Adaptive Region Tracking Control for Autonomous Underwater Vehicle

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Abstract—This paper presents an adaptive region tracking control for Autonomous Underwater Vehicle (AUV). The AUV is required to track a moving region to accomplish a given task. The desired target is specified as a region rather than a point so that the control effort used to track the region is minimal. In the applications where the accuracy is of utmost importance, the desired region can be chosen to be small so that the precision is not lost. The desired region can be scaled up or scaled down so that the AUV can adjust its position to suit the applications. A Lyapunov-like function is presented for the stability analysis. Simulation results on AUV with 6 degrees of freedom are presented to demonstrate the effectiveness of the proposed controller.

I. INTRODUCTION

Autonomous underwater vehicle (AUV) is useful in many underwater missions ranging from undersea oil and gas exploration, pipeline/cable inspection and maintenance, data collection for environmental study, waste cleaning to various military applications. Many research efforts have been devoted to the study of AUV [1]-[14]. Several control schemes have been proposed for the AUV [6]-[13]. Among those control methods, the proportional-derivative (PD) control plus gravity compensation [6] is the simplest setpoint control for the AUV. However, the dynamics of AUV are dependent on the environmental conditions and therefore it is very difficult to obtain the exact model of the gravitational and buoyancy force. To overcome parametric uncertainties, several adaptive control laws [7]-[13] have been proposed.

The aforementioned results are focusing on setpoint control where the desired target is specified as a point. In conventional setpoint control, the position errors are used to drive the AUV toward the desired point. Due to the presence of the wave current, the control input from the propeller is needed to counter balance the force of the current to keep the vehicle at the desired location. Since the desired target is a point, even a small disturbance of the current would move the AUV out of the position and thus activate the propellers to counter balance the current wave. Therefore, the propellers will usually be active and hence consume a lot of energy. Similarly, in a conventional tracking control, motion errors are used to keep the vehicle on the path. A small disturbance of the wave current would drift the vehicle off the trajectory and hence high control effort is needed to keep the vehicle on the trajectory as well as maintaining the velocity of the vehicle.

However, it is interesting to observe that in some applications of AUV, the desired target is a region instead of a point. For example, maintaining the AUV within a minimum and maximum depth in water, AUV traveling inside a pipeline for specific task, station keeping within a specified region for observation and data collection etc. Therefore, it is not necessary for the AUV to dock at a very specific location or travel along a specific path. By defining a desired target as a region rather than a point, it would require less effort (less energy) for the AUV to perform docking or tracking. This is very significant for the case of AUV because in the presence of the wave current, it is very hard to keep the vehicle on the target point or trajectory. Region reaching control has been proposed for AUV in [14], [15]. However, the desired region in these methods is stationary. In some applications such as moving inside a pipeline and travelling along a specific route to collect data, the desired region is not static.

In this paper, we propose an adaptive region tracking for AUV. In the proposed adaptive controller, the desired target is specified as a moving region. Unlike the conventional control method, the proposed controller is only activated when the AUV is outside the desired region. In the presence of disturbance, the controller does not need to be activated if the disturbance is not strong enough to move the AUV out of the region. In an application where precision is of utmost importance, it is possible to define the region to be small so that the precision will not be lost. The region tracking control concept is therefore a generalization of the conventional tracking control problem. The high control effort is only needed when necessary i.e. when precision is important. When the precision is not critical, a rather low control effort is used to minimize the energy consumption. The desired region can be scaled up or scaled down so that the AUV can adjust its position to suit the applications. A Lyapunov-like function is presented for the stability analysis. Simulation results on AUV with 6 degrees of freedom are presented to demonstrate the effectiveness of the proposed controller.

