Blind Amplify-and-Forward Relaying in Multiple-Antenna Relay Networks

Sami (Hakam) Muhaidat*, Murat Uysal** and Raviraj Adve***
* School of Engineering Science, Simon Fraser University, Burnaby, BC, Canada, V5A 1S6
E-mail: muhaidat@ieee.org
** Department of Electrical and Computer Engr., University of Waterloo, Waterloo, Canada, N2L 3G1
E-mail: muysal@ece.uwaterloo.ca
*** Department of Electrical and Computer Engr., University of Toronto, Toronto, Canada, M5S 3G4
E-mail: rsadve@comm.utoronto.ca

Abstract—In this paper, we investigate the performance of a single-relay cooperative scenario where the source, relay, and destination terminals are equipped with multiple transmit/receive antennas. We particularly focus on the so-called blind amplify-and-forward relaying in which the availability of channel state information at the relay terminal is not required. Through the derivation of pairwise error probability, we quantify analytically the impact of multiple antenna deployment assuming various scenarios which involve relay location and power allocation assumptions imposed on the cooperating nodes.

I. INTRODUCTION

Within the last decade, multiple-input multiple-output (MIMO) systems where antenna arrays are deployed at both ends of the transmission link have been a major focus of intensive research [1]. It has been now well established that MIMO systems allow for increase in throughput and reliability because of the additional degrees of freedom offered by the spatial dimension of the wireless channel. Cooperative diversity, also sometimes referred as user cooperation, realizes spatial diversity advantages in a distributed manner and allows cooperating nodes to reap some of the benefits of MIMO transmission [2]–[6]. Taking advantage of the broadcast nature of wireless transmission, the source terminal and its partners (relays) in a cooperative scheme can share their antennas creating a virtual MIMO system.

Although there has been already a rich literature on cooperative diversity, most of the current results are built upon the assumption that user nodes are equipped with single antennas. Some recent results, which exploit further the benefits of multiple antenna deployment, have been recently reported in the literature [7]–[13]. Infrastructure-based fixed relays [7] have the ability to handle multiple antennas, thus, spawning a surge of interest in MIMO relaying. In [7], Adinoyi and Yanikomeroglu examine the impact of multiple antennas on the performance of deploying cooperative fixed relays under the assumption of decode-and-forward (DaF) relaying. In [8], Wang et al. derive upper and lower bounds on the capacity of MIMO relay channels and demonstrate significant capacity gains. Their work, however, is based on some idealistic assumptions such as full-duplex relays and availability of channel state information at the transmitter side. In [9], we have investigated the impact of receive diversity on the error rate performance of amplify-and-forward (AaF) relaying. Specifically, we have shown that the diversity order over the relaying path is governed by the link which has smaller diversity order. In [10], Kim and D. Kim derive a lower bound on the bit error rate of a dual-hop orthogonal space-time block codes (OSTBC) over Rayleigh fading channels. Their work, however, is based on some assumptions such as the availability of channel state information at the relay terminal. Furthermore, it has been assumed in [10] that the relay terminal is equipped with a single antenna. Yiu et al. [11] consider distributed STBC (space-time block codes) with multiple antennas at the relay and destination terminals extending their own work in [12]. Under the assumption that there are \( L \) active relays each of which is equipped with \( M_T \) antennas, a destination node with \( N \) antennas and an underlying distributed STBC of size \( K \times L_1 \) \((K:\) Number of time slots, \( L_1 \geq L)\), it has been shown in [11] that a diversity order of \( \min(L_1 N, M_T L_N) \) can be achieved for DaF relaying. The set-up in [11] assumes an error-free source-to-relay link and only focuses on the relaying phase.

In [13], we have investigated the performance of AaF relaying in a cooperative scenario where the cooperating nodes are equipped with multiple antennas. Our analysis, however, assumes the availability of channel state information (CSI) at the relay terminal for proper scaling. In this paper, we consider the so-called blind AaF relaying where the relay does not have access to CSI and employs a fixed power constraint. The elimination of channel estimation at the relay terminal promises low complexity implementation and makes it attractive from a practical point of view. For a single-relay cooperative network, we quantify analytically the impact of multiple antenna deployment for blind AaF relaying under various scenarios involving relay location and power control rules imposed on cooperating nodes.

