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Abstract—This paper presents a method to estimate the transmission line parameters using synchrophasor measurements from phasor measurement units (PMUs), as well as hybrid measurements from PMU and supervisory control and data acquisition system. It also proposes a method to estimate transmission line parameter uncertainties considering the measurement inaccuracies. Transmission line protection is important for secure and reliable operation of the power system. With the advent of the synchrophasor technology and advances in the communication system, current differential relays can be implemented as a reliable protection system for the transmission lines. In this paper, a new current differential protection scheme has been suggested for transmission line protection using synchronized measurements. The effect of parameter uncertainty on the protection scheme is also investigated.

Index Terms—Current differential protection, parameter uncertainties, phasor measurement unit (PMU), transmission line modeling.

I. INTRODUCTION

Accurate determination of transmission line parameters is important for all types of system simulation studies, such as load flow, transient stability, and state estimation. These are also important to improve the accuracy in relay settings and post-fault location. A method for estimation of transmission line parameters, using measurements of voltage, current, and power or power factor is discussed in [1]. Estimation of positive and zero sequence parameters, using wide area measurements, is reported in [2]. Reference [3] discussed four methods to estimate short transmission line parameters using synchronized phasor measurements. All the methods using synchronous measurements require large number of phasor measurement units (PMUs) to get synchrophasor data from each and every bus, which may not be economically viable. Due to their relatively high cost, the utilities are providing PMUs at a few selected locations only. It is a well-known fact that the PMU measurements have higher resolution and accuracy compared with the conventional supervisory control and data acquisition (SCADA) measurements. Motivation for PMU placement into the existing conventional measurement system is discussed in [4]. The PMU measurements can be used along with the existing SCADA to improve the accuracy of various applications. If one can estimate the line parameters using the combination of PMU and SCADA measurements, the accuracy will be better as compared with that with only SCADA measurements. In this paper, a method is proposed to estimate the transmission line parameters using only synchronous (PMU) measurements as well as using hybrid (combination of PMU and SCADA) measurements.

It is not only sufficient to know the exact value of the line parameters, but also desirable to know the bounds of the parameter values, within which it will vary because of the measurement inaccuracies. The range of the line parameter variation is needed to set the bounds of the restrain region for the current differential relay and to know the uncertainty in the power system state estimation. A methodology to estimate the uncertainties in the estimated states of the power system due to measurement inaccuracies is reported in [5]. Reference [6] proposed a methodology to evaluate the uncertainties associated with the states measured or computed by the PMU. In this paper, a methodology is also proposed to evaluate the uncertainties associated with the estimated transmission line parameters due to the inaccuracies in the measurements.

Transmission line protection is important to ensure stable operation of the power system. Usually, the distance relays are used to protect the transmission lines in a power system. However, sometimes the distance relays mal-operate under dynamic conditions such as power swings and voltage instability. Because of the rapid advancement in the communication technology, current differential relays are being used for protection of the transmission lines [7], [8]. Current differential protection is relatively simple, offers high speed and sensitivity, and is also immune to power swings and external faults. Fig. 1 shows the schematic of a line current differential protection. Neglecting the line capacitances, in the absence of fault, or fault outside the protected zone, the current flowing into the protected short transmission line is equal to the current flowing out of the line.

Under normal condition or external fault

\[
\bar{I}_1 = \bar{I}_2. \tag{1}
\]
If a fault develops inside the protected zone, the currents \( I_1 \) and \( I_2 \) are no longer equal. For the comparison of currents, remote end measurements should be transmitted to the relay. Hence, it requires a high-speed and reliable communication network. Modern high-speed communication networks typically use synchronized optical network or synchronized digital hierarchies standard for communication, having transmission rates in the order of 274.2 or 155.5 Mb/s, respectively [9]. Reference [9] proposes an adaptive current differential protection scheme with equivalent- \( \pi \) model of the transmission line. Current differential protection for ultra-high voltage transmission line, using distributed parameter model is discussed in [10]. In this paper, a simple method of current differential protection scheme using synchronized phasor measurements is proposed, which is based on the distributed parameter model of the line.

This paper is organized as follows. Estimation of the transmission line parameters using PMU and hybrid measurements is discussed in Section II. In Section III, a methodology to evaluate the uncertainties associated with the transmission line parameters due to the inaccuracies in the measurements is described. Section IV explains the new current differential protection algorithm using synchronized measurements. The simulation results are presented in Section V, and Section VI concludes this paper.