II. DYNAMICS

In this section, the structure and properties of an underwater vehicle’s kinematics and dynamics are briefly reviewed. It is convenient to define the underwater vehicle state vectors according to the Society of Naval Architects and Marine
Engineers (SNAME) notation [6]. Two common vectors that are used in defining the underwater vehicle state vector are \(\eta\) and \(v\). The vector \(\eta\) is defined as: 
\[ \eta = [\eta_1^T \eta_2^T]^T, \]
where \(\eta_1 = [x \ y \ z]^T\) is the vehicle position vector in the earth fixed frame and \(\eta_2 = [\phi \ \theta \ \psi]^T\) is the vehicle Euler angle in the earth fixed frame. The vector \(v\) is defined as: 
\[ v = [v_1^T \ v_2^T]^T, \]
where \(v_1 = [u \ v \ w]^T\) is the body fixed linear velocity vector and \(v_2 = [p \ q \ r]^T\) is the body fixed angular velocity vector. Beside the Euler angle representation, Euler parameters or unit quaternions [6], [16] can also be used.

The vehicle’s motion path relative to the earth fixed frame coordinate system is given by the kinematics equation [6] as follows:
\[
\dot{\eta} = J(\eta)v = \begin{bmatrix} J_1(\eta) & 0 \\ 0 & J_2(\eta) \end{bmatrix} v, \tag{1}
\]
where \(J(\eta)\) is a \(6 \times 6\) kinematics transformation matrix or Jacobian matrix.

The dynamics behavior of an underwater vehicle is described through Newton’s laws of linear and angular momentum. The equations of motion of underwater vehicles are highly nonlinear and coupled due to hydrodynamic added mass, lift and drag forces, which are acting on the vehicle. The equations of motion of the underwater vehicle are given as [6]:
\[
M \ddot{v} + C(v)v + D(v)v + g(\eta) = \tau. \tag{2}
\]
The vector \(\tau \in \mathbb{R}^6\) is the vector of generalized forces on the AUV which is supplied by the thrusters. The matrix \(M\) is the inertia matrix of the AUV which includes both the rigid body and the added mass term. The matrix \(C(v)\) denotes the matrix of Coriolis and centripetal forces which includes both the rigid body and the added mass terms. The matrix \(D(v)\) represents the hydrodynamic damping and lift force and the vector \(g(\eta)\) denotes the gravitational and buoyancy force.

The underwater vehicle dynamics described in (2) has the following properties [6]:

**Property 1:** The inertia matrix \(M\) is symmetric and positive definite such that \(M = M^T > 0\).

**Property 2:** The Coriolis and centripetal matrix \(C(v)\) is skew-symmetric matrix such that \(C(v) = -C(v) \quad \forall \ v \in \mathbb{R}^6\).

**Property 3:** The hydrodynamic damping matrix \(D(v)\) is strictly positive such that \(D(v) > 0 \quad \forall \ v \in \mathbb{R}^6\).

**Property 4:** The dynamic model as described by equation (2) is linear in a set of physical parameters \(\theta_d = (\theta_{d1}, ..., \theta_{dm})\):
\[
M \ddot{v} + C(v)v + D(v)v + g(\eta) = Y_d(\eta, v, \dot{v}, \dot{\theta}) \theta_d. \tag{3}
\]
where \(Y_d(\eta, v, \dot{v}, \dot{\theta})\) is called the dynamic regressor.

III. ADAPTIVE REGION TRACKING CONTROL OF AUV

Let us define a desired region as specified by the following inequality functions:
\[
f(\Delta \eta_S) = \begin{bmatrix} f_1(\Delta \eta_{S1}) \\ f_2(\Delta \eta_{S2}) \\ \vdots \\ f_N(\Delta \eta_{SN}) \end{bmatrix} \leq 0, \tag{4}
\]
where \(\Delta \eta_{Si} = S(\eta - \eta_{oi}) = S \Delta \eta, \ \eta_{oi}\) is the reference point of \(f_i(\Delta \eta_{Si})\) \((i = 1, 2, ..., N)\), \(N\) is the total number of objective functions, \(S(t)\) is a time-varying and nonsingular scaling matrix defined as:
\[
S = \begin{bmatrix} S_1 & 0 \\ 0 & S_2 \end{bmatrix}, \tag{5}
\]
In the above equation, \(S_1\) is the scaling matrix of \(\eta_1\), and \(S_2\) is the scaling matrix of \(\eta_2\). Note that scaling of the orientation of AUV is not required in general and hence \(S_2\) can be set as an identity matrix. The functions \(f_i(\Delta \eta_{Si})\) are scalar functions with continuous partial derivatives. Note that the desired region is a single region specified as the intersections of all the objective functions. Therefore, all desired regions must move at the same speed of \(\dot{\eta}_o\) (\(\eta_o\) is a common reference point) so that the desired shape is preserved. Various desired region such as a sphere, a cube, a cylinder etc. can be formed by choosing the appropriate functions.

