The $L_\infty$ ($L_1$) Farthest Line-Segment Voronoi Diagram*

Sandeep Kumar Dey and Evanthia Papadopoulou

Faculty of Informatics
USI - Università della Svizzera italiana
Lugano, Switzerland
email: {deys, evanthia.papadopoulou}@usi.ch

Abstract—We present structural properties of the farthest line-segment Voronoi diagram in the piecewise linear $L_\infty$ and $L_1$ metrics, which are computationally simpler than the standard Euclidean distance and very well suited for VLSI applications. We introduce the farthest line-segment hull, a closed polygonal curve that characterizes the regions of the farthest line-segment Voronoi diagram, and is related to it similarly to the way an ordinary convex hull relates to the farthest point Voronoi diagram. In $L_\infty$ (resp. $L_1$) the farthest line-segment hull, and thus the farthest line-segment Voronoi diagram, has size $O(h)$, where $h$ ranges from $O(1)$ e.g., axis parallel line-segments, to $O(n)$, and it can be constructed in time $O(n \log h)$. Once the $L_\infty$ (resp. $L_1$) farthest line-segment hull is available, the corresponding Voronoi diagram can be constructed in additional $O(h)$ time.

Keywords—farthest Voronoi diagram; line segments; convex hull; $L_p$ metric;

I. INTRODUCTION

Let $S$ be a set of $n$ simple geometric objects in the plane, called sites. The nearest-neighbor Voronoi diagram of $S$ is a subdivision of the plane into regions such that the region of a site $s$ is the locus of points closer to $s$ than to any other site. The farthest-site Voronoi diagram of $S$ is a subdivision of the plane into regions such that the region of a site $s$ is the locus of points farther away from $s$ than from any other site. Farthest-site Voronoi diagrams have received much less attention than their nearest neighbor counterparts. The farthest line segment Voronoi diagram was only recently considered in [2], showing properties surprisingly different than the ones of its counterpart for points. In particular, Voronoi regions may be disconnected and region emptiness is not characterized by convex hull properties. The farthest-polygon Voronoi diagram has been addressed in [5]. An abstract framework is given in [8].

In this note we study the structural properties of the farthest line segment Voronoi diagram in the simple piecewise linear $L_\infty$ metric\(^1\) (equiv. the $L_1$ metric), which is computationally simpler than the standard Euclidean distance and very appropriate for applications related to VLSI design automation. Line segments are arbitrary, i.e., they may cross or abut. We show that in $L_\infty$, the farthest Voronoi region of a single line segment may get disconnected into a constant number of disjoint faces, in particular at most five faces, and the overall size of the diagram need not be constant as in the case of points. We introduce the farthest line-segment hull (for brevity farthest hull), a closed polygonal curve that characterizes the faces and the unbounded bisectors of the farthest line-segment Voronoi diagram similarly to the way an ordinary convex hull relates to the farthest point Voronoi diagram. The concept of the farthest hull is applicable to the general $L_p$ metric, $1 \leq p \leq \infty$. For $p = 1, \infty$, the size of the farthest hull can vary from $O(1)$, as in the case of points, axis-parallel line segments, non-crossing line segments, to $O(n)$, and it can be constructed in $O(n \log h)$ time, where $h$ is the size of the farthest hull, in a simple manner. Once the farthest hull is available, the $L_\infty$ (resp. $L_1$) farthest line-segment Voronoi diagram can be constructed in additional $O(h)$ time.

Voronoi diagrams of line segments in the $L_\infty$ metric have nice properties that can be valuable in applications. Unlike its Euclidean counterpart, the $L_\infty/L_1$ line segment Voronoi diagram consists exclusively of straight line-segments, and Voronoi vertices are on rational coordinates [10] (assuming input on rational coordinates). The well known in-circle test, necessary in the construction of Voronoi diagrams, can be answered with a significantly lower algorithmic degree, namely degree $\leq 5$ compared to degree $\leq 40$ in the Euclidean case [10]; the degree of an algorithm is a measure that reflects the potential of an algorithm for a robust implementation [7]. As a result, the $L_\infty/L_1$ versions of line-segment Voronoi diagrams can result in implementations that are robust, yet considerably simpler and faster than their Euclidean counterpart, while still providing valuable proximity information. The area of VLSI layout and manufacturing is particularly well suited for such diagrams as VLSI shapes are in majority (although not exclusively) axis-parallel. Generalized $L_\infty$ Voronoi diagrams of line segments have proven powerful in addressing problems in this area, see e.g. [14], [12] and references therein. Note also that the $L_\infty$ Voronoi diagram of axis-parallel polygons coincides

