Abstract—Nowadays networks are characterized by the abundance of diverse Radio Access Interfaces (RAI) in the same operating area. The various wireless interfaces can belong to the same Radio Access Technology or not. In such a scenario, a mobile user will be able to select selfishly one of the available radio interfaces in order to enhance its own performance. Therefore, the radio access selection policy is vital and must be designed astutely to avoid resource wastage. In this paper, the RAI selection process is apprehended as a congestion game which is a class of noncooperative games in which users share a common set of limited resources. The cost sustained by a given user depends upon the congestion impact inflicted by other users sharing the same resource. Devising distributed resource sharing schemes that optimize user air interface selection depends crucially on the existence of Nash equilibria for the modelling congestion games. In this paper, we model the downlink access for three main broadband technologies (WiMAX, WiFi and 3.5G HSDPA) and study the existence of pure Nash equilibria for various multi-RAI scenarios involving those technologies.

I. INTRODUCTION

The migration of wireless networking towards the 4G era is distinguished by the proliferation of Radio Access Interfaces that can potentially belong to different technologies. As no existing technology can be surrogated by another one, the coexistence of today wireless networks is the best solution at hand to deal with the incessantly growing user demands. Hence, mobile users will be offered seamless and ubiquitous connectivity for wireless broadband services. The main issue, in this composite radio environment, is which RAI to opt for among all that are available. Furthermore, radio interface selection can be either manoeuvred in a centralised way [19] or in a distributed way. Here, we tackle the problem from the user point of view as each mobile user only strives to minimize its own individual cost. In this competitive environment, resorting to congestion games is quite suitable as they model the congestion externalities that arise when users compete for limited resources. Devising optimal radio access selection schemes depends on the existence of Nash equilibria for these congestion games. In this paper, the downlink channel of major 4G technologies for data applications are considered: WiMAX [1], HSDPA [2] and WiFi [3].

The goal of this paper is twofold: first, we matched existing results of congestion game theory to noncooperative RAI selection, profiting from the existence of pure Nash equilibria for particular wireless interface combinations (WiMAX-WiMAX, WiMAX-HSDPA, HSDPA-HSDPA, WiFi-WiFi). Second, we proved the existence of pure Nash equilibrium for a special two RAI case each belonging to a different technology: WiMAX-WiFi and HSDPA-WiFi.

In ([7], [8], [9], [10]), noncooperative game theory is used for radio resource management. In [7], the available bandwidth is shared in a distributed manner between heterogeneous networks. The approach is based on a two-players finite game that is played by access networks (which is rather a centralised approach). In [8], users in a cell are divided into two groups depending on their location in the cell. The RRM scheme is conducted as a finite two-player game played among these two groups which induces some kind of collaboration between users. In [9], the selection between WiFi and UMTS is formulated as a symmetric noncooperative game that reduces to a threshold policy. The work in [10] presents a load balancing scheme between WiMAX and WiFi and lies on the maximization of some marginal cost pricing where the network notifies users of some reward to enhance global performances.

To our knowledge, this paper is the first to portray distributed RAI selection as a congestion game.

The rest of the paper is organized as follows. The game framework is described in Section II. The system model and cost characterization in WiMAX, HSDPA and WiFi are given in Section III. The radio access selection scheme is presented as a congestion game in Section where many scenarios are handled (unweighted congestion games in IV-A, weighted congestion games in IV-B). Practical issues are considered in Section V and conclusions are given in Section VI.

II. CONGESTION GAME FRAMEWORK FOR RAI SELECTION

Game theory models the interactions between players competing for a common resource. Hence, it is well adapted to radio resource management modelling. For each state of the system, defined by the number of mobile users $n$, we define a multi-player congestion game $G$ between those $n$ mobile users present in an area with $m$ RAI.