![An illustration of desired region](image)

An example of the desired region is shown in Figure 1. The inequality functions that describe this region are specified as:
\[
\begin{align*}
f_1(\Delta \eta_{S1}) &= (x_S - x_o)^2 - b_x^2 \leq 0, \\
f_2(\Delta \eta_{S2}) &= (y_S - y_o)^2 - b_y^2 \leq 0, \\
f_3(\Delta \eta_{S3}) &= (z_S - z_o)^2 - b_z^2 \leq 0,
\end{align*}
\tag{6}
\]
where \(b_x, b_y\) and \(b_z\) are the individual regional bounds for each axis, and
\[
\begin{bmatrix}
x_S - x_o \\
y_S - y_o \\
z_S - z_o
\end{bmatrix} = S_1
\begin{bmatrix}
x - x_o \\
y - y_o \\
z - z_o
\end{bmatrix}.
\tag{7}
\]
The scaling matrix \(S_1\) is given by:
\[
S_1 = \begin{bmatrix}
x_o & 0 & 0 \\
0 & y_o & 0 \\
0 & 0 & z_o
\end{bmatrix},
\]
where \(s_x(t), s_y(t)\) and \(s_z(t)\) are scaling factors. It should be noted when the scaling factors increase, the desired region decreases, and vice versa.

Equation (4) specifies a moving region, where the reference point \(\eta_{oi}\) is time-varying. The size of the moving region can be changed by scaling, which is necessary when the AUV needs to adjust its position to suit the applications. An illustration of scaling region is shown in Figure 2.
The potential energy function for the desired regions described in inequality (4) can be specified respectively as:

\[ P(\Delta \eta_S) = \sum_{i=1}^{N} P_i(\Delta \eta_{Si}), \]  

where

\[ P_i(\Delta \eta_{Si}) = \frac{k_{pi}}{2} [\max(0, f_i(\Delta \eta_{Si}))]^2. \]  

That is,

\[ P_i(\Delta \eta_{Si}) = \begin{cases} \frac{k_{pi}}{2} f_i^2(\Delta \eta_{Si}), & f_i(\Delta \eta_{Si}) > 0, \\ 0, & f_i(\Delta \eta_{Si}) \leq 0, \end{cases} \]

where \( k_{pi} \) are positive constants. The above energy function is lower bounded by zero. Note that \( P(\Delta \eta_S) = 0 \) only if all the inequality functions (8) are satisfied.

Partial differentiating the potential energy function described by (9) with respect to \( \Delta \eta_S \) yields:

\[ \left( \frac{\partial P(\Delta \eta_S)}{\partial \Delta \eta_S} \right)^T = \begin{cases} k_{pi} f_i(\Delta \eta_{Si}) \frac{\partial f_i(\Delta \eta_{Si})}{\partial \Delta \eta_S}, & f_i(\Delta \eta_{Si}) \leq 0, \\ 0, & f_i(\Delta \eta_{Si}) > 0, \end{cases} \]

Therefore,

\[ \left( \frac{\partial P(\Delta \eta_S)}{\partial \Delta \eta_S} \right)^T = \sum_{i=1}^{N} k_{pi} \max(0, f_i(\Delta \eta_{Si})) \frac{\partial f_i(\Delta \eta_{Si})}{\partial \Delta \eta_S} \right)^T = \Delta \xi, \]

where \( \Delta \xi \) denotes the region error which drives the vehicle toward the desired region. After the vehicle is inside the desired region, the gradient of the potential energy is zero and hence \( \Delta \xi \) also reduces to zero.

