Demand Aggregation and the Weak Axiom of Stochastic Revealed Preference

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We address the problem of aggregating demand across a group of consumers, who are identical in terms of wealth and face identical price vectors, but vary in their chosen consumption bundles. We show that, when a stochastic demand function is constructed to aggregate a number of deterministic demand functions, satisfaction of the weak axiom of stochastic revealed preference by this stochastic demand function is weaker than the restriction that every underlying deterministic demand function satisfy Samuelson’s weak axiom of revealed preference. Journal of Economic Literature Classification Number: D11.

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1. INTRODUCTION

The purpose of this note is to examine the structure of a stochastic demand function that one can construct to represent the aggregate demand behavior of a group of competitive consumers who have the same wealth and face the same prices but may differ in their choice behavior. Specifically, we investigate the relation between the weak axiom of stochastic revealed preference (WASRP) formulated for such a stochastic demand function and the requirement of Samuelson’s weak axiom of revealed preference (WARP) for each of the underlying deterministic demand functions.

Many problems in economic analysis require aggregating the consumption behavior of a number of competitive consumers, who may be considered homogeneous in terms of the wealth they own and the prices they face, but are likely to vary in terms of their consumption choices. The standard analytical procedure for generating testable predictions about aggregate demand behavior under such conditions involves the construction of a representative consumer. This representative consumer is endowed with the total wealth of the group, ascribed the total bundle of commodities consumed by the group, and is assumed to satisfy WARP. It is, however, well known that, even if all individuals satisfy WARP, collective demand behavior need not satisfy this restriction, except under quite stringent additional restrictions on individual preferences.\(^2\) The standard approach thus generates predictions that are often known beforehand, on a priori theoretical grounds, as likely to be false.

In this note, we discuss an alternative approach to this problem. Instead of ascribing a deterministic demand function (DDF) to the representative consumer, as in the standard approach, one may alternatively model group consumption behavior by ascribing a stochastic demand function (SDF) to the representative consumer.\(^3\) Suppose, for example, in a group of 1000 competitive consumers who face identical budget sets, 600 individuals choose \(x\) and 400 consumers choose \(y\). Then one could capture this phenomenon analytically by representing the entire group as a single consumer who chooses \(x\) with probability 3/5 and \(y\) with probability 2/5.

What reasonable restrictions can one impose on an SDF constructed to represent, in the sense explained above, aggregate demand behavior of a group of consumers? In a very different context, where an SDF reflects the stochastic choice behavior of a single consumer, Bandyopadhyay et al. (1999) introduced a restriction, WASRP, which was intended to be a rationality property of the stochastic behavior of the consumer under

\(^2\) See, for example, Mas-Colell et al. [7, pp.109–116].

\(^3\) For an early discussion of this approach, see McFadden [8, 9]. See also Barbera and Pattanaik [2].
consideration. In our present context, where an SDF is intended to be a representation of aggregate demand behavior of a group of consumers, it is no longer meaningful to view WASRP as a rationality property. However, in this context, we show that, if each of the underlying DDFs satisfies WARP, then the SDF constructed to represent aggregate demand will necessarily satisfy WASRP, though the converse is not true. Thus, so long as all consumers satisfy WARP, the SDF representation of aggregate demand must necessarily satisfy WASRP, even though its standard deterministic representation may violate WARP. Furthermore, an SDF representation of aggregate demand may satisfy WASRP even if individual consumers violate WARP. Hence, modeling aggregate demand in terms of an SDF, and imposing WASRP on this SDF, constitutes a less restrictive procedure for deriving testable predictions than the standard one.

2. NOTATION AND DEFINITIONS

Let \( m \) denote the number of commodities, \( m \geq 2 \), and let \( M = \{1, 2, ..., m\} \). We assume that \( R^m_+ \) constitutes the consumer’s consumption set. A price–wealth situation is an ordered pair \((p, W)\) where \( p \in R^m_+ \) and \( W \in R_+ \). Let \( Z \) denote the set of all possible price–wealth situations. Given a price–wealth situation, \((p, W)\), the budget set of the consumer is defined to be \( \{x \in R^m_+ | W \geq p \cdot x\} \). The budget sets corresponding to price–wealth situations \((p, W), (p', W')\), \((p^*, W^*)\), etc. will be denoted, respectively, by \( B, B', B^*\), etc.

**Definition 2.1.**
(i) A deterministic demand function (DDF) is a rule \( d \), which, for every \((p, W) \in Z\), specifies exactly one bundle \( x \) belonging to the budget set \( B \) corresponding to \((p, W)\).

