR^4} + \left[ \eta_1 + 1 \right] \frac{1}{R} \frac{d^3 \phi(R)}{dR^3} + \left[ \frac{\eta_2 - \eta_1}{R^2} \right] \frac{d^2 \phi(R)}{dR^2} \left[ \frac{\eta_2 + \eta_1}{R^3} \right] \frac{d\phi(R)}{dR} + \beta_1 N^2 \phi = 0
\]

where \( \Gamma_1 = m_1^2 + \beta_1 - \varepsilon \beta_2, \quad \Gamma_2 = m_1^2 + \beta_1 \eta_1 - \varepsilon \beta_2 \).

Decoupling equations (5.11) and (5.12), we obtain:
\[
(\nabla^2 + \alpha_1^2) (\nabla^2 + \alpha_2^2) (\phi, Z^*) = 0,
\]
where $\alpha_1^2$ and $\alpha_2^2$ are the roots with positive real parts of the biquadratic equation:

$$\alpha^4 + (m_1^2 + \beta_4^2 - \eta_1 \eta_2)\alpha^2 + m_1^2 \beta_4^2 = 0. \quad (5.15)$$

Assuming the regularity conditions for $U$ and $Z^*$ the solutions of equation (5.15) are obtained in terms of spherical Hankel’s function in the form Eq.(5.12) represent ordinary differential equation with variable coefficients of order four, from this equation we can determine $\phi(R)$ and from this we can determine $Z^*$ component of displacement we can determined, finally determine the stress – strain relations.

$$\begin{align*}
(\phi^*, T^*) &= A_1 h_{0}^{(2)}(\alpha_1 R) + A_2 h_{0}^{(2)}(\alpha_2 R),
\end{align*} \quad (5.16)$$

where $A_1$ and $A_2$ are arbitrary constants and $h_{0}^{(2)}$ is the Hankel’s function of its order zero and second kind. From equations (5.1), (5.8), (4.5) and (5.16) the solution for the displacement, temperature and the radial and hoop stresses are found to have the forms:

$$\begin{align*}
U &= \left\{A h_{1}^{(2)}(\alpha_1 R) + B h_{1}^{(2)}(\alpha_2 R)\right\} e^{\Omega \eta}, \\
T &= \left\{A_i h_{0}^{(2)}(\alpha_i R) + A_2 h_{0}^{(2)}(\alpha_2 R)\right\} e^{\Omega \eta}, \\
\sigma_{rr} &= \left\{T_1 h_{0}^{(2)}(\alpha_1 R) + \frac{T_2}{R} h_{1}^{(2)}(\alpha_1 R)\right\} A e^{\Omega \eta} + \\
&+ \left\{T_1 h_{0}^{(2)}(\alpha_2 R) + \frac{T_2}{R} h_{1}^{(2)}(\alpha_2 R)\right\} B e^{\Omega \eta}, \\
\sigma_{\theta\theta} &= \left\{T_4 h_{0}^{(2)}(\alpha_1 R) + \frac{T_5}{R} h_{1}^{(2)}(\alpha_1 R)\right\} A e^{\Omega \eta} + \\
&+ \left\{T_4 h_{0}^{(2)}(\alpha_2 R) + \frac{T_5}{R} h_{1}^{(2)}(\alpha_2 R)\right\} B e^{\Omega \eta},
\end{align*} \quad (5.17, 5.18, 5.19)$$

From (3.1b), we get:

where $S_i = \frac{\alpha_i^2 - m_i^2}{\eta \alpha_i}, \quad A_i = S_i B_i, \quad i = 1, 2$

$$\begin{align*}
B_1 &= -S_2 h_{0}^{(2)}(\alpha_2) \frac{\gamma_1}{\gamma_1}, \\
B_2 &= \frac{\sigma_0}{\gamma_1} S_1 h_{0}^{(2)}(\alpha_1), \\
\gamma_1 &= S_2 h_{0}^{(2)}(\alpha_2) \left\{T_1 h_{0}^{(2)}(\alpha_1) + T_2 h_{1}^{(2)}(\alpha_1)\right\} - S_1 h_{0}^{(2)}(\alpha_1) \left\{T_3 h_{0}^{(2)}(\alpha_2) + T_3 h_{1}^{(2)}(\alpha_2)\right\}. 
\end{align*}$$
\[ T_1 = (1 + i \omega t_0 + R_H^2) \alpha_1 - (1 + i \omega t_2) S_1, \]
\[ T_2 = (1 + i \omega t_0 + R_H^2)(2 \lambda - 2), \]
\[ T_3 = (1 + i \omega t_0 + R_H^2) \alpha_2 - (1 + i \omega t_2) S_2, \]
\[ T_4 = (1 + i \omega t_0 + R_H^2) \alpha_1 - (1 + i \omega t_2) S_1, \]
\[ T_5 = (1 + i \omega t_0 + R_H^2)(1 - \lambda), \]
\[ T_6 = \lambda (1 + i \omega t_0 + R_H^2) \alpha_2 - (1 + i \omega t_2) S_2, \]
\[ \lambda = \frac{\lambda}{\lambda + 2 \mu}, \quad \sigma_0 = \frac{\sigma_0}{\gamma T_0}. \]