Based on the region error, a reference vector is proposed as:

\[ v_r = J^{-1}(\eta)(\dot{\eta}_o - S^{-1}\dot{S}\Delta \eta) - \alpha J^{-1}(\eta)S^{-1}\Delta \xi, \]

where \( J^{-1}(\eta) \) is the inverse of the Jacobian matrix, \( \dot{S} \) is the time derivative of \( S \), \( S^{-1} \) is the inverse of the scaling matrix, and \( \alpha \) is a positive constant.

Differentiating inequality (11) with respect to time, we get:

\[ \dot{v}_r = J^{-1}(\eta)(\ddot{\eta}_o - S^{-1}\dot{S}\Delta \eta) + J^{-1}(\eta)[\ddot{\eta}_o - S^{-1}\dot{S}\Delta \eta] - \alpha J^{-1}(\eta)S^{-1}\Delta \xi - \alpha J^{-1}(\eta)(S^{-1}\Delta \xi + S^{-1}\Delta \xi), \]

where \( \Delta \dot{\eta} = \dot{\eta} - \dot{\eta}_o. \)

A sliding vector is then defined as:

\[ s = v - v_r = v - J^{-1}(\eta)(\dot{\eta}_o - S^{-1}\dot{S}\Delta \eta) + \alpha J^{-1}(\eta)S^{-1}\Delta \xi. \]

Differentiating equation (13) with respect to time yields:

\[ \dot{s} = \dot{v} - \dot{v}_r. \]

Substituting equations (13) and (14) into equation (2) and using Property 3, we have:

\[ M \dot{s} + C(v)s + D(v)s + Y_d(\eta, v, v_r, \dot{v}_r)\theta_d = \tau, \]

where

\[ Y_d(\eta, v, v_r, \dot{v}_r)\theta_d = M\dot{v}_r + C(v)v_r + D(v)v_r + g(\eta). \]

The region tracking controller for AUV is proposed as:

\[ \tau = -J^T(\eta)S\Delta \xi - K_v s + Y_d(\eta, v, v_r, \dot{v}_r)\hat{\theta}_d, \]

where \( K_v \) is a positive constant matrix.

The estimated parameters \( \hat{\theta}_d \) are updated using the following update law:

\[ \dot{\hat{\theta}}_d = -L_d Y_d^T(\eta, v, v_r, \dot{v}_r)s, \]

where \( L_d \) is symmetric positive definite.

The closed-loop dynamic equation is obtained by substituting equation (17) into equation (15):

\[ M \dot{s} + C(v)s + D(v)s + Y_d(\eta, v, v_r, \dot{v}_r)\Delta \theta_d + J^T(\eta)S\Delta \xi + K_v s = 0. \]

Next, a Lyapunov-like function is defined as:

\[ V = \frac{1}{2}s^T M s + P(\Delta \eta_S) + \frac{1}{2}\Delta \theta_d^T L_d^{-1} \Delta \theta_d. \]

Differentiating equation (20) with respect to time:

\[ \dot{V} = s^T M \dot{s} - \theta_d^T L_d^{-1} \Delta \theta_d + \sum_{i=1}^{N} k_{pi} \max(0, f_i(\Delta \eta_{Si}))(\dot{\Delta} \eta_{Si} + S \Delta \eta) \left( \frac{\partial f_i(\Delta \eta_{Si})}{\partial \Delta \eta_S} \right)^T. \]