(ii) A stochastic demand function (SDF) is a rule \( D \) which, for every \((p, W) \in Z\), specifies exactly one finitely additive probability measure \( q \) over the class of all subsets of the budget set \( B \).

**Remark 2.2.** Let \( q = D(p, W) \), where \( D \) is an SDF. Then, for every subset \( A \) of \( B \), \( q(A) \) is to be interpreted as the probability that, given the price–wealth situation \((p, W)\), the consumer’s chosen bundle will belong to the set \( A \). \( D(p, W), D(p', W')\), etc. will be denoted, respectively, by \( q, q' \), etc. The notion of a DDF is the same as the notion of a demand function used in the standard theory of consumers’ choice. DDFs can be identified with degenerate SDFs.
Definition 2.3. (i) A DDF $d$ satisfies the weak axiom of revealed preference (WARP) iff, for all price–wealth situations $(p, W), (p', W') \in Z$, $d(p, W) \neq d(p', W')$ and $d(p', W') \in B$ implies $[p'.d(p, W) > W']$.

(ii) An SDF $D$ satisfies the weak axiom of stochastic revealed preference (WASRP) iff, for all $(p, W), (p', W') \in Z$, and for every $A \subseteq [B \cap B']$, $q(B - B') \geq q'(A) - q(A)^4$.

Remark 2.4. Bandyopadhyay et al. [1], who introduced WASRP, also discuss its intuitive justification as a rationality property of individual stochastic demand behavior. It is weaker than the condition of rationalizability in terms of stochastic orderings discussed, among others, by Barbera and Pattanaik [2], Becker et al. [3], Block and Marschak [4], Falmagne [5], Loomes and Sugden [6], and Nandeibam [10]. Clearly, if an SDF, $D$, is degenerate and satisfies WASRP, then the DDF corresponding to $D$ satisfies WARP, and, conversely, if a DDF $d$ satisfies WARP, then the degenerate SDF corresponding to $d$ must satisfy WASRP.

3. AGGREGATION

We are now ready to address our central problem. We first provide a precise formulation of the way in which an SDF can be constructed to aggregate deterministic demand behavior of a group of competitive consumers facing identical price–wealth situations. We then show that satisfaction of WARP by every member of such a group is a sufficient, but not necessary, condition for the SDF representation of aggregate demand to satisfy WASRP.

Definition 3.1. For all $i \in N = \{1, 2, \ldots, n\}$, let $d_i$ be a deterministic demand function ($d_1, \ldots, d_n$ need not necessarily be all distinct). We say that $(d_1, \ldots, d_n)$ induces the stochastic demand function $D$ iff, for every $(p, W) \in Z$, and every $A \subseteq B(p, W)$,

$$q(A) = (\{i \in N | d_i(p, W) \in A\})/n,$$

where $q = D(p, W)$.

Consider a group of individuals all endowed with the same wealth, $W$, and facing the same price vector, $p$, who exhibit consumption behavior that can be represented by deterministic demand functions but that may not necessarily be identical for all individuals in the group. Suppose that aggregate consumption behavior of this group is modeled in terms of $B - B'$ denotes the set of all consumption bundles that belong to $B$ but not to $B'$. 

representative consumer facing the price–wealth situation \((p,W)\) and choosing according to an SDF, \(D\). Then \(D\) is such that, for any subset \(A\) of the budget set \(B(p,W)\), the probability of choosing in \(A\), i.e., \(q(A)\), is simply the proportion of individuals in the group who choose consumption bundles lying in \(A\).\(^5\)

**Proposition 3.2.** Let \(N = \{1, 2, \ldots, n\}\) be a given (finite) set.

(i) For every \(n\)-tuple of deterministic demand functions, \((d_1, \ldots, d_n)\), such that [for all \(i \in N\), \(d_i\) satisfies WARP], the SDF \(D\) induced by \((d_1, \ldots, d_n)\) satisfies WASRP.

(ii) There exists an \(n\)-tuple of deterministic demand functions, \((d_1, \ldots, d_n)\), such that [for every \(i \in N\), \(d_i\) violates WARP], but the SDF induced by \((d_1, \ldots, d_n)\) satisfies WASRP.

