A New Algorithm for Joint Sensing and Power Allocation in Multiuser Cognitive Radio Networks

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Abstract—In this paper, we propose a new two phase algorithm to maximize the downlink throughput of a cognitive radio network (CRN) using OFDMA. Since we consider a multiuser CRN, subcarrier assignment should be performed for secondary (unlicensed) users. In Phase I of the proposed algorithm, first a coarse spectrum sensing and equal transmitted power allocation are considered and then subcarrier assignment is done. In Phase II, a fine spectrum sensing and power allocation are performed. Instead of applying thresholds, we estimate and use the number of occupied subchannels for fine spectrum sensing. Simulation results show that our proposed algorithm provides better suboptimal solutions with much lower complexity.

Keywords—Cognitive Radio (CR); Power Allocation; Spectrum Sensing; Channel Assignment; OFDMA;

I. INTRODUCTION

To overcome the shortage of frequency spectrum and intense request for bandwidth to be used for new wireless technologies, cognitive radio networks (CRNs) have been proposed. In CRNs, secondary (unlicensed) users opportunistically utilize the spectra that are allocated to the primary (licensed) users. In these networks, besides having reliable communications for primary users, efficient use of spectra would be achieved through efficient spectrum usage by secondary users [1].

Orthogonal frequency division multiple Access (OFDMA) has many advantages, which makes it a good multiple access option for CRNs. First, subcarriers that are occupied by primary users can be nulled by secondary users, and not assigned in subchannel assignment. Additionally, using FFT in this modulation provides an ease of implementation and analysis [2].

Spectrum sensing, as the first fundamental task in a cognitive radio, detects unused spectrum bands. Because of inaccurate spectrum sensing, some of the frequency bands are erroneously detected as occupied or unoccupied channels. Probabilities of being in these cases are called probability of false alarm and misdetection, respectively.

In multiuser CRNs, due to coexistence of secondary and primary users’ spectra beside each other, we have mutual interference. So, using classical algorithms is impossible for power allocation to secondary users. On the other hand, in practical systems imperfect spectrum sensing makes it impossible to achieve the best power allocation to secondary users. In this paper, we propose an efficient two phase algorithm from the viewpoint of throughput maximization, by considering optimization problem of subcarrier assignment, power allocation, and spectrum sensing.

Recently, in the context of OFDM-based CRNs, many power allocation algorithms and cooperative spectrum sensing algorithms are proposed [3-7]. In [3,4], authors proposed optimal and suboptimal power allocation algorithms considering a perfect spectrum sensing. They maximized the capacity of a CRN, subject to meeting constraints such as maximum allowable interference introduced to the primary users, peak transmitted power of secondary access point, and QoS of secondary users. In [5], some different algorithms for spectrum sensing such as matched filter, energy detector and cyclostationary detector are discussed. In most of these algorithms, we should estimate a decision threshold. This threshold is estimated through false alarm probability and this probability is estimated theoretically or using Monte Carlo simulations. In [6], a new algorithm for spectrum sensing is proposed in which, instead of the decision threshold, the number of occupied subchannels is estimated. This estimation is based on information criterion. In this algorithm, subchannels are sorted in terms of their corresponding energy levels and those with highest energy levels are chosen. In [7], the proposed algorithm jointly allocates sensing thresholds at the access point and power levels at the CR transmitters, so that the throughput of the CRN is maximized.

In this paper, we propose a new two phase algorithm, in which first a coarse spectrum sensing, along with equal power allocation and the then subcarrier assignment are performed. Then, in Phase II, both fine spectrum sensing and transmitted power allocation are done. Spectrum sensing is established by estimating the number of occupied subchannels and sorting them in terms of their energy levels. On the other hand, while keeping the interference power introduced to primary users is kept below a given threshold, the throughput of the CRN is maximized. So, the cost function of this problem depends on the number of occupied subchannels and subcarrier assignment to secondary users. We will show that our proposed suboptimal algorithm is sufficiently close to the optimum one with much lower complexity.

The rest of this paper is organized as follows. Section II describes the system model. Section III formulates the optimization problem. Section IV presents suboptimum solution for the problem. Section V discusses simulation results by comparing with those of other algorithms. Finally, Section VI concludes the paper.
II. SYSTEM MODEL

We consider an OFDM-based CRN with \( K \) secondary users as shown in Figure 1. This model is an extension of the one in [3], in which only one secondary user was considered.