\(^{1}\) The $L_\infty$ distance $d(p, q)$ between two points $p = (x_p, y_p)$ and $q = (x_q, y_q)$ is the maximum between the horizontal and the vertical distance between $p$ and $q$ i.e., $d(p, q) = \max\{d_x(p, q), d_y(p, q)\}$, where $d_x(p, q) = |x_p - x_q|$ and $d_y(p, q) = |y_p - y_q|$. The $L_\infty$ distance between a point $p$ and a line $l$ is $d(p, l) = \min\{d(p, q) : q \in l\}$.
with their \textit{straight-line skeleton} \cite{1}.

The $L_\infty$ farthest segment Voronoi diagram is necessary for computing the $L_\infty$ Hausdorff Voronoi diagrams of clusters of line segments, which finds applications, among others, in solving the \textit{geometric min-cut problem} and accurately computing \textit{critical area} in VLSI designs in the presence of redundancy \cite{12}, \cite{13}. Note that the smallest disk overlapping a set of wires on a VLSI layer can model defects responsible for open faults. This work has been motivated exactly by the need for the farthest line-segment Voronoi diagram in the above problems. For a direct application of the $L_\infty$ Hausdorff Voronoi diagram in VLSI critical area extraction for open faults see e.g., \cite{12}.

\section{Structural properties}

Let $S = \{s_1, \ldots, s_n\}$ be a set of $n$ arbitrary line segments in the plane. Segments are allowed to cross or touch at single points. The $L_\infty$ farthest segment Voronoi region of a line segment $s_i \in S$ is defined as

$$reg(s_i) = \{x \in \mathbb{R}^2 \mid d(x, s_i) \geq d(x, s_j), 1 \leq j \leq n\}$$

where $d(x, s_i)$ denotes the $L_\infty$ distance between a point $x$ and a line segment $s_i$, which is the minimum $L_\infty$ distance between $x$ and any point on $s_i$. \cite{2}. The collection of all such regions formed by the line segments in $S$, together with their bounding edges and vertices, defines the $L_\infty$ farthest-segment Voronoi diagram of $S$, for short $FVD(S)$. Fig.1 illustrates an example.

A Voronoi edge bounding regions $reg(s_i)$, $reg(s_j)$ is portion of the bisector $b(s_i, s_j)$, which is the locus of points equidistant from $s_i, s_j$. In $L_\infty$, $b(s_i, s_j)$ consists of a constant number of pieces of straight lines, where each piece is portion of an elementary bisector between the endpoints and the open line-segment portions of $s_i$ and $s_j$. The $L_\infty$ bisector of two non-parallel lines $b(l_1, l_2)$ of slopes $b_1, b_2$ consists of two straight line branches of slopes as given in \cite{10}. If $b_1, b_2$ are both positive (resp. negative) then one branch is always a $-1$-slope (resp. $+1$-slope) line. When two line-segments or their endpoints are aligned along the same vertical or horizontal line their bisector consists of a line segment and two unbounded regions. The equidistant region can be assigned arbitrarily (or lexicographically) to one of the points and consider the region boundary only as their bisector. For more information on $L_\infty$ line-segment bisectors, see \cite{10}. Note that the endpoints and the open line-segment portion of an arbitrary line-segment $s$ can be separated by $\pm1$-slope lines of opposite sign of the slope of $s$.

A maximally connected subset of a region in $FVD(S)$ is called a \textit{face}. A halfplane bounded by an axis-parallel line is called an \textit{axis-parallel halfplane}. The common intersection of two axis parallel halfplanes with perpendicular bounding lines is called a \textit{quadrant}. The following lemma and its corollary is a simple adaptation of Lemma 1 and Corollary 2 of \cite{2} for the $L_\infty$ metric.