In this model, there is a sequence of one-stage games, each corresponding to a given state of the system, defined by the number of mobile users. Whenever a new mobile is admitted in the system, the game is played again with an additional player. We assume that mobiles have complete information on each other. Mobile users are assumed to make their decisions without knowing the decisions of each other. In this paper, players in a congestion game may differ from one another in their intrinsic preferences (e.g., the benefit they get from using a specific RAI), their contribution to congestion, or both.
We present here the general framework:

1) $M = \{1, \ldots, m\}$ is the set of available RAIs.
2) $N = \{1, \ldots, n\}$ is the set of players. Each user is a player that has to pick one RAI among the $m$ available RAIs.
3) $x(k)$: strategy of player $k$. $x(k) = s$ if player $k$ chooses RAI $s$. Hence, $x = (x(k))_{k \in N} \in S = \{1, 2, \ldots, m\}^N$ is a strategy profile. $S$ is the space of all profiles. $N_s \subset N$ is the subset of players connected to RAI $s$.
4) $n_s$ is the number of players that chose RAI $s$.
5) $c_{k,s}(N_s)$ is the cost of player $k$ when choosing RAI $s$ given the subset of players that chose the same RAI (if $k \notin N_s$ then $c_{k,s}(N_s) = 0$).

A. The Cost Function

In the present context, the cost is player specific and defined in a separable form in the sense of [13]. Each player $k$ has a weight or congestion impact $w(k, s)$. Consequently, the cost function is defined as follows:

$$c_{k,s} = a_{k,s} \cdot l(n_w(s))$$

where $n_w(s) = \sum_{k \in N_s} w(k, s)$; $a_{k,s}$ is a positive constant that represents the base cost for user $k$ of selecting RAI $s$ and $l$ is a nondecreasing function in $n_w(s)$ that gives the crowding cost in RAI $s$.

B. The Nash Equilibrium

In a noncooperative game, an efficient solution is that where all players adhere to a Pure Nash Equilibrium (PNE) which is a profile of strategies in which no player will profit from deviating its strategy unilaterally. Hence, it is a strategy profile where each player’s strategy is an optimal response to the other players’ strategies.

In general, finite congestion games are not guaranteed to have a PNE. Nevertheless, they possess a mixed Nash equilibrium where each player has to continually change its RAI selection according to a distribution probability over the strategy set. Mixed equilibria have practical issues. They lead to a situation where each mobile user has to be simultaneously connected to more than one RAI and to split its downlink traffic over those RAIs. Whenever a PNE exists, an equilibrium can be reached where every mobile user is consigned to only one RAI. In this work, we will show that numerous hybrid broadband networks scenarios have a PNE.

C. The Finite Improvement Path Property

A path is a sequence of strategy profiles in which each strategy profile differs from the preceding one in only one coordinate. When the unique player that deviates in each step strictly decreases its cost, the path is called an improvement path. Hence, an improvement path is generated by myopic players. A finite congestion game has the finite improvement path property (FIP) if every improvement path is finite [12].

Obviously, any maximal improvement path (an improvement path that cannot be extended) is terminated by an equilibrium. Therefore, in a finite game with the FIP property, players are guaranteed to reach a PNE when they asynchronously and selfishly switch to a different RAI to reduce their cost.

III. THE SYSTEM MODEL ON THE DOWNLINK

The goal of the article is to enable appropriate RAI selection in a fully distributed fashion. We model the downlink capacity of major technologies in 4G systems: WiMAX termed RAI $WX$, WiFi termed RAI $WF$ and HSDPA termed RAI $PF$ in reference to the Proportional Fair algorithm that governs its behaviour.

The proposed models provide the expression for the mean downlink data rate of a mobile user when connected to a given RAI. The inverse of the mean data rate will be the expected service duration, corresponding to the time necessary to send a unit of data. The latter will be designated as the cost waged by each mobile user in a given technology.

To be compatible with the previous game notations, we use the index $s$ to designate RAI $s$. $n_s$ is then the number of mobile users in RAI $s$. In the next subsections, we compute the mean data rate in each technology.

