Performance Analysis of Cooperative Diversity Using Equal Gain Combining (EGC) Technique over Rayleigh Fading Channels

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Abstract—Cooperative diversity is a promising technology for future wireless networks. In this paper, based on the moment generating function (MGF), we derive exact closed-form expressions for the average bit error rate (BER), the amount of fading (AF), and outage probability ($P_{out}$) for cooperative diversity networks with amplify-and-forward relaying over Rayleigh-fading channel using differentially-non-coherent equal gain combining (EGC) technique. We show that this scheme (differentially-non-coherent EGC) achieves a diversity order of 2 at high SNR. The main advantage of this scheme is that it does not need channel state information of the transmission links at the relay and destination nodes. This feature reduces the design and implementation complexity. Furthermore, we study in this paper the impact of the relay location on the system performance.

Index Terms—Cooperative diversity, single relay networks, Rayleigh fading, error performance, outage probabilities.

I. INTRODUCTION

Cooperative diversity networks technology is a promising solution for the high data-rate coverage required in future cellular and ad-hoc wireless communications systems. There are two main advantages of this relaying technology. The low transmit power requirements and the spatial diversity that can mitigate fading.

Cooperative diversity networks combine the usual power saving with the spatial diversity provided by the antennas of separate nodes [1–6]. The basic idea is that between the transmitter and receiver nodes, there can be another node, which can be used to provide diversity by forming a virtual multi-antenna system. Performance analysis of cooperating relaying networks has yielded many interesting results including signal-to-noise ratio (SNR)-based outage probability, information theoretic metrics, and average error probability expressions over Rayleigh-fading channels [7,8]. Authors in [1] have proposed a variety of low-complexity cooperative protocols using the three-terminal case.

These protocols have been applied on different relaying modes as amplify-and-forward (i.e., non-regenerative relays) and decode-and-forward (i.e., regenerative relays). Also the outage probability, using high SNR approximations, has been analyzed in [1]. The work in [1] has been extended in [6], where a new cooperative protocol is presented realizing maximum degrees of broadcasting and exhibits no receive collision. In another contribution, the authors in [3,4] have proposed the user cooperation concept and considered practical issues related to its implementation.

Most of the proposed schemes assume that the destination and relay nodes have perfect knowledge of the channel state information (CSI) of all transmission links [1–8]. While in some scenarios, e.g., the slow fading channels, the CSI is likely to be acquired by the use of pilot symbols, it might be infeasible to track the CSI in fast fading channels. In [9], the authors studied the outage and the error performance over Rayleigh fading channels of the dual-hop systems (not cooperative diversity networks) equipped with non-regenerative blind relays (i.e. relays with fixed gain). In the same paper, the authors propose a specific fixed gain relay, called “semi-blind”, that benefits from the knowledge of the first hop average fading power. The main advantage of the fixed gain introduced in [9] is that the relay and destination nodes do not need to estimate the CSI of the involved channels, and as a result, this will reduce the complexity of the receivers.

Non-coherent cooperative diversity has been proposed in [10] for the decode-and-forward protocol employing frequency shift keying modulation (FSK). Authors in [18] examined the error performance for BDPSK and find closed-form expression for error probability using amplify-and-forward protocol.

This paper presents a completely analytical approach in studying the performance of the amplify-and-forward protocol in a two-user cooperative communication system over Rayleigh-fading channel using differentially-
non-coherent equal gain combining (EGC) technique. We obtain a closed form expression for moment generating function (MGF) and this MGF is then applied to study the end-to-end performance of the cooperative diversity networks in terms of BER for Differential M-ary phase shift keying (DMPSK), outage probability ($P_{\text{out}}$) and amount of fading ($\alpha$). Simulation results are used to validate the analytical analysis. The remainder of this paper is organized as follows. Section II introduces the system and channel models under investigation. Section III evaluates the end-to-end performance of the cooperating communication systems. Numerical results are presented in Section IV and concluding remarks are given in Section V.