**Proof.**

(i) Consider two arbitrarily given price–wealth situations \((p,W)\) and \((p',W')\). Let \(A\) be any given subset of \(B \cap B'\). To show that \(D\) satisfies WASRP, it is enough to show that

\[
q(B - B') \geq q'(A) - q(A),
\]

where \(q = D(p,W)\) and \(q' = D(p',W')\). Since \(D\) is induced by \((d_1, \ldots, d_n)\),

\[
q(B - B') = \frac{|\{i \in N \mid d_i(p,W) \in (B - B')\}|}{n};
\]

\[
q'(A) = \frac{|\{i \in N \mid d_i(p',W') \in A\}|}{n}; \quad \text{and}
\]

\[
q(A) = \frac{|\{i \in N \mid d_i(p,W) \in A\}|}{n}.
\]

For every \(i \in N\), \(d_i\) satisfies WARP. Therefore, noting \([B = (B - B') \cup (B \cap B')]\) and \(A \subseteq (B \cap B')\), it is clear that, if \(d_i(p',W') \in A\), then either \([d_i(p',W') = d_i(p,W)\), and, hence, \(d_i(p,W) \in A\)] or \([d_i(p,W) \in (B - B')\)]. Therefore,\(\{i \in N \mid d_i(p',W') \in A\} \subseteq [\{i \in N \mid d_i(p,W) \in A\} \cup \{i \in N \mid d_i(p,W) \in (B - B')\}]\). It follows that \([|\{i \in N \mid d_i(p',W') \in A\}| \leq |\{i \in N \mid d_i(p,W) \in (B - B')\}| + |\{i \in N \mid d_i(p,W) \in (B \cap B')\}|\) and, hence,

\[
\frac{|\{i \in N \mid d_i(p',W') \in A\}| - |\{i \in N \mid d_i(p,W) \in A\}|}{n} \leq \frac{|\{i \in N \mid d_i(p,W) \in (B - B')\}|}{n}.
\]

Equations (3.2) and (3.3), together, imply (3.1).

\(^5\) Inducing an SDF by a finite number of DDFs forces every choice probability to be a rational number. Thus, such SDFs cannot allow the probability of choosing in some set \(A\) to vary continuously in prices or wealth.
Let $a_1 = 0$, $a_{n+1} = \infty$, and consider $a_2, \ldots, a_n \in \mathbb{R}_{++}$ such that, for all $j \in \{1, \ldots, n\}$, $a_j < a_{j+1}$. Partition the interval $[0, \infty)$ into $[a_1, a_2)$, $[a_2, a_3)$, ..., $[a_n, a_{n+1})$. For every $i \in N$, define a deterministic demand function $d_i$ as follows: for every price–wealth situation $(p, W)$, if $p_1/p_2 \in [a_i, a_{i+1})$, then
\[ d_i(p, W) = (W/p_1, 0, \ldots, 0) \quad \text{and} \quad \text{if } p_1/p_2 \notin [a_i, a_{i+1}), \quad \text{then} \quad d_i(p, W) = (0, W/p_2, 0, \ldots, 0). \quad (3.4) \]
(Note that, since all prices are positive, $p_1/p_2 \in (0, \infty)$).

It is easy to check that, for every $i \in N$, $d_i$ violates WARP. Consider the SDF, $D$, induced by $(d_1, \ldots, d_n)$. Since, for every $i \in N$, (3.4) holds, it is clear that, for every price–wealth situation $(p, W)$ and for every $A \subseteq B(p, W)$,
\[ q(A) = \begin{cases} 1/n & \text{if } ((W/p_1, 0, \ldots, 0) \notin A \text{ and } (0, W/p_2, 0, \ldots, 0) \notin A), \\ (n-1)/n & \text{if } ((W/p_1, 0, \ldots, 0) \notin A \text{ and } (0, W/p_2, 0, \ldots, 0) \notin A), \\ 1 & \text{if } ((W/p_1, 0, \ldots, 0) \notin A \text{ and } (0, W/p_2, 0, \ldots, 0) \notin A), \\ 0 & \text{if } ((W/p_1, 0, \ldots, 0) \notin A \text{ and } (0, W/p_2, 0, \ldots, 0) \notin A), \end{cases} \]
where $q = D(p, W)$. \quad (3.5)

Given part (i), to prove that $D$ satisfies WASRP, it is enough to show that there exists an $n$-tuple, $(d'_1, \ldots, d'_n)$, of deterministic demand functions satisfying WARP, which induces $D$. Consider the deterministic demand functions $d'_1, \ldots, d'_n$ such that, for every price–wealth situation $(p, W)$,
\[ [d'_i(p, W) = (W/p_1, 0, \ldots, 0), \text{ and, for every } i \in N-\{1\}, \quad d'_i(p, W) = (0, W/p_2, 0, \ldots, 0)]. \]
It can be checked that each of $(d'_1, \ldots, d'_n)$ satisfies WARP, and $(d'_1, \ldots, d'_n)$ induce $D$. 

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