In WiFi, WiMAX and HSDPA standards, the set of achievable rates is not continuous. Indeed, coding constraints result in a discrete set of achievable peak rates $\chi_{1,s} < \chi_{2,s} < \ldots < \chi_{M_s,s}$ where $M_s$ is the maximum number of achievable rates for RAI $s$ belonging to a given technology (see Table I).

The instantaneous rate $R_{k,s}(t)$ that user $k$ can obtain when connected to a given RAI $s$ depends on its location and varies with time $t$ due to fading effects (mobility is not taken into account). Hence, we have the following:

$$R_{k,s}(t) = x_k(t) \cdot \chi_{k,s}$$

where $x_k$ are i.i.d. random variables (of unit mean) that represent the impact of fast fading experienced by mobile user $k$. They follow an exponential distribution as we consider Rayleigh fading [4]. As we assume stationarity, the time index will be omitted in what follows.

1) WiMAX ($s=WX$): The wireless resource is time-shared between all the $n$ mobile users. We consider a Fair Time sharing model where active users are given the same chance to access resources. Therefore, the data rate of mobile user $k$ assigned to RAI $WX$ is given by:

$$P_{k,WX} = \frac{R_{k,s}}{\sum_{i=1}^n \mathbb{1}_{\{i \in WX\}}} = x_k \frac{\chi_{k,s}}{n_s}$$

As the fading mean equals 1, the mean data rate becomes:

$$E[P_{k,WX}] = \frac{\chi_{k,s}}{n_s}$$

2) HSDPA ($s=PF$): Here, the wireless resource is time-shared in an opportunistic manner among the $n$ mobile users. At time slot $t$, the PF algorithm schedules the user with the highest instantaneous rate relative to its average throughput,

$$k^* = \arg \max_k \left[ \frac{R_{k,s}}{T\chi_{k,s}} \right]$$
The average throughput is proportional to the data rate $T_h_{k,s} = \kappa \cdot \chi_{k,s}$, where $\kappa$ is some constant (see [11] for details).

Thus, the data rate of mobile user $k$ assigned to RAI $P_F$ is given by:

$$R_{k}^{P_F} = \frac{R_{k,s} \cdot \Pi \{ \frac{R_{k,s}}{T_h_{k,s}} = \max_{1 \leq n, s} \frac{R_{k,s}}{T_h_{k,s}} \}}{\Pi \{ \frac{R_{i,s}}{T_h_{i,s}} = \max_{1 \leq n, s} \frac{R_{i,s}}{T_h_{i,s}} \}}$$

We deduce the mean data rate $R_{k}^{P_F}$ for user $k$:

$$\mathbb{E}[R_{k}^{P_F}] = \mathbb{E}\{\chi_{k,s} \cdot x_k \cdot \Pi \{ x_k = \max_{1 \leq n} x_l \} \}$$

$$= \mathbb{E}\{\chi_{k,s} \cdot \mathbb{E}[x_k | x_k = \max_{1 \leq n} x_l] \}$$

$$\cdot \mathbb{P}\{ x_k = \max_{1 \leq n} x_l \}$$

$$= \frac{\chi_{k,s}}{n_s} \cdot \mathbb{E} [\max_{1 \leq n} x_1, \ldots, x_{n_s}]$$