Notation: \((\cdot)^*\), \((\cdot)^T\), and \((\cdot)^H\) denote conjugate, transpose, and Hermitian transpose operations, respectively. \(I_Q\) denotes the identity matrix of size \( Q \times Q \), \( tr\{\cdot\}\) denotes a trace of a matrix, \( A < 0 \) means \( A \) is negative definite, \( E[\cdot] \) denotes
II. Transmission Model

We consider a wireless communication scenario where the source terminal $S$ transmits information to the destination terminal $D$ with the assistance of a single relay terminal $R$ (Fig. 1). The source and destination terminals are equipped with $M_S$ and $N$ antennas, respectively. The relay terminal is equipped with $M_R$ receive and $M_T$ transmit antennas. We further assume $M_S=M_R=M_T=M$ \(^1\) and employ conventional STBC for source-to-relay ($S \rightarrow R$) and source-to-destination ($S \rightarrow D$) links. We assume that the relay terminal does not have the CSI of $S \rightarrow R$ link. The destination terminal has the CSI of $S \rightarrow D$ and concatenated $S \rightarrow R \rightarrow D$ links, but does not need explicit access to $S \rightarrow R$ link fading coefficients.

Our transmission model is built upon Protocol II [3], [14]: In the first $K$ time slots, i.e., broadcasting phase, the source terminal communicates with the relay and destination terminals relying on an orthogonal STBC [1] designed for $M$ transmit antennas. In the next $M_TK$ time slots, i.e., relaying phase, only the relay terminal communicates with the destination. The destination then performs maximum likelihood (ML) decoding on the signals received from the source and relay over $K(M_T + 1)$ time slots.

Let $E_{SD}$, $E_{SR}$, and $E_{RD}$ represent the average energies available at the destination and relay terminals taking into account for possibly different path loss and shadowing effects in $S \rightarrow D$, $S \rightarrow R$, and $R \rightarrow D$ links, respectively. Let $h_{SD,j}^i$, $h_{SR,m}$, and $h_{RD,j}^m$ denote the complex fading coefficients over the $S \rightarrow D$ link from the $i^{th}$ transmit antenna to the $j^{th}$ receive antenna, $S \rightarrow R$ link from the $i^{th}$ transmit antenna to the $m^{th}$ receive antenna, and the $R \rightarrow D$ link from the $m^{th}$ transmit antenna to the $j^{th}$ receive antenna.

\(^1\) In practical scenarios, the same antenna elements can be used for transmission and reception, therefore, it is reasonable to assume $M_R=M_T$. It makes practical sense to further assume $M_R=M_T=M_S$ since source and relay terminals are cooperating nodes designed for a given application, therefore typically share the same physical features. The purpose of assigning different variables for antenna numbers is solely to distinguish their effect on the diversity order.

These channel coefficients are modeled as zero-mean complex Gaussian with variance 0.5 per dimension leading to the well-known Rayleigh fading channel model. Unless otherwise indicated, the variables $n$ (regardless of index) are independent and identically distributed (i.i.d.) zero mean complex Gaussian samples with $N_0/2$ per dimension and model the additive noise. The received signals during the broadcasting phase at the $j^{th}$ ($j = 1, 2, ..., N$) receive antenna of the destination terminal are given by

$$r_{D,j}^k = \sqrt{E_{SD}/M_S} \sum_{i=1}^{M_S} h_{SD,j}^i x_{i,k} + n_{D,j}^k, \quad k = 1, 2, ..., K$$

(1)

where $x_{i,k}$ is the STBC-encoded modulation symbol sent from the $i^{th}$ transmit antenna in time interval $k$. Similarly, the received signals at the $m^{th}$ ($m = 1, 2, ..., M_R$) receive antenna of the relay terminal are given by

$$r_{R,m}^k = \sqrt{E_{SR}/M_S} \sum_{i=1}^{M_S} h_{SR,m}^i x_{i,k} + n_{R,m}^k, \quad k = 1, 2, ..., K.$$  (2)