Substituting update law (18) and equation (19) into equation (21) and using Property 2, we have:

\[ \dot{V} = s^T[-C(v)s - D(v)s - K_v s - J^T(\eta)S\Delta \xi] + \sum_{i=1}^{N} k_{pi} \max(0, f_i(\Delta \eta_{Si}))(\dot{\Delta} \eta_{Si} + S \Delta \eta) \left( \frac{\partial f_i(\Delta \eta_{Si})}{\partial \Delta \eta_S} \right)^T - s^T K_v s - s^T D(v)s - s^T J^T(\eta) \left( \frac{\partial f_i(\Delta \eta_{Si})}{\partial \Delta \eta_S} \right)^T \Delta \xi + \alpha J^{-1}(\eta)S^{-1} \Delta \xi^T \left( \frac{\partial f_i(\Delta \eta_{Si})}{\partial \Delta \eta_S} \right)^T \Delta \xi = -s^T K_v s - s^T D(v)s - \alpha \Delta \xi^T \Delta \xi. \]

We are now in a position to state the following theorem:

**Theorem:** The adaptive control law (17) and the update laws (18) for the AUV system (2) guarantee the convergence of \( \Delta \xi \to 0 \) and \( s \to 0 \) as \( t \to \infty \).

**Proof:** Since \( M(q) \) is uniformly positive definite, \( V \) in equation (20) is positive definite in \( s, \Delta \theta_d \). Since \( V > 0 \) and \( \dot{V} \leq 0, V \) is bounded. Hence, \( s, \Delta \theta_d \) and \( P(\Delta \eta_S) \) are bounded. From
equation (9), \( f_i(\Delta \eta_{S_i}) \) is also bounded. The boundedness of \( f_i(\Delta \eta_{S_i}) \) ensures the boundedness of \( \eta \). Hence, \( \frac{\partial f_i(\Delta \eta_{S_i})}{\partial \Delta \eta_{S_i}} \) is also bounded. Therefore, from equation (10), \( \Delta \xi \) is bounded. Since \( \eta \) is bounded, \( v_r \) is bounded if \( \eta_0 \) is bounded. Since \( s \) is bounded, from equation (13), \( v \) is bounded. The boundedness of \( v \) guarantees the boundedness of \( \dot{\eta} \) since \( \dot{\eta} = J(\eta)v \) and \( J(\eta) \) is trigonometric function. Since \( \frac{\partial f_i(\Delta \eta_{S_i})}{\partial \Delta \eta_{S_i}} \), \( \eta \), \( \dot{\eta} \) are bounded, \( \Delta \xi \) is bounded. Then \( \dot{v}_r \) is bounded. From the closed-loop equation (19), we can conclude that \( \dot{s} \) is bounded. Thus, \( \dot{v} \) is bounded since \( \dot{s} = \dot{v} - \dot{v}_r \). Differentiating equation (22) with respect to time we get:

\[
\dot{V} = -2(\dot{s}^T K_v s + s^T D(v)s + \alpha \Delta \xi^T \Delta \xi) - s^T \dot{D}(v)s, \quad (23)
\]

Hence, \( \dot{V} \) is bounded since \( \dot{s} \), \( s \), \( \Delta \xi \), \( \Delta \xi \) are bounded. Therefore, \( V \) is uniformly continuous. Applying Barbalat’s lemma [17], we have \( \dot{V} \rightarrow 0 \) which also indicates: \( \Delta \xi \rightarrow 0 \) and \( s \rightarrow 0 \). From equation (10), \( \Delta \xi \rightarrow 0 \) indicates that \( f_i(\Delta \eta_{S_i}) \leq 0 \) or \( \frac{\partial f_i(\Delta \eta_{S_i})}{\partial \Delta \eta_{S_i}} \rightarrow 0 \). Therefore, \( \eta \) converges to the moving desired region.

### IV. SIMULATION

This section presents some simulation results to illustrate the effectiveness of the proposed controller. The simulations were performed on the Omni Directional Intelligent Navigator (ODIN) [18] with full 6 DOF. ODIN is a spherical underwater vehicle designed in the University of Hawaii. The details of the dynamic model of ODIN can be found in [18].