We denote by $G(n_s)$ the ratio of what the user receives in PF as compared to a plain fair access scheduling. For Rayleigh fading, $G(n_s) = \mathbb{E} [\max_{1 \leq n} x_1, \ldots, x_{n_s}] = \sum_{i=1}^{n_s} \frac{1}{\chi_{s,i}}$. We conclude that the mean data rate in PF is:

$$\mathbb{E}[R_{k}^{P_F}] = \frac{\chi_{k,s}}{n_s} \cdot G(n_s)$$

3) WiFi ($s=WF$): In this paper, the uplink traffic is neglected which leads to a fair access scheme on the downlink channel. However, when a low rate user captures the channel, it will use it for a long time which penalizes high rate users and reduces the fair access strategy to a case of fair rate sharing (assuming a constant MAC frame size and neglecting the 802.11 waiting times (i.e., DIFS, SIFS, ...) in comparison with transmission times). Consequently, the data rate of a WiFi user is given by:

$$R_{k,s}^{WF} = \left[ \sum_{i=1}^{n_s} \frac{\Pi \{ i \ in \ WF \}}{\Pi \{ i \ in \ WF \}} \right]^{-1} \left[ \sum_{i=1}^{n_s} \frac{1}{\chi_{i,s} \cdot x_i} \right]^{-1}$$

Using the Jensen inequality, the mean data rate in RAI $WF$ has the following lower bound:

$$\mathbb{E}[R_{k}^{WF}] \geq \left[ \sum_{i=1}^{n_s} \frac{1}{\chi_{i,s}} \right]^{-1}$$

In the remaining, the mean data rate in RAI $WF$ will be approximated by its lower bound.

We conclude this section by deriving the cost $c_{k,s}$ suffered by a user $k$ in RAI $s$ as the inverse of its mean data rate:

$$c_{k,s} = \frac{1}{\mathbb{E}[R_{k}^{s}]}$$

IV. MULTI-ACCESS SCENARIOS

We consider in this section various scenarios entailing a hybrid radio environment where $m$ RAI’s cover, with partial or total overlapping, a given geographical area (see Figure 1) containing $n$ mobile users. Each mobile user can be connected to at least one RAI.

All multi-access Scenarios considered are portrayed by congestion games with separable cost function. They can be divided into two classes depending on the form of the cost function.

- Unweighted congestion games, where the cost depends on the number $n_s$ of other players connected to the same RAI $s$.
- Weighted congestion games, where the cost not only depends on $n_s$ but on the different impact each player has in a given RAI.

### TABLE I

<table>
<thead>
<tr>
<th>$k$</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\chi_{k,WX}$</td>
<td>7.9</td>
<td>16.3</td>
<td>24.4</td>
<td>32.5</td>
<td>48.8</td>
<td>65.1</td>
<td>73.2</td>
</tr>
<tr>
<td>$\chi_{k,PF}$</td>
<td>1.2</td>
<td>1.8</td>
<td>3.6</td>
<td>7.2</td>
<td>10.1</td>
<td>14.0</td>
<td>-</td>
</tr>
<tr>
<td>$\chi_{k,WF}$</td>
<td>-</td>
<td>2</td>
<td>5.5</td>
<td>11</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

TABLE I

THE SET OF DISCRETE PEAK RATES (MBITS/S) IN WiMAX, HSDPA, WiFi

A. Unweighted Congestion Games

The cost a player receives for playing a particular strategy (selecting a particular RAI) depends only on the total number of players playing the same strategy and decreases with that number in a manner which is specific to the particular player. It is shown in [15] that each game in this class possesses at least one Nash equilibrium in pure strategies as they have the FIP property. This class of games is handled in [15] which is a generalisation of congestion games first presented by Rosenthal [16] (where costs are not player specific).

1) WiMAX-WiMAX Scenario: we consider the case where all $m$ RAI’s are WiMAX BSs (Base Stations). Equations 2 and 7 yield the following cost function for player $k$ choosing WiMAX BS $s$:

$$c_{k,s} = \frac{1}{\chi_{k,s} \cdot n_s}$$

where the basic cost is $a_{k,s} = 1/\chi_{i,s}$, $\forall i \in N_s$.
2) **HSDPA-HSDPA Scenario**: we consider the case where all m RAIs are HSDPA BSs. Equations 5 and 7 yield the following cost function for player k choosing HSDPA BS s:

\[ c_{k,s} = \frac{1}{\chi_{k,s} \cdot G(n_s)} \cdot n_s \tag{9} \]

where the base cost \( a_{k,s} = \frac{1}{G(n_s)} \cdot \chi_{i,s}, \forall i \in N_s \).