This is the solution of the current problem for the case of homogeneous isotropic visco-elastic unbounded body with spherical cavity without the effect of magnetic field, that coincides with pervious published.

6- Numerical results and discussion

The copper material was used chosen for purposes of numerical evaluations. The constants of the problem are given as:
\[ \mu = 3.86 \times 10^{10} \text{ kg/s}^2, \quad \lambda = 7.76 \times 10^{10} \text{ kg/s}^2, \quad \rho = 8954 \text{ kg/m}^3, \]
\[ c_v = 383.1 \text{ J/kg/K}, \quad \alpha_t = 1.78 \times 10^{-5} \text{ K}^{-1}, \quad k = 386 W/(mK), \]
\[ K = 3.14 \times 10^2 \text{ col/mK}, \quad T_0 = 293. \]
The numerical technique outlined above was used to obtain the temperature, radial displacement, radial stress and hope stress inside the sphere. These distributions are shown in figs. 1-16 respectively. The computational were carried out for three values of thermal relaxation time, namely \( \tau_t = 0.2, \tau_t = 0.5, \text{ and } \tau_t = 0.8 \), magnetic field, namely \( H = 2 \times 10^1, H = 3 \times 10^2, H = 4 \times 10^2 \) mechanical relaxation time, namely \( \tau_m = 0.1, \tau_m = 0.4, \tau_m = 0.7 \) and the frequency, namely \( \omega = 1 \times 10^5, \omega = 2 \times 10^5, \omega = 3 \times 10^5 \).

For the sake of brevity some computational results are not being presented here. Figs. 1-16 show the solution corresponding to the use of the non-homogeneous material \( m=0.5 \), while the figs. 1,5,9 and 13, respectively, show the radial displacement in generalized thermo elastic media. Both figures indicate that the medium along \( r \) undergoes expansion deformation because of thermal relaxation time, magnetic field, mechanical relaxation time and frequency. The radial displacement decreases with increasing thermal relaxation time and frequency and it increases with increasing mechanical relaxation time and magnetic field. Figs. 4,6,10 and 14, the radial stress in generalized thermo elastic. From both figures, the radial stress increases with increasing magnetic field, mechanical relaxation time and frequency and it is decreases with increasing thermal relaxation time. Figs. 3,7,11 and 15, respectively, the hoop stress in generalized thermo elastic. From both figures, the hoop stress decreases with increasing thermal relaxation time, magnetic field and frequency and it is increases with increasing
mechanical relaxation time. Figs. 4, 8, 12 and 16 show the temperature distribution, where the figure 4 represents the solution corresponding to the use of the effect of thermal relaxation time $\tau_1$ while the figure 8 represents the solution corresponding to the use of the effect magnetic field $H_0$ also, figure 12 represents the solution corresponding to the use of the effect mechanical relaxation time $\tau_2$ and figure 16 represents the solution corresponding to the use of the effect of frequency. It was found that near the surface cavity where the boundary conditions domain the coupled and the generalized theories give very close results. Inside the sphere, the solution is markedly different. This is due to the fact that thermal waves in the coupled theory travel is not identically zero (though it may be very small) for any small of time. At different instant, the non-zero region removes forward correspondingly with the passage of time. This indicates that the heat propagates as a wave with finite velocity. In both figures, it is clear that distribution have a non-zero value only in a bounded region of space. Outside this region the values vanish identically. This indicates that the equation satisfied by the concentration predicts finite speed of propagation of matter from one medium to the other.