In the first simulation, the desired region was specified as a cube given by the following inequality functions:

\[
\begin{align*}
(x_S - x_o)^2 &\leq b_x^2, \\
(y_S - y_o)^2 &\leq b_y^2, \\
(z_S - z_o)^2 &\leq b_z^2.
\end{align*}
\]

The allowable orientation errors were given by:

\[
\begin{align*}
(\phi_S - \phi_o)^2 &\leq b_{\phi}^2, \\
(\theta_S - \theta_o)^2 &\leq b_{\theta}^2, \\
(\psi_S - \psi_o)^2 &\leq b_{\psi}^2,
\end{align*}
\]

where \([b_x, b_y, b_z, b_{\phi}, b_{\theta}, b_{\psi}]^T = [0.3, 0.3, 0.3, 0.05, 0.05, 0.05]^T\). The initial position and orientation of the AUV were given as \([7, -3, 1, \pi/18, -\pi/18, \pi/12]^T\). The reference point \( \eta_o = [x_o, y_o, z_o, \phi_o, \theta_o, \psi_o]^T \) was specified as follow:

\[
\begin{align*}
x_o &= 5 + r \sin(\omega t), \\
y_o &= -2 + r \cos(\omega t), \\
z_o &= 3 + 0.05t, \\
\phi_o &= 0, \quad \theta_o = 0, \quad \psi_o = 0,
\end{align*}
\]

where \( r = 1 \) and \( \omega = 0.1 \text{ rad/s} \). The scaling factor was set as \( S = I_{6 \times 6} \), where \( I_{6 \times 6} \) was the identity matrix. Figure 3 and Figure 4 showed the simulation results for the tracking controller with \( k_p = \text{diag}\{1000, 1000, 1000, 3000, 3000, 3000\} \), \( K_v = \text{diag}\{500, 500, 500, 10, 10, 10\} \), \( L_d = 10 I_{9 \times 9} \) and \( \alpha = 1 \). From Figure 3 and Figure 4, it was observed that all position and orientation errors converged to desired region in about 10s. The yaw error took slightly longer to converge due to larger initial error. Figure 5-8 showed the positions of AUV at various time instances.

In the second simulation, the AUV was required to pass through a connected pipe, where the ending part was smaller than the beginning part. The smaller pipe had a diameter of 2 m while the bigger pipe had a diameter of 3.8 m. The desired region in this case was specified as a sphere characterized by the following inequality function:

\[
(x_S - x_o)^2 + (y_S - y_o)^2 + (z_S - z_o)^2 \leq R^2, \quad (26)
\]

where \( R = 1.2 \text{ m} \). The desired trajectory was specified as:

\[
\begin{align*}
x_o &= 5 - 0.1t, \\
y_o &= -2, \\
z_o &= 3, \\
\phi_o &= 0, \quad \theta_o = 0, \quad \psi_o = 0,
\end{align*}
\]

The size of the desired region was decreased so that the AUV could pass through without colliding with the pipe. The scaling factor was set to be \( S_1 = (1 + 0.1t) I_{3 \times 3} \) and hence the size of the desired region was decreasing. The control parameters were set as: \( k_p = \text{diag}\{2000, 2000, 2000, 700, 700, 700\} \), \( K_v = \text{diag}\{300, 300, 300, 50, 50, 50\} \), \( L_d = 10 I_{9 \times 9} \) and \( \alpha = 1 \). Figure 9-13 showed the positions of AUV at various time instances.

### V. CONCLUSION

In this paper, we have proposed a region tracking control of AUV. It has been shown that the AUV is able to track the moving region even when the region changes its size. Lyapunov-like function has been proposed for the stability analysis of the system. Simulation results have been presented to illustrate the performance of the proposed controller.
Fig. 4. Tacking errors of AUV position

Fig. 5. The initial position of AUV and desired region \((t = 0)\)

Fig. 6. The path of AUV in rising to surface \((t = 5s)\)

Fig. 7. The path of AUV rising to surface \((t = 35s)\)

Fig. 8. The path of AUV in rising to surface \((t = 65s)\)

Fig. 9. The initial position of AUV and desired region \((t = 0)\)
REFERENCES