II. TRANSMISSION LINE PARAMETER ESTIMATION

The equivalent \( \pi \) network of a long line [11] is shown in Fig. 2, which provides

\[
\begin{align*}
\hat{V}_s &= (1 + ZY)\hat{V}_r + Z\hat{I}_r \\
\hat{I}_s &= (2\hat{Y} + ZY^2)\hat{V}_r + (1 + Z\hat{Y})\hat{I}_r
\end{align*}
\]

where \( \hat{V}_s, \hat{V}_r \) are the sending and receiving end voltages and \( \hat{I}_s, \hat{I}_r \) are the sending and receiving end currents, respectively. \( Z \) and \( Y \) are the series impedance and shunt admittances of the equivalent \( \pi \) network of the long line.

After solving (2) and (3), one can obtain

\[
\begin{align*}
\hat{Z} &= \frac{\hat{V}_s^2 - \hat{V}_r^2}{I_sV_r + V_sI_r} \\
\hat{Y} &= \frac{\hat{I}_s - \hat{I}_r}{V_s + V_r}.
\end{align*}
\]

Using the distributed parameter model of the long transmission line, the expressions for the sending end voltage \( \hat{V}_s \) and sending end current \( \hat{I}_s \) are given by

\[
\begin{align*}
\hat{V}_s &= \cosh(\gamma L) \cdot \hat{V}_r + \hat{Z}_0, \sinh(\gamma L) \cdot \hat{I}_r \\
\hat{I}_s &= \frac{\sinh(\gamma L)}{\hat{Z}_0} \cdot \hat{V}_r + \cosh(\gamma L) \cdot \hat{I}_r
\end{align*}
\]

where \( \hat{Z}_0 = \sqrt{\frac{Z}{Y}} \) is the characteristic impedance, \( \gamma = \sqrt{\frac{Z}{Y}} \) is the propagation constant, \( L \) is the length of the transmission line, and \( \hat{Z} \) and \( \hat{Y} \) are the distributed series impedance and shunt admittance of the transmission line per unit length.

Comparing (2) and (3) with (6) and (7), one can obtain

\[
\hat{\gamma} = \frac{\cosh^{-1}(1 + \frac{Z\hat{Y}}{L})}{\hat{Z}_0} \quad \hat{\tilde{Z}}_0 = \frac{\hat{Z}}{\sinh(\gamma L)}.
\]

The line distributed parameters, i.e., resistance \( r \), inductance \( l \), and capacitance \( c \) of the long transmission line are obtained from (9) and (10).

A. Synchronous Measurements

Consider a transmission line with PMUs installed at both the ends, as shown in Fig. 3. Using a PMU, the voltage phasor of the bus and current phasors on the incident lines can be measured [12]. The measurements coming from each PMU are time stamped and synchronized to the universal time coordinated. The aim is to estimate the distributed positive sequence parameters, i.e., resistance, inductance, and capacitance of the line per unit length. The measurements with the same time stamp can be used to estimate the transmission line parameters.

The phasors measured are

\[
\begin{align*}
\hat{V}_s &= V_i \angle \delta_s = V_i(\cos\delta_s + j\sin\delta_s) \\
\hat{V}_r &= V_i \angle \delta_r = V_i(\cos\delta_r + j\sin\delta_r) \\
\hat{I}_s &= I_i \angle \theta_s = I_i(\cos\theta_s + j\sin\theta_s) \\
\hat{I}_r &= I_i \angle \theta_r = I_i(\cos\theta_r + j\sin\theta_r).
\end{align*}
\]

Using these measurements, the line parameters can be obtained through the following steps.

1) Substituting \( \hat{V}_s, \hat{V}_r, \hat{I}_s, \) and \( \hat{I}_r \) measured by PMUs with same time stamp in (4) and (5) and separating the real
and imaginary parts, the following four equations can be obtained:

\[ R = \frac{ac + bd}{c^2 + d^2} \]  
\[ X = \frac{bc - ad}{c^2 + d^2} \]  
\[ G = \frac{eg + fh}{g^2 + h^2} \]  
\[ B = \frac{fg - eh}{g^2 + h^2} \]

The variables \( a, b, c, d, e, f, g, \) and \( h \) are defined in Appendix A.

2) Calculate \( \tilde{\gamma} \) and \( \tilde{Z}_0 \) by substituting \( \tilde{Z} = R + jX \) and \( \tilde{Y} = G + jB \) obtained from previous step in (8).

3) Evaluate distributed parameters \( \tilde{r} = r + jx \) and \( \tilde{\gamma} = g + jh \) using (9) and (10) and calculate the inductance and capacitance using \( l = x/2\pi F \) and \( c = b/2\pi F \), respectively, where \( F = \) system frequency.