II. SYSTEM MODEL

As shown in Fig. 1, an information source node (S) and a destination node (D) communicate over a channel with a Rayleigh-flat-fading coefficient (g). A relay node (R) cooperates by providing the destination with a second copy of the original signal through the two-hop link (S – R and R – D), with Rayleigh-flat-fading coefficients h and g, respectively. It is assumed that all fading coefficients are not known at the transmitter, relay, and receiver. Also, the channel fading coefficients are assumed to be approximately fixed over two symbol periods (quasi-static). Without any loss of generality, we assume that all the additive white Gaussian noise (AWGN) terms $n_1(t)$, $n_2(t)$ and $n_D(t)$, associated with the three links S – D, S – R and R – D, respectively, have equal variance ($N_0$).

For differential transmission, the information is conveyed in the difference of the phases of two consecutive symbols. Specifically, information symbols to be conveyed in the direction of the relaying among source and relay. Relaxing the symmetry condition allows us to analyze the impact of the relay location by including the distance-based path-loss in the channel model. We use the log-distance model, i.e., the received power decreases linearly with distance, on a logarithmic scale. An interesting approach proposed in [1, 11] suggests modeling the effects of path loss into the variance of the fading variables by setting $E(h^2) \propto d_{SR}$, $E(g^2) \propto d_{RD}$ and $E(f^2) \propto d_{SD}$ where $d_{ij}$ is the distance between terminal $i$ and $j$, $\alpha$ is the power exponent path, and $E$ is the statistical average operator.

III. PERFORMANCE ANALYSIS

This section studies the performance of the relay link first. Then, we include the direct link to analyze the performance of the combined signal at the destination.

A. Relay Link Only

For fading channel, the instantaneous $\text{SNR}$ for the relaying path can be written as

$$\gamma_R = \frac{(hAg)^2}{(Ag)^2 + 1} N_0$$

(2)

It is clear from (2) that the choice of the relay gain ($A$) affects the equivalent end-to-end $\text{SNR}$ of the relaying path. The relaying mobile terminal introduces fixed gains to the received signal regardless of the fading amplitude on the first hop. Let $D = 1/(A^2 N_0)$ then (2) can be written as

$$\gamma_{R,Fixed} = \frac{\gamma_h \gamma_g}{D + \gamma_g}$$

(3)

where $\gamma_h = h^2/N_0$ and $\gamma_g = g^2/N_0$. Now, the probability density function (PDF) of $\gamma_{R,Fixed}$ can be written as [9]

$$p_{\gamma_{R,Fixed}}(x) = \frac{2}{\gamma_h} e^{-x/\gamma_h} \left[ \sqrt{\frac{D}{\gamma_h}} K_1 \left( 2 \sqrt{\frac{D}{\gamma_h}} \right) + \frac{D}{\gamma_g} K_0 \left( 2 \sqrt{\frac{D}{\gamma_h}} \right) \right]$$

(4)

where $K_1(.)$ and $K_0(.)$ are the first and zero order modified Bessel function of the second kind defined in
\[ SNR = \gamma \] can be written as \[ \frac{1}{\gamma} = BER \gamma AF \] is the complex AF of the differentially coherent EGC. \[ + p^\infty \] is defined as the ratio of the variance power where \( a \) is independent from the relaying path, \[ \hat{h} \] is an important tool can so ft h e fading power. Hence, the SNR of the fading to the square of its mean, and hence it is a measure of the severity of the fading independent of the fading power. Hence, the \( R,\)Fixed can be written as \[ M_{\gamma R,\text{Fixed}}(s) = \mathbb{E}\left( e^{-s\gamma R,\text{Fixed}} \right). \] Hence, the MGF can be written as

\[ M_{\gamma R,\text{Fixed}}(s) = \int_0^\infty \frac{2}{\gamma h} e^{-s/\gamma h} \left[ \frac{\gamma h K_0(2\sqrt{\gamma h})}{2} + \frac{D^2}{\gamma h} K_0(2\sqrt{\gamma h}) \right] e^{-sx} \] \] With the help of [12, 13], (6) can be simplified as