Secondary users cooperatively sense spectrum at their receivers and coordinate with each other through one access point. The number of primary users is denoted by \( J \), which is unknown for CRN. \( h_{k,i}^{\text{ps}} \) and \( h_{k,i}^{\text{ss}} \) represent the instantaneous channel gain between \( k \)’th secondary user’s transmitter and receiver at the \( i \)’th subcarrier and the instantaneous channel gain between \( l \)’th primary user’s transmitter and \( k \)’th secondary user’s receiver, respectively, and are assumed to be known at the secondary user’s transmitter.

Figure 2 shows spectral domain occupied by primary users and allocated to secondary users among them. We have \( N \) subcarriers that should be assigned to \( K \) secondary users. Each primary user’s subband consists of a number of subchannels, which depends on the primary user’s spectral activity. \( M \) denotes the total number of subchannels occupied by primary users and \( J \) is its estimated value.

Now, we want to have the mutual interference introduced to the primary users, below an interference power threshold \((I_{\text{th}})\). For this purpose, we consider each of interference powers introduced to primary user \( l \) from \( k \)’th secondary user transmitting in \( i \)’th subcarrier, denoted by \( I_{k,i}^{(l)} \) and is given by [8]:

\[
I_{k,i}^{(l)} = P_{k,i} \cdot \Phi_{k,i}^{(l)},
\]

where,

\[
\Phi_{k,i}^{(l)} = \left| h_{k,i}^{\text{ps}} \right|^2 \int_{-T_s/2}^{T_s/2} \sin \left( \frac{\pi f T_s}{f_{\text{Th}}} \right)^2 \left( \frac{\sin \pi f T_s}{\pi f T_s} \right)^2 \, df.
\]

Here, \( T_s \) specifies OFDM symbol duration and \( d_{i,j} \) denotes the spectral distance between the \( i \)'th secondary user’s subcarrier and \( l \)'th primary user’s subband. The integral limits in (2) show why exact specification of spectral distance between secondary subcarriers and primary subbands is too important. In the next section, we consider a cost function in optimization problem for precision in spectrum sensing.

The transmission rate of \( k \)’th secondary user at the \( i \)’th subcarrier, with transmit power \( P_{k,i} \) is given by:

\[
R_{k,i} = \Delta f \log_2 \left( 1 + \frac{\left| h_{k,i}^{\text{ss}} \right|^2 P_{k,i}}{\sigma^2} \right).
\]

For large number of primary users, the interference introduced to the \( k \)’th secondary user by primary users is approximately AWGN [3]. Therefore, \( \sigma^2 \) represents the variance of the environmental received noise and the summation of the variance of interferences introduced by primary users.

III. PROBLEM FORMULATION

Here, the optimization problem is to maximize downlink throughput of a CRN by estimating the total number of occupied subchannels \((J)\) and two other parameters of subcarrier assignment to secondary users. In this problem, some constraints should be considered, such as keeping the total interference below a threshold and the total transmitted power (assigned to subcarriers) should be less than peak transmitted power \((P_{\text{p}})\) of the access point. Mathematically, maximization problem of the downlink throughput can be written as:

\[
C = \max_{J, P_{\text{p}}} \sum_{i=1}^{K} \sum_{l=1}^{N} \gamma_{k,i} \left( 1 - p_{\mu}(J) \right) \log_2 \left( 1 + \frac{\left| h_{k,i}^{\text{ss}} \right|^2 P_{k,i}}{\sigma^2} \right),
\]

subject to,

\[
\sum_{k=1}^{K} \sum_{i=1}^{N} \gamma_{k,i} P_{k,i} \Phi_{k,i}^{(l)} \leq I_{\text{th}}, \quad \text{for all } l,
\]

\[
p_{\mu}(J) < \alpha,
\]

\[
\sum_{k=1}^{K} \sum_{i=1}^{N} P_{k,i} \leq P_{\text{p}},
\]

\[
\sum_{k=1}^{K} \gamma_{k,i} = 1, \quad \text{for all } i \in \{1, \ldots, N\},
\]
where $P_{kj}$ represents the power of $k$'th secondary user transmitted on the $i$'th subcarrier, and $\gamma_{k,i} = 1$ denotes that $i$'th subcarrier is occupied by $k$'th secondary user and $\gamma_{k,i} = 0$ denotes vice versa,
\[
\gamma_{k,i} \in \{0,1\}, \quad \text{for all } k,i .\tag{9}
\]