**B. Asynchronous Measurements**

Consider the transmission line shown in Fig. 4, where a PMU is installed at bus \( j \). The voltage phasor \( \bar{V}_r \) and current phasor \( \bar{I}_r \) are, therefore, known. The bus \( i \) is provided with SCADA measurement, which is able to measure the power flow \( P_r \), voltage magnitude \( V_r \), and current magnitude \( I_r \) at bus \( i \). The phase angle \( \phi_r \) between the voltage and current at bus \( i \) can be calculated from these SCADA measurements. These measurements are refreshed every 4–5 s. In the case of PMU measurements, refresh rate can be up to a few kilohertz, and each measurement from a PMU is aligned with a time stamp [12]. The conventional measurements used by the SCADA system also carry local time stamps. Using the two time stamps, the synchronous PMU measurements can be combined with the asynchronous conventional measurements. In case there is no common instant corresponding to the measurements, one set of measurements can be interpolated [4].

The phasors at the buses \( i \) and \( j \) are not synchronized. A synchronization angle \( \delta \) is used to synchronize the voltage phasors at both the terminals. Using the synchronization angle \( \delta \), the phasors at the bus \( i \) with respect to global positioning system (GPS) reference can be defined as

\[ \bar{V}_i = V_i \angle \delta = V_i (\cos \delta + j \sin \delta) \]  
\[ \bar{I}_i = I_i \angle \delta - \phi_i = I_i (\cos (\delta - \phi_i) + j \sin (\delta - \phi_i)). \]

The phasors obtained from PMU are

\[ \bar{V}_r = V_r \angle \delta_r = V_r (\cos \delta_r + j \sin \delta_r) \]  
\[ \bar{I}_r = I_r \angle \delta_r = I_r (\cos \delta_r + j \sin \delta_r). \]

Substituting \( \bar{V}_i, \bar{V}_r, \bar{I}_i, \) and \( \bar{I}_r \) in (5) and separating the real and imaginary parts gives the real part as

\[ G = \frac{K M + L N}{M^2 + N^2} \]

where

\[ K = I_r \cos (\delta - \phi_r) - I_r \cos \theta_r \]  
\[ L = I_r \sin (\delta - \phi_r) - I_r \sin \theta_r \]  
\[ M = V_r \cos \delta + V_r \cos \phi_r \]  
\[ N = V_r \sin \delta + V_r \sin \phi_r. \]

Since the power lost in the insulation resistance and corona is extremely small in comparison with other line losses, the value of the line conductance parameter is normally very low, and can be neglected, i.e., \( G = 0 \). Therefore

\[ K M + L N = 0 \]

\[ (I_r \cos (\delta - \phi_r) - I_r \cos \theta_r)(V_r \cos \phi_r + V_r \cos \delta_r) + (I_r \sin (\delta - \phi_r) - I_r \sin \theta_r)(V_r \sin \phi_r + V_r \sin \delta_r) = 0. \]

Equation (20) is a nonlinear equation with one unknown \( \delta \), and it can be easily solved by using any iterative numerical method. In this paper, Newton Raphson method has been used to solve the nonlinear equation. With this synchronization angle \( \delta \), the phasors at bus \( i \) are now synchronized with the phasors at bus \( j \). Thus, the synchronized phasors are obtained at both the terminals of the transmission line. The steps described above using synchronous measurements can be used to estimate the parameters of the transmission lines.

## III. TRANSMISSION LINE PARAMETER BOUNDS ESTIMATION

In this section, estimation of the transmission line parameter uncertainty bounds by considering uncertainties in the measurements is discussed. Among the various methods of evaluation of measurement uncertainty, the one proposed in Guide to the expression of uncertainty in measurement [13] is the most widely used [14], and adopted in this paper.

### A. Synchronous Measurements

The upper and lower bounds, within which the line parameters lie, are estimated by considering the uncertainty in the PMU measurements. The uncertainties in the PMU measurements are mainly due to the instrument transformers, the A/D converters, the cables connecting them, and the computational logic [15]–[17]. The uncertainties due to the instrument transformers and the cables can be compensated by calibration of the PMU. The uncertainties due to the A/D converter and the associated computational algorithm are difficult to compensate, which results in uncertainties in the magnitudes and angles of the voltage phasors and current phasors computed by the PMU. To evaluate the uncertainty intervals in the transmission line parameters, the maximum measurement uncertainties in the voltage magnitude, current magnitudes, and phase angle measurements by PMU must be...
known. The range of intervals, within which the measured quantity lies, is usually specified by PMU manufacturers [18].

A nonlinear constrained multivariable optimization problem is formulated, to evaluate the bounds of the transmission line parameters. The formulation of the optimization problem is as follows.

To obtain the lower bound of the parameters

$$\min f_j(x) \text{ subject to } x - \Delta x \leq x \leq x + \Delta x.$$  \hspace{1cm} (21)

To obtain the upper bound of the parameters

$$\max f_j(x) \text{ subject to } x - \Delta x \leq x \leq x + \Delta x.$$  \hspace{1cm} (22)

where $f(x) = [RXGB]^T$, $x = [V_s V_r I_s I_r \delta_s \delta_r \theta_s \theta_r]^T$, and $\Delta x$ is the maximum uncertainty in the corresponding measurement specified by the manufacturer.