During the relaying phase, the relay terminal forwards the received signals after proper normalization. Each of the received signal, i.e., $r_{R,m}^k$, is first normalized by a factor of $\sqrt{E_{SR}/M_S + N_0}$ to ensure the average energy is unity then transmitted through $M_T$ antennas consecutively during the next $M_TK$ time slots. The received signals at the destination terminal from $t^{th}$ ($t = 1, ..., M_T$) antenna are given by

$$r_{D,j}^{t,l} = \frac{E_{RD}}{M_T} h_{RD,j}^l r_{R,m}^{t,l} \sqrt{E_{SR}/M_S + N_0} + n_{D,j}^{t,l},$$  (3)

for $l = tK + 1, ..., t(K+1)$, $k = l - tK$ and $j = 1, ..., N$. Replacing (2) in (3) and noting $m = t$, we obtain

$$r_{D,j}^{t,l} = \frac{E_{SR}E_{RD}}{M_T M_S (E_{SR}/M_S + N_0)} \sum_{i=1}^{M_S} h_{SR,t}^i x_{i,k} + \tilde{n}_{D,j}^{t,l},$$  (4)

where the effective noise term $\tilde{n}_{D,j}^{t,l}$ is given as

$$\tilde{n}_{D,j}^{t,l} = \frac{E_{RD}}{M_T (E_{SR}/M_S + N_0)} h_{RD,j}^{t,l} r_{R,t}^{t-lK} + n_{D,j}^{t,l}$$

(5)

which is complex Gaussian (conditioned on $h_{RD,j}^{t,l}$) with zero mean and variance of

$$E \left[ |\tilde{n}_{D,j}^{t,l}|^2 \right] = N_0 \left( 1 + \frac{E_{RD}}{M_T (E_{SR}/M_S + N_0)} |h_{RD,j}^{t,l}|^2 \right)$$

(6)

Let $n = [\tilde{n}_{D,j}^{t,l} \ldots \tilde{n}_{D,N}^{t,l}]^T$, it can be shown that the covariance matrix $Q_n = E[nn^H]$ is not diagonal. Hence, the noise terms at different receive antennas are still Gaussian, but no longer spatially white. This makes the analytical analysis in this case quite involved. However, noting that $Q_n$
\[ P(\mathbf{x}, \hat{\mathbf{x}}|h_{SR,m}^{m}, h_{RD,j}^{m}, h_{SD,j}^{m}) \leq \exp\left(-\frac{1}{4} \sum_{t=1}^{M_T} \text{tr}\left\{ \text{EH}_t^t Q_n^{-1} (\text{EH}_t^t)^H \right\} - \frac{1}{4N_0} \text{tr}\left\{ \text{EH}_{SD} \text{H}_{SD}^H \text{E}_H^H \right\} \right) \] \hspace{1cm} (9)

\[ P(\mathbf{x}, \hat{\mathbf{x}}|h_{SR,m}^{m}, h_{RD,j}^{m}, h_{SD,j}^{m}) \leq \exp\left(-\frac{1}{4} \sum_{t=1}^{M_T} \text{tr}\left\{ \text{H}_t^t (\text{H}_t^t)^H \right\} + \frac{1}{4N_0} \text{tr}\left\{ \text{H}_{SD} \text{H}_{SD}^H \right\} \right) \] \hspace{1cm} (10)

\[ P(\mathbf{x}, \hat{\mathbf{x}}|h_{SR,m}^{m}, h_{RD,j}^{m}, h_{SD,j}^{m}) \leq \exp\left(-\frac{1}{4} \sum_{t=1}^{M_T} \text{tr}\left\{ \text{H}_t^t (\text{H}_t^t)^H \right\} - \frac{1}{4N_0} \text{tr}\left\{ \text{H}_{SD} \text{H}_{SD}^H \right\} \right) \] \hspace{1cm} (11)

is a semi positive definite matrix, one can easily show that \( Q_n \leq N_0 \left( \frac{E_{RD}}{M_T(E_{SR}/M_S + N_0)} \sum_{j=1}^{N} |h_{RD,j}| + 1 \right) I_N \) for \( t = 1, \ldots, M_T \). Here, \( \text{H}_t^t \) represents the signals received at the destination terminal with size \( K \times N \), \( \text{X} \) is the data matrix with size \( K \times M_S \), \( \text{H}^t \) is the channel matrix with size \( M_S \times N \), and \( \text{N}_t \) denotes the noise matrix with size \( K \times N \).