\begin{lemma}
All faces of $FVD(S)$ are bounded in one of the eight possible directions that are implied by rays of slope $\pm1, 0, \infty$.
\end{lemma}

\begin{proof}
Let $t$ be a point in an arbitrary face $f$ of $FVD(S)$ belonging to the region of segment $s_i$. Then there is a square disk $D(t)$ centered at $t$, touching $s_i$, that is also intersecting (or touching) all segments $s_j \in S - \{s_i\}$. If $D(t)$ touches $s_i$ with a vertical (resp. horizontal) side let $R$ be a horizontal (resp. vertical) ray starting at $t$ directed away from $s_i$. If $D(t)$ touches $s_i$ with a corner at a single point $p$, let $R$ be the $\pm1$-slope ray through the incident diagonal of $D(t)$, starting at $t$ and directed away from $p$. Since for any point $y$ along $R$, $D(t) \subset D(y)$, and $D(y)$ touches $s_i$ in exactly the same way as $D(t)$, $D(y)$ must continue to intersect all other segments in $S$ and thus, $R$ must be entirely contained in $reg(s_i)$ and in particular $f$. Thus, face $f$ must be unbounded along the direction of $R$.
\end{proof}

\begin{corollary}
The interior of $reg(s_i)$ is non-empty if and only if there exists an axis-parallel halfplane or a quadrant $L$ which touches $s_i$ and intersects (or touches) all other segments in $S$. If $L$ is an axis-parallel halfplane then $reg(s_i)$ is unbounded in the direction in $L$ normal to the bounding line of $L$. If $L$ is a quadrant touching $s_i$ by its corner then $reg(s_i)$ is unbounded in the $\pm1$ direction away from the corner of $L$.
\end{corollary}

It is clear by Corollary 1 that $FVD(S)$ must always contain four faces, denoted as \textit{north}, \textit{south}, \textit{east} and \textit{west}, such
that each face is unbounded in one of the four axis-parallel directions respectively. Each one of these faces is induced by a halfplane bounded by an axis-parallel line as defined below.

**Definition 1.** Let $l_i, i = n, s, e, w$ be four axis parallel bounding lines of $S$ defined as follows: Let $l_n$ (resp. $l_s$) be the horizontal line passing through the bottommost upper-endpoint (resp. the topmost lower-endpoint) of all segments in $S$. Let $l_e$ (resp. $l_w$) be the vertical line passing through the leftmost right-endpoint (resp. the rightmost left-endpoint) of all segments in $S$.

Clearly the north, south, east, and west face of $FVD(S)$ is induced by line $l_n, l_s, l_e, l_w$ respectively; the $L_\infty$ distance within each face simplifies to the vertical or horizontal distance from the respective bounding line. Fig. 2 illustrates the bounding lines. The bounding lines partition the plane into four quadrants, labeled 1–4, in counterclockwise order as follows (see Fig. 2): Quadrant 1 is formed by $(l_s, l_w)$ and faces north-east, Quadrant 2 is formed by $(l_s, l_e)$, Quadrant 3 by $(l_n, l_e)$, and Quadrant 4 by $(l_n, l_w)$ facing south-east. The closed rectangular regions induced by the four bounding lines is denoted by $R$.

It is clear, by Corollary 1, that no segment $s$ that lies partially or entirely in the closed rectangular region $R$ can have a non-empty Voronoi region in $FVD(S)$. Thus, these segments can be immediately discarded. Among the remaining segments, it is clear, by definition, that no such segment can have endpoints in the four quadrants. The quadrants are thus, either empty of the remaining segments, or there are segments that straddle them entirely.

**Definition 2.** Let $E_i, i = 1, 2,$ denote the upper envelope of the set of line segments straddling Quadrant $i$. Let $E_j, j = 3, 4$ denote the lower envelope of the segments straddling quadrant $j$. In case no segments straddle quadrant $i$, $i = 1, \ldots , 4$, let $E_i = O_i$, where $O_i$ is the corner point (origin) of Quadrant $i$.

We now define the **farthest line-segment hull** of $S$, a closed polygonal curve that characterizes the regions and the unbounded bisectors of $FVD(S)$. Figure 3 illustrates $FH(S)$ for an arbitrary set of segments in its standard form.

**Definition 3.** The farthest line-segment hull of $S$ (in short farthest hull, denoted $FH(S)$) is the closed polygonal curve obtained by $E_1, l'_s, E_2, l'_e, E_3, l'_n, E_4, l'_w$, where $l'_i$ denotes the segment along bounding line $l_i$ between its two incident envelopes.

For a set of points or a set of axis-parallel segments, the farthest-hull simplifies to the rectangle $R$ of the bounding lines. In the case of points, $R$ coincides with the minimum enclosing rectangle of the given set of points, which is well known to characterize the $L_\infty$ farthest point Voronoi diagram (see Fig. 4). For a set of axis parallel segments, the farthest-hull simplifies to the rectangle $R$ induced by the bounding lines, however, the placement of the bounding lines need not always be standard, i.e., $l_n$ may lie above $l_s$ or left to the right of $l_w$ as in Fig. 5. Fig. 12 in the Appendix illustrates similar non standard situations. The farthest hull is defined in the same way in all cases.