The steps to find the bounds on the distributed parameters of the long line are as follows.

1) The minimum and maximum values of equivalent pi parameters $R$, $X$, $G$, $B$ of the long line are computed by substituting (11)–(14) in (21) and (22), respectively.

2) Using the maximum values of $R$, $X$, $G$, and $B$, the upper bounds of the distributed parameters $r$, $l$, and $c$ are calculated using (8)–(10).

3) Similarly, the lower bounds of the $r$, $l$, and $c$ are calculated using minimum values of $R$, $X$, $G$, and $B$ in (8)–(10).

A numerical verification based on Monte Carlo simulation is presented in Appendix C to justify the assumption used in steps 2 and 3 above that the maximum and minimum values of $r$, $l$, and $c$ correspond to the maximum and minimum values of $R$, $X$, $G$, and $B$.

B. Asynchronous Measurements

Consider a transmission line provided with PMU at one end, and with SCADA system on the other end. From SCADA system, one gets voltage magnitude $V_s$, current magnitude $I_s$, and active power and reactive power flows $P_r$, $Q_s$, as shown in Fig. 4. The power factor angle $\phi_s$ is given by

$$\phi_s = \sin^{-1}\left(\frac{Q_s}{V_s I_s}\right).$$  \hspace{1cm} (23)

To calculate $\phi_s$, reactive power $Q_s$ is used, because one can distinguish lagging and leading power factor by the sign of $Q_s$. The power factor is lagging if the sign of $Q_s$ is $-ve$ and it is leading if $Q_s$ is $+ve$.

It is assumed that the multiple measurements by a PMU are independent of each other. The standard uncertainty in the power factor angle $\phi_s$ and the synchronization angle $\delta$ is calculated using classical uncertainty propagation theory, and is given by [13]

$$u(\phi_s) = \sqrt{\sum_{k=1}^{3} \left[\frac{\partial \phi_s}{\partial p(k)}\right]^2 \left[u(p(k))\right]^2}$$  \hspace{1cm} (24)

$$u(\delta) = \sqrt{\sum_{k=1}^{7} \left[\frac{\partial \delta}{\partial q(k)}\right]^2 \left[u(q(k))\right]^2}$$  \hspace{1cm} (25)

where $p = [V_s, I_s, Q_s]$, $q = [V_s, I_s, \phi_s, V_r, I_r, \delta_r, \theta_r]$ and $u(p(k))$, $u(q(k))$ are the standard uncertainties in the measurements $p(k)$, $q(k)$, respectively. The partial derivative of $\phi_s$ with respect to $V_s$, $I_s$, and $Q_s$ are given in Appendix B. The synchronization angle $\delta$ is calculated by solving (20). The partial derivative of $\delta$ with respect to $V_s$, $I_s$, $\phi_s$, $V_r$, $I_r$, $\delta_r$, and $\theta_r$ are also given in Appendix B.

To evaluate the standard uncertainty in $\phi_s$, the standard uncertainties in the conventional measurements $V_s$, $I_s$, and $P_r$ measured by meters must be known. Usually, the maximum measurement uncertainty is specified by the meter manufacturers. In the absence of any probability distribution of the measurement uncertainty specified by the manufacturer, a uniform distribution may be assumed. The standard uncertainty in the measurement can then be expressed in terms of the maximum measurement uncertainty, as given below [13]

$$u(q(k)) = \frac{\Delta q(k)}{\sqrt{3}}$$  \hspace{1cm} (26)

where $\Delta q(k)$ is the maximum specified uncertainty by the manufacturer in the measurement of $q(k)$.

Similarly, to evaluate the standard uncertainty in $\delta$ in (25), the standard uncertainties in $V_s$, $I_s$, $V_r$, $I_r$, $\delta_r$, and $\phi_s$ must be known. The standard uncertainty in $\phi_s$ is already calculated using (24) and the standard uncertainties in $V_s$, $I_s$, $V_r$, $I_r$, $\delta_r$, and $\theta_r$ can be calculated using the maximum measurement uncertainty specified by the meter manufacturers.

1) Finding the Expressions for $R$, $X$, $B$: Substituting (15)–(18) in (4) and (5) and separating the real part and the imaginary part, the following equations are obtained (shunt conductance is neglected, i.e., $G = 0$):

$$R = \frac{a_1 c_1 + b_1 d_1}{c_1^2 + d_1^2}$$  \hspace{1cm} (27)

$$X = \frac{b_1 c_1 - a_1 d_1}{c_1^2 + d_1^2}$$  \hspace{1cm} (28)

$$B = \frac{f_1 g_1 - e_1 h_1}{g_1^2 + h_1^2}.$$  \hspace{1cm} (29)

The variables $a_1$, $b_1$, $c_1$, $d_1$, $e_1$, $f_1$, $g_1$, and $h_1$ are defined in Appendix B.