In appendix, we will show that false alarm probability of each subchannel can be derived as:
\[
P_{fa} = \left( \frac{Q \left( \frac{h_{0,j}^2}{2}\right)}{2\sigma_n^2 \sqrt{\sigma_v^4 + \left( \sigma_v^2 + 2|h_{0,j}^0| \right)^2}} \right)^j \left( \frac{1}{2} \right)^{N-1}. \tag{10}
\]

For ease of analysis and decreasing system complexity, we determine false alarm threshold $\alpha$ and two other threshold values of the constraints to ensure convexity of the optimization problem. Similar to [7], if $p_{fa}(J) < 0.5$, then the Q-function would be convex for positive values. Therefore, it can be shown that false alarm probability function is convex.

IV. PROPOSED ALGORITHM

In this section, we explain the details of our proposed two phase algorithm. In Phase I, a coarse spectrum sensing is done first. Then equal transmitted power is assigned to each subcarrier. Finally, subcarrier assignment is performed. The flowchart of Phase I of the proposed algorithm is shown in Figure 3.

We initially assume equal transmitted power is allocated to secondary users, so the algorithm becomes suboptimum. According to classical algorithms, the secondary user with higher channel to noise ratio (CNR) obtains more subcarriers. In CRNs, we also consider mutual interference. So, we define channel to noise cross mutual interference ratio ($\xi_{k,i}$) of $i$'th subcarrier for $k$'th secondary user as:
\[
\xi_{k,i} = \frac{1}{\sigma_v^2} \sum_{j=1}^{N} \Phi_{k,i}^{(j)} ,
\tag{11}
\]
which $\Phi_{k,i}^{(j)}$ is obtained by coarse spectrum sensing. To assign $i$'th subcarrier to $k$'th secondary user, $\xi_{k,i}$ must be less than pre-specified threshold.

We want to assign the subcarriers to those having higher $\xi$ so that maximizing the total rate of the CRN. As it can be seen in the flowchart, first, we consider equal power for all subcarriers. Then, for each secondary user, we find the subcarrier with the largest $\xi$ and eliminate index of this subcarrier. So, the eliminated subcarrier wouldn’t be assigned to the other users. In the lower part of the flowchart, we assign the rest of subcarriers by updating the rate of secondary users and selecting the subcarrier with largest $\xi$ to the desired user.

In this algorithm, we assign subcarriers by determining $\gamma_{k,i}$ for $1 < k < N$ and $1 < i < N$. This eliminates a long set of unknown parameters from optimization problem and so reduces system complexity. Therefore, (9) will be omitted and the maximization would only based on $P_{kj}$ and $J$.

The Phase II of the proposed algorithm is to jointly optimize throughput of CRN by estimating the number of occupied subchannels and allocating powers to secondary users. In this phase, first, we apply Lagrange multipliers for equation (5) to (7) and then we use Karush-Kuhn-Tucker (KKT) conditions for convex optimization [9].

We assume that the subchannels occupied by primary users have higher energy levels in contrast with unoccupied ones. So,

<table>
<thead>
<tr>
<th>Initialize by allocating equal power to all secondary users $k = 0, I = {1,2,\ldots,N}$</th>
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<tbody>
<tr>
<td>(number of secondary users) $k = k + 1$</td>
</tr>
<tr>
<td>find $i$, that $\xi_{k,i} &gt; \xi_{k,j}$ (†) for $1 &lt; j &lt; N$</td>
</tr>
<tr>
<td>$\gamma_{k,i} = 1$</td>
</tr>
<tr>
<td>$I = I - {i}$</td>
</tr>
<tr>
<td>no</td>
</tr>
<tr>
<td>Is $k &lt; K$ ?</td>
</tr>
<tr>
<td>yes</td>
</tr>
<tr>
<td>Is $I = \emptyset$ ?</td>
</tr>
<tr>
<td>no</td>
</tr>
<tr>
<td>Update rate of secondary user (R$_k$)</td>
</tr>
<tr>
<td>find $i$, that $R_{k,i} &gt; R_{f,j}$ for $1 &lt; j &lt; N$</td>
</tr>
<tr>
<td>For found $i$, obtain $k$ satisfies (†)</td>
</tr>
<tr>
<td>$\gamma_{k,i} = 1$</td>
</tr>
<tr>
<td>$I = I - {i}$</td>
</tr>
<tr>
<td>End</td>
</tr>
</tbody>
</table>

Figure 3. The flowchart of Phase I (subcarrier assignment).
by choosing $J$ subchannels with the highest energy, the occupied subchannels are determined [6].