### III. DIVERSITY GAIN ANALYSIS

In this section, we investigate the achievable diversity order for the considered relaying technique through the derivation of pairwise error probability (PEP) expression. Defining the transmitted codeword vector from the source and the erroneously-decoded codeword vector at the destination terminal, respectively, as \( \text{X} \) and \( \hat{\text{X}} \), a Chernoff bound on the PEP, conditioned on CSI, is given as in (9) on top of the page, where

\[
\text{H}_{SD} = \begin{bmatrix}
\sqrt{\frac{E_{SR}}{M_S^2}} h_{SD,1}^1 & \cdots & \sqrt{\frac{E_{SR}}{M_S^2}} h_{SD,N}^1 \\
\vdots & \ddots & \vdots \\
\sqrt{\frac{E_{SR}}{M_S^2}} h_{SD,1}^N & \cdots & \sqrt{\frac{E_{SR}}{M_S^2}} h_{SD,N}^N 
\end{bmatrix},
\]

\( \text{E} = \text{X} - \hat{\text{X}} \) is the codeword error matrix, and \( \lambda \) is the eigen value of \( \text{E} \). Using (7), we can further upper bound the right-hand side of (9), resulting in (10) which is given on top of the page. Alternatively, we can re-write (10) as given in (11), where

\[
\gamma_t = \frac{E_{SR}}{N_0} \left( \frac{M_T E_{SR}}{N_0} + M_T M_S + M_S \frac{E_{RD}}{N_0} \sum_{j=1}^{N} |h_{RD,j}|^2 \right)^{-1}.
\] \hspace{1cm} (12)

In the following, we consider various scenarios and analyze the resulting diversity order.

**Scenario 1 (High SNR in S \rightarrow R link and balanced S \rightarrow D and R \rightarrow D links):** In this scenario, we assume that \( S \rightarrow R \) link experiences a high SNR which corresponds to a practical scenario where relay is located close the source.
Following steps detailed in the Appendix, we obtain the PEP expressions as

\[ P(x, \hat{x}) \leq \left( \frac{(N-M_S)}{1}(N) \right)^{M_T} \left( \frac{\lambda E_{SD}}{4MN_0} \right)^{-M_S(N+M_T)} \quad N > M_S \]  
\[ P(x, \hat{x}) \leq \left( \frac{(M_S-N)}{1}(M_S) \right)^{M_T} \left( \frac{\lambda E_{SD}}{4MN_0} \right)^{-N(M_S+M_T)} \quad M_S > N \]  
\[ P(x, \hat{x}) \leq \log^{M_T}(E_{SD}/N_0) \left( \frac{\lambda E_{SD}}{4MN_0} \right)^{-M_S(M_S+M_T)} \quad M_S = N \]

It can be observed from (13)-(15) that the maximum achievable diversity order is given by \( M_T \min(M_S, N) + M_S N \). This illustrates that the smaller of diversity orders experienced in \( S \to R \) and \( R \to D \) links become the performance bottleneck for the relaying path.

**Scenario 2 (High SNR in \( R \to D \) link and balanced \( S \to D \) and \( S \to R \) links):** Here, we assume that \( R \to D \) link experiences a high SNR which is likely to occur in practical scenarios when the relay is close to the destination terminal. We further assume that \( S \to R \) and \( S \to D \) links are balanced, i.e., \( E_{RD}/N_0 >> E_{SD}/N_0 = E_{SR}/N_0 \). Under these assumptions, \( \gamma^t \) in (12) can be approximated as \( \gamma^t = E_{SR}/N_0/(M_S \sum_{j=1}^{N} |h^i_{RD,j}|^2 E_{RD}/N_0) \). Hence, \( \Omega \) in (11) can be written as

\[ \Omega = \frac{\lambda E_{SR}}{M_S} \sum_{i=1}^{M_T} \sum_{j=1}^{M_S} |h^i_{SR,j}|^2 + \frac{\lambda E_{SD}}{M_S} \sum_{j=1}^{N} \sum_{i=1}^{M_S} |h^i_{SD,j}|^2. \]  