\[ M_{\gamma R,\text{Fixed}}(s) = \frac{1}{1 + \gamma h s} + \frac{D \gamma h s (D/\gamma h)(\gamma h s + 1)}{\gamma h (\gamma h s + 1)^2} E_1 \left( \frac{D}{\gamma h (\gamma h s + 1)} \right) \] where \( E_1(x) = \int_0^\infty e^{-s/\gamma h} dt \) [12]. \( E_1(x) \) is called the exponential integral. Average SNR is an important tool used in analyzing the performance over fading channels [14]. Using (7) the average SNR can be written as

\[ \bar{\gamma}_{\text{R,Fixed}} = -\frac{\partial}{\partial s} M_{\gamma R,\text{Fixed}}(s) \bigg|_{s=0} = \gamma_h - \frac{D^2}{\gamma_h} E_1 \left( \frac{D}{\gamma h} \right) \] The AF is defined as the ratio of the variance power of the fading to the square of its mean, and hence it is a measure of the severity of the fading independent of the fading power. Hence, the AF can be written as [14]

\[ AF = \frac{VAR(\gamma_{\text{R,Fixed}})}{(\bar{\gamma}_{\text{R,Fixed}})^2} \] where \( VAR() \) denotes the variance. Hence the AF of relay link equals to

\[ AF = \frac{\partial^2}{\partial s^2} M_{\gamma R,\text{Fixed}}(s) \bigg|_{s=0} - \left( \frac{\partial}{\partial s} M_{\gamma R,\text{Fixed}}(s) \bigg|_{s=0} \right)^2 \] Finally after some algebraic manipulation the AF can be calculated as

\[ AF = \frac{\gamma_g^2 (2D + \gamma_g)}{\left[ \frac{D e^{D/\gamma_R} E_1 \left( \frac{D}{\gamma_g} \right)}{\gamma_g} - \gamma_g \right]^2} - \frac{D e^{D/\gamma_R} E_1 \left( \frac{D}{\gamma_g} \right)}{\gamma_g} \left[ 2 \left( \gamma_g + D \right) + \frac{D e^{D/\gamma_R} E_1 \left( \frac{D}{\gamma_g} \right)}{\gamma_g} \right] \]

For BDPSK, it is very simple to find out the exact error probability which equals to \( 0.5 M_{\gamma R,\text{Fixed}}(1) \) [14], so the BER can be written in compact closed form as

\[ P_{BDPSK}(e) = \frac{1}{1 + \gamma_h} + \frac{D^2 e^{\gamma_D/(\gamma_h+1)}}{2 \gamma_g (1 + \gamma_h)^2} E_1 \left( \frac{D}{\gamma_g (1 + \gamma_h)} \right) \] The error probability of general MDPSK using differential coherent detection can be evaluated by evaluating

the following integral using a simple numerical analysis [14, eq. (8.166)]

\[ P(e) = \frac{1}{\pi} \int_0^{(M-1)\pi/M} M_{\gamma R,\text{Fixed}} \left( \frac{\sin^2(\pi/M)}{1 + \sin^2(\pi/M)\cos(\varphi)} \right) d\varphi \] B. Combined Relay and Direct Links

At the destination, the received signals from the source and from the relay are combined together, and then the combined output is differentially decoded. Based on the multichannel differentially non-coherent detection [17], the combined signal prior to differential decoding is

\[ y = a_1 y_{D}(t - 1) y_{D}(t) + a_2 y_D(t - 1) y_D(t) \]

where \( a_1 = 1 \) , \( a_2 = \frac{D}{D + \gamma_D} \) and * is the complex conjugate. The destination decodes information as \( \hat{m} = \arg \max_{m=0,1,...,M-1} \text{Re} \{ d_{m}^{\text{out}} \} \). Even though the optimum value of \( a_2 \) is not practical since it relies on the CSI of \( \gamma_R \), the performance evaluation based on such optimum weight can be used as a performance benchmark. In this paper we provide a performance analysis for an ideal EGC that is obtained by using these optimum weights. In practice, we can use suboptimal value \( a_2 = \frac{D}{D + \gamma_D} \), which do not need CSI at the destination.