By Corollary 1 and the definition of the farthest hull we conclude.

**Lemma 2.** A line segment $s_i$ in $S$ has a non-empty Voronoi region in $FVD(S)$ if and only if it appears on the farthest hull, either explicitly as a segment on some envelope or implicitly by defining a bounding line. Voronoi faces are circularly ordered following the order of the farthest hull.

Let $e_i, e_j$ be two edges of the farthest hull corresponding
to segments \( s_i, s_j \). Let \( l_i, l_j \) denote the lines through \( s_i \) and \( s_j \) respectively and let \( H(e_i), H(e_j) \) denote the corresponding halfplane on the side of \( l_i, l_j \) containing \( R \). The unbounded bisector of \( e_i, e_j \), denoted \( b'(e_i, e_j) \), is the portion of \( b(l_i, l_j) \) that is relevant to \( FVD(S) \), that is, \( b'(e_i, e_j) \) is the ray of \( b(l_i, l_j) \) in \( H(e_i) \cap H(e_j) \). In Fig. 6, unbounded bisectors of consecutive farthest hull edges are indicated by arrows.

**Lemma 3.** There is a 1-1 correspondence between the unbounded Voronoi edges of \( FVD(S) \) and the vertices of the \( L_\infty \) farthest hull. All unbounded Voronoi edges are rays of slope \( \pm 1 \) partitioned into four groups, one group for each envelope of the farthest hull, each group being a set of parallel rays.

**Proof:** Let \( v_1, v_2, \ldots, v_h \) denote the list of vertices along \( FH(S) \) in, say, counterclockwise order, starting at \( E_1 \cap l_w \). Each vertex \( v_i \) corresponds to an unbounded bisector \( b'(e_i, e_{i+1}) \), where \( e_i, e_{i+1} \) are the farthest hull edges incident to \( v_j \) (see Fig. 6). By definition of the farthest hull, the quadrant \( L \) induced by any vertex \( v \) on the farthest hull that contains \( R \) must touch the two segments that explicitly or implicitly define \( v \) and it must intersect all other segments in \( S \). Thus, every vertex of \( FH(S) \) corresponds to an unbounded Voronoi edge. Conversely, for any unbounded Voronoi edge of \( FVD(S) \), portion of bisector \( b(s_i, s_j) \), there is a quadrant \( L \) that touches \( s_i, s_j \) and intersects all other segments in \( S \). In all cases the corner point of \( L \) corresponds to a vertex of the farthest hull (by the definition of the farthest hull).

As shown in [2] for the euclidean case, the graph structure of \( FVD(S) \) is connected and corresponds to a tree. This property clearly remains valid in \( L_\infty / L_1 \) following the same arguments. By the above discussion we conclude.

**Theorem 1.** The \( L_\infty \) farthest segment Voronoi diagram consists of exactly one unbounded face for each edge of the farthest hull and has size \( O(h) \), where \( h \) is the size of the farthest hull that can vary from \( O(1) \) to \( O(n) \).

**Remarks.** The \( L_\infty \) \( FVD(S) \) always consists of four faces, facing north, south, east, and west, each induced by one bounding line, \( l_n, l_s, l_e, l_w \) respectively. In addition, for any non-trivial envelope, it contains one face for each segment of the envelope, always bounded by parallel slope-\( \pm 1 \) rays. A segment may contribute to at most two different envelopes and thus, it may appear twice on the farthest hull. It may also contribute to both a bounding line and its incident envelope or induce a number of the bounding lines. Degenerate instances of the farthest-hull may arise, when parallel bounding lines coincide (see e.g Fig. 7). For non-crossing segments the envelopes can clearly consist of at most one segment each, thus, the \( L_\infty \) \( FVD(S) \) has \( O(1) \) size. Fig. 8 illustrates the farthest hull of non-crossing segments forming a simple polygon.

**Corollary 2.** The region of a segment in \( FVD(S) \) may consist
of a constant number of disjoint faces. The maximum number of disjoint faces for a single segment is five.

Proof: A segment may contribute an edge to at most two envelopes and it may induce at most four bounding lines. However, any two edges of the farthest hull that are adjacent produce neighboring regions in FVD(S), which, if they are induced by the same segment, correspond to a single face. By definition, a segment may contribute to two envelopes only if it contributes their first or last edge. Thus, the maximum number of disjoint faces for a single segment is five, at most four for the bounding lines, plus one for a single potential envelope edge that is not incident to bounding lines.