Defining \( Y_1 = \sum_{j=1}^{N} \sum_{i=1}^{M_S} |h^i_{SR,j}|^2 \), \( Y_2 = \sum_{i=1}^{M_T} \sum_{j=1}^{M_S} |h^i_{SD,j}|^2 \), and noting that \( |h^i_{SD,j}| \) and \( |h^i_{SR,j}| \) are Rayleigh distributed, their characteristic functions are readily available, c.f. (24). Replacing those in (23) and assuming \( E_{RD}/N_0 >> 1 \), we find PEP as

\[ P(x, \hat{x}) \leq \left( \frac{E_{SD}}{4MN_0} \right)^{-M_S(M_T+N)} \].  

The diversity order in this scenario is \( M_S (M_T + N) \) which is obviously either equal or larger than the diversity order observed for the previous scenario, i.e., \( M_T \min(M_S, N) + M_S N \).

**Scenario 3 (Non-fading \( R \to D \) link):** Now, we assume the case where the channel between the relay and the destination terminals is AWGN, i.e., \( h_{RD} = 1 \). Physically, this assumption corresponds to a case where the destination and relay terminals are static and have a very strong line-of-sight connection [14]. Under the assumptions of \( E_{RD}/N_0 = E_{RD}/N_0 \) and \( E_{SR}/N_0 >> E_{SD}/N_0 \), \( \gamma^t \) in (12) can be approximated as \( \gamma^t = 1/M_T \). In this case, \( \Omega \) can be written as

\[ \Omega = \lambda N \frac{E_{SD}}{M_T} \sum_{i=1}^{M_T} \sum_{j=1}^{M_S} |h^i_{SR,j}|^2 + \lambda \frac{E_{SD}}{M_S} \sum_{j=1}^{N} \sum_{i=1}^{M_S} |h^i_{SD,j}|^2. \]  

Substituting (18) in (11), taking the expectation with respect to \( |h^i_{SD,j}| \) and \( |h^i_{SR,j}| \) which are Rayleigh distributed, and assuming \( E_{SD}/N_0 >> 1 \), we find PEP as

\[ P(x, \hat{x}) \leq \frac{M_S(M_T+N)}{N^{M_T} M_S} \left( \frac{\lambda E_{SD}}{4N_0} \right)^{-M_S(M_T+N)} \].  

It can be observed that the diversity order for a non-fading \( R \to D \) link is \( M_S (M_T + N) \).

**IV. SIMULATION RESULTS AND DISCUSSION**

In this section, we present results of Monte-Carlo simulations for cooperative transmission systems which have been described and analyzed in this paper. We employ 4-PSK modulation and consider the aforementioned scenarios with the following numerical values:

- Scenario 1: \( E_{SR}/N_0 = 35\text{dB} \) and \( E_{SD} = E_{RD} \).
- Scenario 2: \( E_{RD}/N_0 = 35\text{dB} \) and \( E_{SD} = E_{SR} \).
- Scenario 3: \( E_{SR}/N_0 = 35\text{dB} \), \( E_{SD} = E_{RD} \), non-fading \( R \to D \) link.

In Fig. 2, we illustrate the SER (symbol error rate) performance of the blind AaF scheme assuming \( M_S = M_T = M_R = M = 2 \) and \( M_S = M_T = M_R = M = N = 1 \). For \( M = 2 \) and \( 3 \), we consider the Alamouti and G3-STBC schemes [1], respectively, as the underlying space-time codes. For Scenario 1, where \( S \to D \) and \( R \to D \) links are balanced, it is observed that the diversity orders for \( (M = 2, N = 1) \) and \( (M = 3, N = 1) \) are 4 and 6, respectively, confirming the diversity order of \( M_T \min(M_S, N) + M_S N \) obtained through our PEP expressions in (13)-(15). To further confirm our diversity order analysis, we also include the performance for \( M_T = M_R = 1 \) while keeping \( M_S = 2 \). It is clearly seen that the achievable diversity order in this case is 3. For Scenario 2, where \( S \to D \) and \( S \to R \) links are balanced, it is observed that the diversity order is 6 for \( (M = 2, N = 1) \), confirming our PEP derivation of (17). On the other hand, in
the presence of non-fading $R \to D$ link with $M = 2$ and $N = 1$, it is observed that the diversity order is 6 confirming our observation in (19) for Scenario 4.