Note that the instantaneous SNR of the direct path \( \gamma_f = f^2/N_0 \) is independent from the relaying path, and the PDF of the direct link is given by \( p_{\gamma_f}(x) = (1/\gamma_f) e^{-x/\gamma_f} \), where \( \gamma_f = \mathbb{E}(f^2/N_0) \). Hence, the instantaneous SNR of the differentially coherent EGC combiner is the sum of the two independently links and it can be written as \( \gamma_{eq} = \gamma_{R,\text{Fixed}} + \gamma_f \).

Since we have two independent paths, the MGF of \( \gamma_{eq} \) is simply the product of the MGFs of the SNRs of
The outage probability can be expressed as [14]

$$P_{\text{out}} = L^{-1} \left( \frac{M_{\gamma_{\text{eq}}}(s)}{s} \right)_{s=\gamma_0}$$

(15)

where $L^{-1}(\cdot)$ denotes the inverse Laplace transform. Outage probability can be obtained by using any numerical technique for the Laplace transform inversion such as the numerical technique used in [15, 16].

Using the same method as in the relay link analysis, the AF of the combined signal can be written as

$$AF = \frac{\frac{\gamma_{\text{h}} \gamma_{\text{f}}}{\gamma_{\text{h}} \gamma_{\text{f}}} \left( 2D + \gamma_{\text{g}} \right) + \gamma_{\text{g}}^2 \gamma_{\text{f}}^2}{\left[ \gamma_{\text{h}} \gamma_{\text{f}} \gamma_{\text{g}} \gamma_{\text{f}} \right] \left( 2 \gamma_{\text{g}} + D \right) + D e^{D/\tau_{\text{g}}} \gamma_{\text{g}} \left( \frac{D}{\tau_{\text{g}}} \right) - \gamma_{\text{g}}(\gamma_{\text{h}} + \gamma_{\text{f}})}$$

$$D e^{D/\tau_{\text{g}}} \gamma_{\text{g}} \left( \frac{D}{\tau_{\text{g}}} \right) - \gamma_{\text{g}}(\gamma_{\text{h}} + \gamma_{\text{f}})^2$$

(16)

Finally, by using the error probability expression for two-channel DMPSK in [14, eq. 9.110], the unconditional error probability is given by

$$P(e) = \frac{1}{6\pi} \int_{-\pi}^{\pi} \frac{\psi(\varphi)}{1 + 2\beta \sin \varphi + \beta^2} M_{\gamma_{R,\text{Fixed}}}(\lambda) M_{\gamma_{f}}(\lambda) d\varphi$$

(17)

with $\psi(\varphi) = (1 - \beta^2) \left( 3 + \cos(2\varphi) - (\beta + 1/\beta) \sin \varphi \right)$, $0^\circ \leq \angle(a/b) = \beta \leq 1$ and $\lambda = (a^2 + b^2 + 2ab \sin \varphi)/2$, where $a$ and $b$ are constants and depend on modulation size, specifically, $a = 10^{-3}$ and $b = \sqrt{2}$ for DQPSK modulation, $a = \sqrt{2} - \sqrt{2}$ and $a = \sqrt{2} + \sqrt{2}$ for DQPSK modulation [14]. The values for larger modulation sizes can be obtained by using the result in [17].

While the expression found in (17) is exact, it still requires numerical evaluation of the integral. By using the same method in [14, eq. (9.113)] and after some algebraic manipulation, a closed form upper bound on the error performance can be written as

$$P(e) \leq \frac{1/(15\pi)}{\beta(1 - \beta)} \left[ (6\pi \beta(1 - \beta) + 3) M_{\gamma_{f}}(\zeta) M_{\gamma_{R,\text{Fixed}}}(\zeta) - (2\pi \beta(1 - \beta) + 3) M_{\gamma_{f}}(\upsilon) M_{\gamma_{R,\text{Fixed}}}(\upsilon) \right]$$

(18)

where $\zeta = b^2(1 - \beta)^2/2$ and $\upsilon = b^2(1 + \beta)^2/2$.