Fig. 13 illustrates an example of a segment having exactly five disjoint faces, where four of those five faces are the standard faces of the bounding lines. This segment induces all four bounding lines plus it contributes one edge to an envelope that is not adjacent to any bounding line.

In the presence of multiple segments or multiple segment endpoints aligned along the same axis-parallel line, there may be a number of segments inducing a bounding line. Let S(l_i) be the set of segments inducing l_i, where i = n, s, e, w. Then reg(l_i) is equidistant from all segments in S(l_i). In this case, a convention regarding the ownership of the region may be adapted. For example, we can assign reg(l_i) to one segment in S(l_i) arbitrarily or to one segment according to a lexicographic order. Alternatively, we can attribute reg(l_i) to l_i and characterize it as a region common to all segments in S(l_i). The choice of convention depends on the needs of the application. Our choice is the latter convention as it avoids any artificial partitioning of equidistant regions.

III. CONSTRUCTION ALGORITHM

Following [2], the algorithm to construct FVD(S) proceeds in two steps:

1) Construct the farthest hull FH(S). Given FH(S), construct the circular list C of all unbounded bisectors
2) Given C, construct FVD(S).

Step 1 takes $O(n \log h)$ time, where $h$ is the size of the farthest hull. In particular, the bounding axis parallel lines $l_n$, $l_s$, $l_e$, and $l_w$ can be constructed in $O(n)$ time. The envelopes of the segments straddling the four quadrants can be constructed by any convex hull type algorithm, thus, in $O(n \log h)$ time following [6]. The circular list $C$ of all unbounded bisectors is derived in additional $O(h)$ time. Step 2 can be constructed in $O(h)$ time as a simplification of the algorithm in [2] for the $L_\infty$ metric.

The algorithm in [2] is based on the observation that vertices of FVD(S) can be discovered in order of decreasing weight, where the weight of any point $v$ along FVD(S) is the radius of the disk (a square in $L_\infty$) centered at $v$ passing through the line segments whose bisector induces $v$ (see also [11]). Intuitively, the tree structure of FVD(S), denoted $T(S)$, can be regarded as a rooted tree, rooted at the point of minimum weight $v_{\min}$ (in $L_\infty$, $v_{\min}$ may be an entire axis-parallel segment), where $v_{\min}$ is the locus of centers of the minimum size disk intersecting all segments in $S$. Along any path of the rooted $T(S)$ (except the root in $L_\infty$) the weight of nodes is strictly increasing. Thus, we can compute $T(S)$ in a bottom up fashion, always proceeding in order of decreasing weight, until the node of minimum weight is reached.

In the $L_\infty$ metric the implementation is particularly simple: There are only four pairs of neighboring bisectors that may intersect; all other unbounded bisectors are parallel rays of slope ±1. As a result, the intersection of maximum weight can be determined in $O(1)$ time. We simply maintain a set $V$ of size four of all possible intersections. Given a node of maximum weight $v$ let $b'(e_i, e_k)$ and $b'(e_k, e_j)$ be the neighboring bisectors inducing $v$. Once $v$ is selected, delete $b'(e_i, e_k)$ and $b'(e_k, e_j)$ from $C$ and substitute them with the new bisector $b'(e_i, e_j)$; compute the intersections of $b'(e_i, e_j)$ with its two neighboring bisectors in $C$, choose the one of larger weight, and insert it in $V$. The algorithm is summarized in Algorithm 1. Its asymptotic complexity is clearly optimal in the worst case.

Theorem 2. Given a set of arbitrary segments $S$, the farthest hull of $S$ can be constructed in $O(n \log h)$ time. Given the farthest hull, the $L_\infty$ farthest segment Voronoi diagram of $S$ can be constructed in additional $O(h)$ time, where $h$ is the number of edges on the farthest hull. $h$ can vary from $O(1)$ to $O(n)$.

It is well known that the $L_1$ metric is equivalent to $L_\infty$ under $45^\circ$ rotation. Figure 9 illustrates the farthest hull of $S$ in the $L_1$ metric. Unbounded bisectors in the $L_1$ version of FVD(S) are axis-parallel rays. The construction is equivalent.