3) **WiMAX-HSDPA Scenario**: in this scenario, there are \( m_1 \) WiMAX BSs and \( m_2 \) HSDPA BSs such as \( m = m_1 + m_2 \). The cost function amounts to:

\[ c_{k,s} = \begin{cases} \frac{n_s}{\chi_{k,s} \cdot G(n_s)} & \text{for } s = \{1, \ldots, m_1\} \\ \frac{n_s}{\chi_{k,s} \cdot G(n_s)} & \text{for } s = \{1, \ldots, m_2\} \end{cases} \tag{10} \]

Here, the base cost varies across resources:

- \( a_{k,s} = \frac{1}{\chi_{i,s} \cdot G(n_s)} \) if user \( i \) selects a HSDPA RAI
- \( a_{k,s} = \frac{1}{\chi_{i,s}} \) if user \( i \) selects a WiMAX RAI

### B. Weighted Congestion Games

Unlike standard congestion games, weighted congestion games do not necessarily possess pure Nash equilibria. However, such equilibria always exist in certain subclasses of these games, in which, moreover, any sufficiently long sequence of myopic unilateral moves by players is bound to lead to an equilibrium.

1) **WiFi-WiFi Scenario**: This scenario involves \( m \) RAIs that are WiFi access points (APs). Each user can connect to one or more AP. The cost function of a player \( k \) choosing AP \( s \) can be directly derived from equations 6 and 7:

\[ c_{k,s} = \sum_{i=1}^{n_s} \frac{1}{\chi_{i,s}} \tag{11} \]

Therefore, the base costs are all equal to 1 while the cost of player \( i \) connected to AP \( s \) is given by \( w(i,s) = \frac{1}{\chi_{i,s}} \). It is obvious that players connected to the same AP \( s \) have the same cost.

Our game is then a weighted congestion game with separable preferences and player-independent costs. In [17], it is proved that this game has a PNE and moreover has the FIP property. More precisely, our case corresponds to the restricted assignment case handled in [17] where a user can choose between \( q < m \) RAIs. In our context, because of coverage issues, a mobile user cannot normally switch to all available RAIs in the system, which corresponds indeed to the restricted assignment case. Applying the FIP, the system reaches a PNE in at most \( m^q \) steps.

2) **Hierarchical WiFi-WiMAX/HSDPA Coverage Scenario**: we consider a geographical area serviced by a WiMAX or HSDPA BS. This area contains a number of WiFi hotspots with limited coverage in comparison with the area dimensions (see Figure 2).

Depending on their location, users in this area can either be covered by the BS solely, or can be by both the BS and the AP. In the latter case, they have the choice between connecting to the BS or to the hotspot. The first category of users are statically connected and thus, do not participate in the game. They only contribute in adding a constant load to the BS. The first category of users will be the players of the congestion game, each one trying to chose whether to stick to the AP or to connect to the BS.

For simplicity sake, \( s = 1 \) designates the RAI WX or RAI PF and \( s = 0 \) designates RAI WF (e.g. the base cost of player \( k \) connected to the BS is denoted by \( a_{k,1} \)). In addition a strategy profile \( x \) will be simply a binary vector where \( x(k) = 0 \) if player \( k \) is in RAI WF, and 1 otherwise. Equations 8, 9, and 11 give the cost of a user depending on the selected RAI. In particular, the cost formulas show that mobile users connected to the same hotspot have the exact same cost. In addition, for any two users \( i \) and \( k \) connected to the BS and whose base cost verify \( a_{i,1} \leq a_{k,1} \), their costs obey the same order, hence, \( c_{i,1} \leq c_{k,1} \).