7- Concluding
Due to the complicated nature of the governing equations for the generalized thermo-visco-elastic theory, the work done in this field is unfortunately limited in number. The method used in this study provides a quite successful approach in dealing with such problems. This approach gives exact solution in the Hankel’s transform domain without any assumed restriction on the actual physical quantities that appear in the governing equations of the problem considered. Important phenomena are observed in all these computations:

1- It was found that for large values of time the coupled and the generalized give close results. The case is quite different when we consider small value of time. The coupled theory predicts infinite speeds of wave propagation. This is evident from the fact that the obtained solutions are not identically zero for any values of time but fade gradually very small values at points for removed from the surface. The solutions obtained in the context of GL theory, however, exhibit the behavior of finite speeds of wave propagation.

2- By comparing Figs. 1, 5, 9 and 13, it was found that $U$ has the same behavior in both media. But the values of $u$ in generalized thermo elastic medium are larger in comparison with those in thermo elastic medium. The same remark for $\sigma_{RR}$ in comparing Figs. 2, 6, 10 and 14. This is due to the influence of relaxation time, magnetic field and frequency.

3- The results presented in this project should prove useful for researchers in material science, designers of new materials, low-temperature physicists as well as for those working on the development of a theory of a theory magneto-thermo-visco-elastic.
Fig. 1: Variation of radial displacement versus $R$ at varies values of thermal relaxation time $\tau_1$ at $H_0 = 3 \times 10^3$, $\tau_0 = 0.4$ and $w = 2 \times 10^3$.

Fig. 2: Variation of radial stress versus $R$ at varies values of thermal relaxation time $\tau_1$ at $H_0 = 3 \times 10^3$, $\tau_0 = 0.4$ and $w = 2 \times 10^3$.

Fig. 3: Variation of hoop stress versus $R$ at varies values of mechanical relaxation time $\tau_0$ at $\tau_1 = 0.5$, $H_0 = 3 \times 10^2$ and $w = 2 \times 10^3$.

Fig. 4: Variation of temperature versus $R$ at varies values of thermal relaxation time $\tau_1$ at $H_0 = 3 \times 10^3$, $\tau_0 = 0.4$ and $w = 2 \times 10^3$. 

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Fig. 5: Variation of radial displacement versus $R$ at varies values of magnetic field $H_0$ at $\tau_1 = 0.5$, $\tau_0 = 0.4$ and $w = 2 \times 10^3$

Fig. 6: Variation of radial stress versus $R$ at varies values of magnetic field $H_0$ at $\tau_1 = 0.5$, $\tau_0 = 0.4$ and $w = 2 \times 10^3$

Fig. 7: Variation of hoop stress versus $R$ at varies values of magnetic field $H_0$ at $\tau_1 = 0.5$, $\tau_0 = 0.4$ and $w = 2 \times 10^3$

Fig. 8: Variation of temperature versus $R$ at varies values of magnetic field $H_0$ at $\tau_1 = 0.5$, $\tau_0 = 0.4$ and $w = 2 \times 10^3$
Fig. 9: Variation of radial displacement versus $R$ at varies values of mechanical relaxation time $\tau_0$ at $\tau_1 = 0.5$, $H_0 = 3 \times 10^2$ and $w = 2 \times 10^3$

Fig. 10: Variation of radial stress versus $R$ at varies values of mechanical relaxation time $\tau_0$ at $\tau_1 = 0.5$, $H_0 = 3 \times 10^2$ and $w = 2 \times 10^3$

Fig. 11: Variation of hoop stress versus $R$ at varies values of mechanical relaxation time $\tau_0$ at $\tau_1 = 0.5$, $H_0 = 3 \times 10^2$ and $w = 2 \times 10^3$

Fig. 12: Variation of temperature versus $R$ at varies values of mechanical relaxation time $\tau_0$ at $\tau_1 = 0.5$, $H_0 = 3 \times 10^2$ and $w = 2 \times 10^3$
Fig. 13: Variation of radial displacement versus $R$ at varies values of $w$ at $\tau_1 = 0.5$, $\tau_0 = 0.4$ and $H_0 = 3 \times 10^2$

Fig. 14: Variation of radial stress versus $R$ at varies values of $w$ at $\tau_1 = 0.5$, $\tau_0 = 0.4$ and $H_0 = 3 \times 10^2$

Fig. 15: Variation of hoop stress versus $R$ at varies values of $w$ at $\tau_1 = 0.5$, $\tau_0 = 0.4$ and $H_0 = 3 \times 10^2$

Fig. 16: Variation of temperature versus $R$ at varies values of $w$ at $\tau_1 = 0.5$, $\tau_0 = 0.4$ and $H_0 = 3 \times 10^2$
References


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