V. CONCLUSION

In this paper, we have investigated performance of blind AaF relaying in a cooperative scenario in which the cooperating terminals are equipped with multiple antennas. Specifically, we have analyzed the diversity order of blind AaF in a single relay assisted transmission scenario where the source, relay, and destination terminals are equipped with $M_S$, $M_R = M_T$, and $N$ antennas, respectively. Under the assumption of balanced $R \to D$ and $S \to D$ links and sufficiently large SNR for the $S \to R$ link, we have shown that the maximum achievable diversity order is $M_T \min\{M_S, N\} + M_SN$. For the scenario where $S \to D$ and $S \to R$ links are balanced and sufficiently large SNR for the $R \to D$ link, we have demonstrated that blind AaF scheme achieves a diversity order of $M_S (N + M_T)$. We have also found out that, under the non-fading $R \to D$ link assumption, the maximum achievable diversity order is $M_S (N + M_T)$.

APPENDIX A

DERIVATION OF (13)-(14)-(15)

In this Appendix, we derive PEP expressions for blind AaF scheme assuming $E_{RD}/N_0 = E_{SD}/N_0$ and $E_{SR}/N_0 >> E_{SD}/N_0$. Under these assumptions, $\gamma^i$ in (12) can be approximated as $\gamma^i \approx 1/M_T$. Thus, $\Omega$ in (11) reduces to

$$\Omega = \frac{\lambda E_{SD}}{M_T} \sum_{i=1}^{N} \sum_{j=1}^{M} |h_{RD,j}^i|^2 \sum_{i=1}^{M_S} |h_{SR,t}^i|^2 + \frac{\lambda E_{SD}}{M_S} \sum_{j=1}^{N} \sum_{i=1}^{M} |h_{RD,j}^i|^2 . \tag{20}$$

Introducing $Y_1 = \sum_{j=1}^{N} \sum_{i=1}^{M} |h_{RD,j}^i|^2$ and $Y_2 = Z_1^i Z_2^i$ with $Z_1 = \sum_{j=1}^{N} |h_{RD,j}^i|^2$ and $Z_2 = \sum_{i=1}^{M} |h_{SR,t}^i|^2$, we can rewrite (20) as

$$\Omega = \frac{\lambda E_{SD}}{M_S} Y_1 + \frac{\lambda E_{SD}}{M_T} \sum_{i=1}^{M_T} Y_2^i . \tag{21}$$

After substituting (21) in (11), we have the PEP expression as

$$P(x, \hat{x}|Y_1, Y_2) \leq \exp \left( -\frac{\lambda E_{SD}}{4N_0 M_S} Y_1 \right) \prod_{i=1}^{M_T} \exp \left( -\frac{\lambda E_{SD}}{4M_T N_0} Y_2^i \right) . \tag{22}$$

The unconditional PEP can be obtained as [15]

$$P(x, \hat{x}) \leq \Phi_{Y_1}(\omega) \prod_{i=1}^{M_T} \Phi_{Y_2^i}(\omega) \tag{23}$$

where $\Phi_{Y_1}(\omega)$ and $\Phi_{Y_2^i}(\omega)$ are the characteristic functions of $Y_1$ and $Y_2^i$, respectively. Since $|h_{RD,j}^i|$ is Rayleigh distributed, the characteristic function of $Y_1$ can be readily found as

$$\Phi_{Y_1}(\omega) \bigg|_{\omega = -\frac{\lambda E_{SD}}{4N_0 M_S}} = \left( 1 + \frac{E_{SD}}{4M_S N_0} \right)^{-M_SN} \tag{24}$$

where the first term can be further ignored under high SNR assumption. In the following, we will derive the PEP expression for three cases:

Case 1 ($M_S < N$): The characteristic function of $Y_2$ can be evaluated as [16]

$$\Phi_{Y_2}(\omega) = \int_0^\infty f_{Z_1^i(z_1^i)} \Phi_{Z_1^i(\omega z_1^i)} dz_1^i \tag{25}$$

where $f_{Z_1^i(z_1^i)}$ is the probability density function (pdf) of $Z_1^i$ and $\Phi_{Z_1^i(\omega z_1^i)}$ has the similar form as in (24). Here, $Z_1^i$ is a chi-squared random variable with $2N$ degrees of freedom with the pdf $f_{Z_1^i(z_1^i)} = z_1^{i-1} e^{-z_1^i} / \Gamma(N)$ [17] where $\Gamma(.)$ denotes the gamma function [18]. This leads to

$$\Phi_{Y_2}(\omega) \bigg|_{\omega = -\frac{\lambda E_{SD}}{4M_T N_0}} = \frac{1}{\Gamma(N)} \left( \frac{E_{SD}}{4M_T N_0} \right)^{-M_S} \frac{\lambda^{-M_S}}{\lambda^{-M_S}} \times \int_0^\infty \left( \frac{AM_T N_0}{\lambda E_{SD}} + z_1^i \right)^{-M_S} e^{-z_1^i} dz_1^i \tag{26}$$

Assuming $E_{SD}/N_0 >> 1$ and using the integral form given by [18], we obtain

$$\Phi_{Y_2}(\omega) \bigg|_{\omega = -\frac{\lambda E_{SD}}{4M_T N_0}} = \frac{\Gamma(N - M_S)}{\Gamma(N)} \left( \frac{E_{SD}}{4M_T N_0} \right)^{-M_S} \lambda^{-M_S} . \tag{27}$$

Substituting (27) and (24) in (23), we find the final PEP expression as given in (13).

Case 2 ($M_S > N$): Noting that this case is similar to the previous case with $N$ and $M_S$ now interchanged, we simply follow similar steps and find the PEP expression as given in (14).

Case 3 ($M_S = N$): Following the same argument in Case 1 and further defining $u = v + z_1^i$ where $v = 4M_S N_0 / (\lambda E_{SD})$, we write (26) as

$$\Phi_{Y_2}(\omega) \bigg|_{\omega = -\frac{\lambda E_{SD}}{4M_T N_0}} = \frac{1}{\Gamma(N)} \left( \frac{E_{SD}}{4M_T N_0} \right)^{-M_S} \lambda^{-M_S} \times \exp(v) \int_v^\infty u^{-1} (1 - v/u)^{N-1} e^{-u} du \tag{28}$$

Under $E_{SD}/N_0 >> 1$ assumption, we have $1 - v/u \approx 1$. Hence, we can rewrite (28) as

$$\Phi_{Y_2}(\omega) \bigg|_{\omega = -\frac{\lambda E_{SD}}{4M_T N_0}} = \frac{1}{\Gamma(N)} \left( \frac{E_{SD}}{4M_T N_0} \right)^{-M_S} \lambda^{-M_S} \times \exp(v) \Gamma(0, v) \tag{29}$$

where $\Gamma(a, b) = \int_b^\infty q^{a-1} \exp(-q) dq$ [18] denotes the incomplete gamma function. Using the limiting approximation
\[ \Gamma(0, v) \approx -\log(v) \text{ for } v \to 0 \text{ [18], (29) reduces to } \]

\[ \Phi_{Y}(\omega)\bigg|_{j\omega=-\frac{\lambda_{\text{ESD}}}{4M_{T}N_{0}}} = \frac{1}{\Gamma(N)} \left(\frac{E_{\text{SD}}}{4M_{T}N_{0}}\right)^{-M_{S}} \lambda^{-M_{S}} \times \log\left(\frac{E_{\text{SD}}}{N_{0}}\right) \] (30)

Substituting (30) and (24) in (23), we find the final PEP expression as given in (15).

REFERENCES