IV. EXTENSIONS IN THE EUCLIDEAN METRIC

The concept of the farthest hull can extend to the Euclidean metric, more generally to the $L_p$ metric, $1 \leq p \leq \infty$. 
Algorithm 1 CONSTRUCT-FVD(S)

INPUT: A set S of n arbitrary line segments.
OUTPUT: FVD(S).
CONSTRUCT-FVD(S)
1. Compute the farthest-hull of S, FH(S).
2. From FH(S) construct the circular list C of unbounded bisectors.
3. Identify the four pairs of intersecting neighboring bisectors, compute their intersection point, and insert it in the intersection set V.
4. While (C is not empty)
   4.a. Report the intersection point (Voroni vertex) in V with maximum weight. Let b_1, b_2 be the two bisectors in C inducing the intersection
   4.b. Compute a new bisector b_3 as defined by the two non-common edges of the farthest hull segment inducing of b_1 and b_2.
   4.c. Compute the intersection points of the new bisector with its two neighboring bisectors, choose the one of maximum weight and insert it in V.
   4.d. Delete b_1, b_2 from C and insert b_3 in their position.
end While

Figure 9. Farthest hull in the $L_1$ metric

It corresponds to a closed polygonal curve that encodes the unbounded bisectors of the corresponding Voronoi diagram maintaining their cyclic order. Information on the Euclidean farthest line-segment hull will appear in a companion paper (see e.g. [9] for a short abstract). For completeness and comparison purposes with the $L_\infty$ metric we review here the Euclidean definition. The Euclidean definition remains identical in $L_\rho$, $1 < \rho < \infty$, due to properties of $L_\rho$ bisectors shown in [3].

An endpoint $p$ of a segment in $S$ is a farthest hull vertex if there is an open halfplane induced by a line $l$ through $p$, denoted $H(l)$, that intersects all segments in $S$, except $s$ (and possibly except additional segments incident to $p$). Similarly, a line segment $s \in S$ is a farthest hull segment if a halfplane $H(l)$ induced by the line $l$ through $s$ intersects all segments in $S \setminus \{s\}$. The line segment $\overline{pq}$ joining two hull vertices $p, q$ is a supporting segment of $S$ if and only if an open halfplane induced by the line $l$ through $\overline{pq}$, denoted $H(\overline{pq})$, intersects all segments in $S$ except segments incident to $p, q$. The Euclidean farthest line-segment hull of $S$ is the closed polygonal curve obtained by the sequence of hull segments in $S$, ordered according to their slope, interleaved by chains of supporting segments that join their endpoints. Fig. 10(a) illustrates the farthest line-segment hull in thick dashed lines.

The cyclic ordering of the Euclidean farthest hull is revealed by its Gaussian map. Let the unit vector of a farthest hull edge $e$ be the unit normal of the line $l$ through $e$ pointing away from halfplane $H(l)$. The Gaussian map is a mapping of the farthest hull onto the unit circle $K_o$, such that every edge $e$ is mapped to a point on $K_o$ as obtained by its unit vector, and every vertex is mapped to one or more arcs as delimited by the unit vectors of the incident edges (see Fig. 10). Using the Gaussian map, efficient algorithms to compute the Euclidean farthest line-segment hull, and thus the farthest line-segment Voronoi diagram, can be derived.

The Gaussian map can be useful in identifying the cyclic order of the farthest hull in $L_\infty$ as well in case of degenerate cases. Fig. 11 shows the Gaussian map of the degenerate $L_\infty$ farthest hull of Fig 7b.

V. CLOSING REMARKS

We presented the structural properties of the farthest line-segment Voronoi diagram in the $L_\infty$ (resp. $L_1$) metric.
Our study was motivated by applications in VLSI design automation. We introduced the $L_\infty$ farthest line segment hull, a closed polygonal curve that fully characterizes the unbounded bisectors of the $L_\infty$ farthest line-segment Voronoi diagram. Combinatorial results and algorithms to compute farthest line-segment hull in the $L_p$ metric, $1 < p < \infty$, are given in a companion paper.

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**REFERENCES**


**APPENDIX**

Figure 12. The $L_\infty$ farthest hull for different positions of $l_n$, $l_s$, $l_e$ and $l_w$. (a) Standard position (b) $l_s$ lies above $l_n$ (c) $l_e$ lies to the right of $l_w$ (d) $l_s$ lies above $l_n$, as well as $l_e$ lies to the right of $l_w$

Figure 13. Example showing a segment (in purple) whose farthest Voronoi region consists of five disconnected faces.