To find the bounds for the transmission line distributed parameters, similar steps can be followed as described using synchronous measurements in Section III-A. Only difference is that, the variable set $x = [V_s V_r I_s I_r \delta_s \phi_s \theta_s]^T$ is used in (21) and (22) and for $R$, $X$, and $B$, (27)–(29) are used ($G$ is neglected).

IV. CURRENT DIFFERENTIAL PROTECTION

A. Principles

In principle, current differential protection scheme using a positive sequence network representation can alone detect all possible faults [9], as it is excited by both the ground and the phase faults.

Consider an uncompensated three-phase transposed transmission line of length $L$ km, as shown in Fig. 5. Assume that two PMUs are installed at buses S and R.
If there is no internal fault on the line, for an arbitrary point \( k \), which is \( x \) km away from bus \( R \) on the transmission line
\[
\tilde{I}_{skp} = \tilde{I}_{rkp}
\]
(30)
where \( p \) indicates the phase current a, b, and c, \( \tilde{I}_{skp} \) is the current calculated at point \( k \) from the sending end data set \((V_{sp}, I_{sp})\) using (31) and \( \tilde{I}_{rkp} \) is the current calculated at point \( k \) from the receiving end data set \((V_{rp}, I_{rp})\) using (32)
\[
\tilde{I}_{skp} = \frac{-\tilde{V}_{sp} \cdot \sinh(\tilde{\gamma}x) + \tilde{I}_{sp} \cdot \cosh(\tilde{\gamma}x)}{\tilde{Z}_0} + \tilde{I}_{rp} \cdot \cosh(\tilde{\gamma}x) \quad (31)
\]
\[
\tilde{I}_{rkp} = \frac{\tilde{V}_{rp} \cdot \sinh(\tilde{\gamma}x)}{\tilde{Z}_0} + \tilde{I}_{rp} \cdot \cosh(\tilde{\gamma}x).
\]
(32)
From (30), one can define
\[
\text{ratio} = R^p = \frac{\tilde{I}_{skp}}{\tilde{I}_{rp}} = 1 \quad (33)
\]
angle difference \( \lambda^p = \angle \tilde{I}_{skp} - \angle \tilde{I}_{rkp} = 0 \). \quad (34)

If (33) and (34) are satisfied, there is no internal fault or the fault may be outside the transmission line. This can be visualized in the current differential plane by point \( \bullet \) at \((0^\circ, 1)\), as shown in Fig. 6. If a fault occurs between buses \( S \) and \( R \), any one of the phase currents will not satisfy (33) or (34) and the differential relay will send trip signal to the associated circuit breaker and isolate the line under fault. In practice, a threshold would be considered for the ratio and angle difference to allow for the tolerance in the measurement and the parameters [9]. In the proposed method, for calculation of the ratio and angle difference in (33) and (34), the currents at the fault location from the both ends are used. It involves calculation of the exact fault location.

**B. Calculation of Fault Location Index [19]**

Consider a three-phase transposed transmission line, as shown in Fig. 5. Assume that a fault occurred at a point \( k \) in the line, which is located \( x = DL \) km away from bus \( R \), where \( D \) is the per unit fault location index. The voltage at the fault point \( k \) can be calculated using the positive sequence data sets \((V_{s}, I_{s})\) and \((V_{r}, I_{r})\), as follows:
\[
\tilde{V}_{sk} = 0.5(V_r + \bar{Z}_0 I_r) e^{\tilde{\gamma}x} + 0.5(V_{s} - \bar{Z}_0 I_s) e^{-\tilde{\gamma}x} \quad (35)
\]
\[
\tilde{V}_{rk} = 0.5(V_r + \bar{Z}_0 I_r) e^{-\tilde{\gamma}(L-x)} + 0.5(V_{s} - \bar{Z}_0 I_s) e^{\tilde{\gamma}(L-x)}.
\]
(36)
Solving (35) and (36), the per-unit fault location index can be obtained as following [19]:
\[
\tilde{D} = \frac{\ln(N/M)}{2\tilde{\gamma}L} \quad (37)
\]
where \( M \) and \( N \) are given by
\[
M = 0.5e^{-\tilde{\gamma}L}(\tilde{V}_r + \bar{Z}_0 I_t) - 0.5(\tilde{V}_s - \bar{Z}_0 I_t)
\]
\[
N = 0.5(\tilde{V}_r - \bar{Z}_0 I_t) - 0.5e^{\tilde{\gamma}L}(\tilde{V}_s - \bar{Z}_0 I_t).
\]
The absolute value of \( \tilde{D} \) gives the fault location index \( D \). If there is a fault external to the transmission line (external fault), then the fault location index \( D \) assumes a value outside the range \((0–1 \text{ p.u.})\) [20], [21]. To find the fault location index, positive sequence symmetrical components of the voltages and currents, having the same time stamp, have been used.