The performance analysis for fixed gain relay system given above can be used for any arbitrary fixed gains. In this paper we consider that the relay can measure the statistical information of the instantaneous SNR of the relaying path and has in particular the average fading power of the first hop. This does not need a continuous monitoring of the instantaneous SNR of the channel; and as in [9] this value of $D$ can be written as $D = \gamma_{\text{h}} e^{1/\tau_{\text{g}}} E_1(1/\gamma_{\text{h}})$. We will use this value in studying the cooperative communication networks performances in the results section.

IV. SIMULATIONS AND RESULTS

In this section, we compare the probability of error, the amount of fading and outage probability results developed in section III in various regimes. We use simulation to verify the derived analytical expressions. To make it clear in which context certain observations hold, we have divided this section into two subsections. Subsection IV-A focuses on the special case where the fading variances are identical, e.g. $E(f^2) = E(h^2) = E(g^2) = 1$ (without loss of generality). Subsection IV-B considers general asymmetric networks in which fading variances may be distinct. The primary observation of this sub-section is to analyze the effect of the location of the relaying terminal on $P_{\text{out}}$, $AF$ and error performance, which helps in future research for routing algorithms relay(s) selection.

A. Symmetric Networks

Fig. 2 compares the performance of $AF$ for the direct link, the relay link only, and the combined link. Note from this figure that as $\gamma_{R,\text{Fixed}}$ goes to infinity $AF$ of relay link only gets closer to 1. For the combined links as goes to infinity, $AF$ gets close to 0.5. It can be seen that in all cases $AF_{\text{Combined}} < AF_{\text{Direct Only}} < AF_{\text{Relay Only}}$. For instance, at $SNR = 20$ dB the combined link outperforms the direct link and the relay link by 2.53 dB and 3.47 dB, respectively. If the $SNR$ goes to infinity the combined links outperforms both the direct and relay links by 3 dB.

Fig. 3 shows the outage performance. This figure reveals that the diversity order of the combined link is 2. Also, Fig. 3 shows that the combined link outperforms both the direct and relay links. Fig. 4 shows that $BER$ performance. Again, this figure shows that the combined link outperforms both the direct and relay links. Fig. 4 reveals that the diversity order of the combined link is 2. This can be translated to a significant reduction in the required $SNR$. For example to achieve $10^{-3}$ error probability, you need approximately $SNR=15$ dB (31.6) in combined link and $SNR=27$ dB (501.2) in the direct link and $SNR=30$ dB (1000) in the relay link. It should be noted that due to symmetry the direct link outperforms the relay in error and outage performances.

B. Asymmetric Networks

Without any loss of generality we assume the location of the source at (0) and the location of the destination...
for the relaying terminal will be located in ($r$) point where $0 < r < 1$ and the power exponent $\alpha = 4$. Fig. 5 shows the effect of the location of intermediate relay terminal on the $AF$ at $SNR = 10$ dB. Fig. 5 reveals that as the relay terminal gets closer to the destination, there is a reduction in the $AF$. This behavior is due to the fact the factors $\gamma_h$ and $D$ decreases and only $\gamma_g$ increases. This figure also reveals that $AF$ of direct link outperforms the combined link when the relay terminal is closer to the source and vice versa.

Fig. 6 and Fig. 7 show $P_{out}$ and error performance for different location of the relay terminal at $SNR = 10$ dB. The interesting result is that the $BER$ and $P_{out}$ are more resistible to increase if the terminal relay is closer to the source until the midway point and this is due to the fact that there is a good channel between the source and relay so it will have also a high gain.

V. CONCLUSIONS
This paper examined the performance of a differential amplify-and-forward relay scheme for single relay cooperative wireless communication systems. We have obtained analytical closed-form expressions of the distribution for the instantaneous SNR, the amount of fading ($AF$), the outage probability and the error perfor-
We have shown that differential EGC technique for amplify-and-forward relay scheme offers remarkable diversity advantage (despite its simplicity) over direct transmission and conventional non-cooperative relaying over Rayleigh-fading channels. Also in this paper we have investigated the effect of the location of the relay terminal on the performance.

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