![Hierarchical WiFi-WiMAX/HSDPA Coverage Scenario](image_url)

**Lemma 1**: The Hierarchical WiFi-WiMAX/HSDPA with only one wifi hotspot has the FIP property.

**Proof**:

Assume that there is an infinite improvement path. This means that there exists a finite series of series of profiles, denoted \( x_1, x_2, \ldots, x_L \), which form an improvement path and such that \( x_1 = x_L \). Without loss of generality, players can be numbered according to the increasing order of their base costs \( c_{k,1} \). Therefore \( i < k \Rightarrow c_{i,1} \leq c_{k,1} \). In each step, only one player ameliorates its cost by migrating either from 0 to 1 or from 1 to 0. Migration from 0 to 1 means the player left the hotspot to connect to BS.

We first make the following general observation. The improvement path can be seen as an alternating series of migrations. A given series of migrations is composed of consecutive migrations of different players in the same direction. A series of migrations from 0 to 1 (resp. 1 to 0) is called 1-series (resp. 0-series).

Let \( i \) be the player that makes the last migration in a 0-series, which means that \( c_{i,0} < c_{i,1} \). Then any player \( k \) migrating in the following 1-series must verify \( i < k \). In fact, let us assume that a player \( k \geq i \) can ameliorate its cost by migrating from 0 to 1. This means that \( c_{k,1} < c_{k,0} \). However,
players in strategy 0 have all the same cost which means that $c_{k,0} = c_{i,0}$. By the specified ordering of players we have finally that $c_{k,1} < c_{k,0} = c_{i,0} < c_{i,1}$ which contradicts the fact that $k \geq i$.

Since our cyclic improvement path has finite number of profiles, all players that migrated once, must migrate at least once more in order to complete the cycle. Let $N_m$ be the subset of players that migrate in the path. Other players are static and do not move. Let $k$ be the mobile user in $N_m$ with the greatest base cost $a_{k,0}$. Consider the 0-series in which player $k$ migrated from 1 to 0. The user $i$ that ends this 0-series verifies necessarily $i \leq k$. From, then, the previous observation, players that migrate to 1 in the following 1-series have all indexes such as $< i \leq k$. Then $k$ cannot return to 1 in the following 1-series. In all subsequent 0-series, the player ending the series will be $\leq k$ which, by be same recursive reasoning, implies that $k$ will not have the possibility to migrate to 1 in any future 1-series. This contradicts the fact that $k \in N_m$ and concludes the proof. 

V. PRACTICAL ISSUES

In what followed, we showed the existence of a PNE in numerous multi radio interfaces scenarios. However, implementing a practical distributed RAI selection policy is not straightforward and must be carried out carefully. The main issues are relative to the FIP property. It guarantees in theory that any sufficiently long sequence of selfish unilateral moves by players is bound to lead to a PNE. However, in practice, mobile users will not have the exact cost incurred in each RAI to properly make their migration decision, but they will have a measure-based estimation of that cost. Furthermore, the FIP compels, by construction, mobile users to migrate one at a time. Simultaneous migrations damage the process and convergence to PNE is no longer assured. Therefore, some mechanism must be devised to advertise user costs and to prevent simultaneous RAI migrations.

Additionally, we need to evaluate the performance of the RAI selection policy in a dynamic setting with new arrivals in the system. If so, the game must be played upon every new arrival, and consequently, we need to identify the arrival intensity that enables the system to attain a PNE prior to a new arrival (which modifies the game).

VI. CONCLUSION

The goal of this paper is to associate known results from congestion game theory to the issue of wireless interface selection in a hybrid broadband network. Besides the existence of PNE, the FIP property is valuable in order to devise distributed RAI selection. In fact, it suffices that mobile users, once at a time, iteratively and in random order, switch to any RAI that reduces their cost until convergence is reached. More importantly, we proved the convergence of the WiFi-WiMAX/WiFi-HSDPA case to pure Nash equilibria.

REFERENCES