**C. Proposed Algorithm**

The following steps have been used to detect the fault on the transmission line.

1. Choose the threshold parameters \( R_{\text{min}}^p, R_{\text{max}}^p, \lambda_{\text{min}}^p \) and \( \lambda_{\text{max}}^p \) to detect differential current. This paper has used \( \pm 20\% \) phase error and \( \pm 20\% \) magnitude error in the current differential plane, as shown in Fig. 6.
2. Compute the fault location index \( D \), with synchronized positive sequence measurements coming from PMUs at both the ends, using (37). If \( 0 \leq D \leq 1 \), take \( x = DL \) otherwise, take \( x = 0.5L \).
3. Compute \( \tilde{I}_{skp} \) and \( \tilde{I}_{rkp} \) at the fault location using synchronized voltage and current phasors coming from PMUs \((V_{sp}, I_{sp})\) and \((V_{rp}, I_{rp})\) and using (31) and (32), respectively.
4. Check if
\[
R_{\text{min}}^p \leq \frac{\tilde{I}_{skp}}{\tilde{I}_{rkp}} \leq R_{\text{max}}^p \quad (38)
\]
and
\[
\lambda_{\text{min}}^p \leq \angle \tilde{I}_{skp} - \angle \tilde{I}_{rkp} \leq \lambda_{\text{max}}^p. \quad (39)
\]
If the above is true, it indicates no fault on the phase-\( p \) of the transmission line. Otherwise, there is a fault on the phase-\( p \) of the transmission line and the relay issues trip signal to the associated circuit breakers, where the phase current violates one of the conditions, given by (38) or (39). The trip decision can be taken if the ratio or the angle difference exceeds the threshold values consecutively for four samples.
V. SIMULATION RESULTS

The case studies presented in this section are organized in three parts: 1) the transmission line parameter evaluation using only PMU and as well as hybrid (combination of PMU and conventional) measurements and comparison of the results obtained by both the methods are discussed; 2) the upper and lower bounds of the transmission line parameters are computed by considering uncertainty in the measurements; and 3) the proposed current differential protection, with and without considering transmission line parameter uncertainties, is investigated.

For reporting the results, a simple two-area system has been taken [22]. The system consists of two similar areas connected by two weak tie-lines of 220 km each (Fig. 7). Each area consists of two coupled units, each having a rating of 900 MVA. The transmission system base voltage is 230 kV. The detailed load, line and generator data on a 100-MVA base are given in [22]. To obtain the results, this two-area system was simulated in MATLAB-Simulink. It is assumed that the samples, coming from MATLAB-Simulink, correspond to the same instant of time.

A. Estimation of Transmission Line Parameters

The proposed scheme for the estimation of transmission line parameters using synchronous measurements has been applied to one of the tie-lines, L1 between buses 7 and 8 in Fig. 7. Table I shows the estimated transmission line parameter values using PMU measurements only and the percentage error in the estimated parameters from the actual values, computed as

\[
\text{% error} = \frac{\text{estimated value} - \text{actual value}}{\text{actual value}} \times 100. \quad (40)
\]

The method proposed for the estimation of the transmission line parameters using asynchronous measurements or hybrid measurements has been applied to the same line L1 between buses 7 and 8, and the results are shown in Table II.

It is observed that the parameters estimated using both the methods are quite close to the actual values. It is also observed that the parameters estimated using only synchronous (PMU) measurements have more accuracy as compared with the parameters estimated using the hybrid (PMU plus SCADA) measurements.

B. Estimation of Transmission Line Parameter Bounds

The proposed methods for estimating the transmission line parameter bounds have been applied on the same transmission line L1 between buses 7 and 8. Using the maximum measurement uncertainty specified by the manufacturer, the line parameter bounds are calculated using (21) and (22). The maximum uncertainties in the measurements assumed in this paper are shown in Table III [18]. Table IV shows the estimated lower and upper bounds of the line parameters, when conventional SCADA measurements are present along with the PMU measurements.


C. Current Differential Protection

The performance of current differential protection is limited by current transformer saturation, sampling asynchronization, and communication channel delay. In this paper, CTs are assumed to be identical and having no saturation. Using a high-speed (<1 ms) and reliable communication channel, the data exchange function can be easily maintained. The problem of sampling synchronization can be overcome if a timing signal is available from an external source. The GPS provides time synchronization with an accuracy of 1 μs. The proposed current differential protection scheme has been applied to the line L1 between buses 7 and 8 and simulated for all type of faults (LG, LLG, LL, and LLLG) with and without consideration of parameter uncertainties. The value of the ratio of the current magnitudes of different phases $|\bar{I}_{ckp}|/|\bar{I}_{kp}|$ and the angle differences $\angle\bar{I}_{ckp} - \angle\bar{I}_{kp}$ are calculated for all types of faults at different locations on the line and plotted. The + point indicates the relay operating point in the current differential plane in Figs. 8–11.

1) With Actual Values of Line Parameters: From Fig. 8, for internal faults on the line, all the operating points fall outside of the restrain region, so the relay treats it as a fault and sends trip signal to the associated circuit breakers.

For no fault and external faults, all the operating points fall inside the restrain region, as shown in Fig. 9. As all the points are inside the restrain region, the relay will not pick up.

The sensitivity of the current differential protection scheme can be evaluated by its ability to detect a high impedance internal fault. The proposed scheme has worked satisfactorily up to a fault resistance of 1300 Ω for all types of faults.

2) With Consideration of Line Parameter Uncertainties: Fig. 10 shows the performance of the proposed current differential protection scheme for internal faults under transmission line parameter uncertainties. As all points fall outside the restrain region, the relay will pick up and send trip signal to the associated circuit breakers. Similarly, the values of the ratio of the current magnitudes of different phases and the angle differences are calculated for no fault or external faults. These points are located in the current differential plane, as in Fig. 11. The simulation results show that the proposed scheme is working satisfactorily under consideration of line parameter uncertainties up to a fault resistance of 1100 Ω for all types of faults.

VI. CONCLUSION

The synchrophasor technology is an emerging technology, which is being now deployed in several utility systems.

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TABLE V

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Actual value</th>
<th>Maximum value</th>
<th>Minimum value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Resistance (pu/km)</td>
<td>1x10^4</td>
<td>1.68697x10^4</td>
<td>0.38725x10^4</td>
</tr>
<tr>
<td>Inductance (pu/km)</td>
<td>2.653x10^4</td>
<td>3.13782x10^4</td>
<td>2.19049x10^4</td>
</tr>
<tr>
<td>Capacitance (pu/km)</td>
<td>4.64202x10^4</td>
<td>6.522478x10^6</td>
<td>2.80663x10^6</td>
</tr>
</tbody>
</table>

Fig. 8. Performance of the proposed current differential protection scheme for internal faults on the line.

Fig. 9. Performance of the proposed scheme for external faults.

Fig. 10. Performance of the proposed current differential protection scheme for internal faults under transmission line parameter.

Fig. 11. Performance of the proposed current differential protection scheme for external faults under transmission line parameter uncertainties.
This paper has mainly focused on estimating the uncertainty in the transmission line modeling using synchrophasor technology, and the effect of the uncertainty on the line current differential protection.

An analysis of the uncertainty in the estimated transmission line parameters with PMU measurements has been presented. The uncertainty is modeled via deterministic upper and lower bounds on measurement errors. A methodology to determine the uncertainties in the estimated transmission line parameters, when conventional measurements are present along with PMU measurements, is also suggested. Furthermore, a new current differential protection scheme for transmission line protection using the PMU measurements at both the ends has been proposed. This method is based on the distributed parameter model of the transmission line. The scheme includes the effect of the distributed capacitive current. The proposed scheme has been tested with and without considering the line parameter uncertainties. The results show that the proposed scheme is sensitive, robust and can successfully discriminate between the internal and external faults. The scheme does not lose its sensitivity for high resistance faults (up to 1100 Ω). The scheme is quite fast and accurate as the phasors can be updated 50–60 times in each second and the current phasors provided by the PMUs are used for comparison. The proposed scheme also works satisfactorily under the consideration of transmission line parameter uncertainties.

**APPENDIX A**

**Expressions for the Variables Used in Section II-A**

The expressions for the variables a, b, c, d, e, f, g, and h are as follows:

\[ a = V_s^2 \cos 2\delta_r - V_s^2 \cos 2\delta_r \]
\[ b = V_s^2 \sin 2\delta_r - V_s^2 \sin 2\delta_r \]
\[ c = I_s V_r \cos (\theta_r + \delta_r) + V_i I_r \cos (\delta_r - \theta_r) \]
\[ d = I_s V_r \sin (\theta_r + \delta_r) + V_i I_r \sin (\delta_r + \theta_r) \]
\[ e = I_s \cos \theta_r - I_r \cos \theta_r \]
\[ f = I_s \sin \theta_r - I_r \sin \theta_r \]
\[ g = V_s \cos \delta_r + V_r \cos \delta_r \]
\[ h = V_s \sin \delta_r + V_r \sin \delta_r. \]

**APPENDIX B**

**Partial Derivatives and Variables Used in Section III-B**

The partial derivatives of \( \phi_s \) with respect to \( V_s, I_s \) and \( Q_s \) are given below:

\[ \frac{\partial \phi_s}{\partial V_s} = \frac{1}{\sqrt{1 - w^2}} I_s \]
\[ \frac{\partial \phi_s}{\partial I_s} = \frac{1}{\sqrt{1 - w^2}} V_s \]
\[ \frac{\partial \phi_s}{\partial Q_s} = \frac{-1}{\sqrt{1 - w^2}} V_s. \]

**APPENDIX C**

**Relationship Among R, X, G, B and r, l, c**

The steps 2 and 3 in Section III-A assume that the maximum and the minimum values of \( r, l, \) and \( c \) correspond to the maximum and minimum values, respectively, of \( R, X, G, \) and \( B \). As it is difficult to analytically prove such assumption, a numerical verification using Monte Carlo simulation was performed. 1000 Monte Carlo trials were run by taking random samples of \( R, X, G, \) and \( B \) within their respective maximum and minimum values obtained by solving (21) and (22), and evaluating \( r, l, \) and \( c \) using (9) and (10). Table A.1 shows the results of one thousand Monte Carlo simulations, considering uncertainties in PMU measurements only. The assumption used in Section III-A, step 2 and 3 is, thus, validated by the Monte Carlo trials.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Evaluated for maximum values of R, X, G, and B</th>
<th>Evaluated for minimum values of R, X, G, and B</th>
<th>Maximum from Monte Carlo simulation</th>
<th>Minimum from Monte Carlo simulation</th>
</tr>
</thead>
<tbody>
<tr>
<td>r</td>
<td>1.0201x10^4</td>
<td>1.0196x10^4</td>
<td>1.0196x10^4</td>
<td>0.98659x10^4</td>
</tr>
<tr>
<td>l</td>
<td>2.6571x10^4</td>
<td>2.6570x10^4</td>
<td>2.6496x10^4</td>
<td>2.6496x10^4</td>
</tr>
<tr>
<td>c</td>
<td>4.6522x10^4</td>
<td>4.6332x10^4</td>
<td>4.6522x10^4</td>
<td>4.6332x10^4</td>
</tr>
</tbody>
</table>

The partial derivatives of the synchronization angle \( \delta \) with respect to \( V_s, I_s, \phi_s, V_r, I_r, \delta_r, \) and \( \theta_r \) are given below:

\[ \frac{\partial \delta}{\partial V_s} = \frac{1}{\sqrt{1 - w^2}} I_s \cos \phi_s - I_r \cos (\delta - \theta_r) \]
\[ \frac{\partial \delta}{\partial I_s} = \frac{1}{\sqrt{1 - w^2}} V_s \cos \phi_s + V_r \cos (\delta - \theta_r) \]
\[ \frac{\partial \delta}{\partial V_r} = \frac{1}{\sqrt{1 - w^2}} I_r \cos (\delta - \phi_s - \delta_r) - I_s \cos (\delta - \theta_r - \phi_s) \]
\[ \frac{\partial \delta}{\partial I_r} = \frac{1}{\sqrt{1 - w^2}} V_s \cos (\delta - \phi_s - \delta_r) - V_r \cos (\delta - \theta_r - \phi_s) \]
\[ \frac{\partial \delta}{\partial \phi_s} = \frac{1}{\sqrt{1 - w^2}} I_s \sin (\delta - \phi_s - \delta_r) - I_r \sin (\delta - \theta_r - \phi_s) \]
\[ \frac{\partial \delta}{\partial \phi_r} = \frac{1}{\sqrt{1 - w^2}} V_s \sin (\delta - \phi_s - \delta_r) - V_r \sin (\delta - \theta_r - \phi_s) \]
\[ \frac{\partial \delta}{\partial \theta_r} = \frac{1}{\sqrt{1 - w^2}} I_r \sin (\delta - \phi_s - \delta_r) - V_r \sin (\delta - \theta_r - \phi_s) \]

The expressions for the variables \( a_1, b_1, c_1, d_1, e_1, f_1, g_1, \) and \( h_1 \) are as follows:

\[ a_1 = V_s^2 \cos 2\delta - V_r^2 \cos 2\delta_r \]
\[ b_1 = V_s^2 \sin 2\delta - V_r^2 \sin 2\delta_r \]
\[ c_1 = I_s V_r \cos (\delta - \phi_s + \delta_r) + V_i I_r \cos (\delta - \theta_r) \]
\[ d_1 = I_s V_r \sin (\delta - \phi_s + \delta_r) + V_i I_r \sin (\delta - \theta_r) \]
\[ e_1 = I_s \cos (\delta - \phi_s) - I_r \cos \theta_r \]
\[ f_1 = I_s \sin (\delta - \phi_s) - I_r \sin \theta_r \]
\[ g_1 = V_s \cos \delta + V_r \cos \delta_r \]
\[ h_1 = V_s \sin \delta + V_r \sin \delta_r. \]
REFERENCES


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