V. SIMULATION AND PERFORMANCE RESULTS

In this section, simulation results of the proposed algorithm are presented and discussed. The simulation parameters are listed in Table 1. In this table, $M$ represents the actual number of occupied subchannels by primary users and $J$ is its estimation. Here, we consider an OFDM-based CRN. Secondary receivers cooperatively sense spectrum and feedback data to secondary transmitters. The channel models between any two secondary users, between primary to secondary and secondary to primary users are assumed to be Rayleigh fading and average channel gain of the channels is \(-10\) dB. We compare our proposed algorithm with Algorithm I and Algorithm II given in [10] and [4], respectively. Algorithm I is a classical power allocation algorithm for multiuser OFDM systems, which is proposed in [10]. In this paper, the authors have not considered cognitive environment. In other words, they have not considered mutual interference introduced to the primary users. So, they have applied water-filling algorithm as a classical solution for power allocation. Here, we don’t assume the fairness constraint that is considered in [10]. Algorithm II, [4] considers a coarse spectrum sensing and then a subcarrier allocation for secondary users is performed. Algorithm II, similar to Algorithm I and our proposed algorithm considers a multiuser OFDM-based CRN.

Figure 4 represents the throughput of the CRN in terms of peak transmitted power ($P_T$). In this figure, $I_{th}$ is assumed to be fixed. By increasing the peak transmitted power, the throughput of the system increases. It can be observed from this figure that our proposed algorithm is more efficient than Algorithm II in terms of throughput. As can be seen from Figure 4, for $P_T = 5\mu\text{Watts}$, our proposed algorithm has 21% more throughput in comparison with that of Algorithm II and 7.5% less throughput in comparison with that of Algorithm I. Since in Algorithm I, mutual interference introduced to the primary users is not considered, it will have higher throughput in comparison with other algorithms. Although Algorithm I has higher throughput, the interference introduced to the primary users will exceed the interference threshold. This is shown in Figure 5. This interference causes Algorithm I to be inefficient in CRNs. As can be seen from Figure 5, in our proposed algorithm, the interference introduced to the primary users, remains below the interference threshold.

Figure 6 shows the throughput in terms of interference threshold ($I_{th}$). In this figure, we assume a fix peak transmitted power. By increasing interference threshold, the throughput of the CRN increases. It can be seen that, by increasing the interference threshold, our proposed algorithm provides a higher throughput in comparison with that of Algorithm II. For example, for $I_{th} = 6\mu\text{Watts}$, our proposed algorithm has 16% more throughput than Algorithm II.

<table>
<thead>
<tr>
<th>TABLE I. SIMULATION PARAMETERS</th>
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<tbody>
<tr>
<td>parameter</td>
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<tr>
<td>----------</td>
</tr>
<tr>
<td>$T_s$</td>
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<tr>
<td>$N$</td>
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<tr>
<td>$\Delta f$</td>
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<td>$K$</td>
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<td>$\sigma^2$</td>
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<tr>
<td>$\alpha$</td>
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<tr>
<td>normalized $f_D$</td>
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</tbody>
</table>

Figure 4. Throughput vs. $P_T$ (assuming $I_{th} = 4 \times 10^{-7}$ W).

Figure 5. Interference introduced to the $l$'th primary user’s subband vs. peak transmitted power $P_T$.

VI. CONCLUSION

In this paper, we achieved considerable improvement in the performance of a multiuser OFDM-based CRN, by employing our proposed two phase algorithm. In Phase I, the algorithm assigns subcarriers to different secondary users based on their
channel to noise cross mutual interference ratio. Then, in Phase II, by joint optimization of spectrum sensing and power allocation we have maximized the throughput. It’s notable that spectrum sensing was based on the number of occupied subchannels. Simulation results indicate the efficiency of this proposed algorithm in comparison with those of two former algorithms.

APPENDIX

If we sort subchannels in descending order in terms of their energy levels, the index of the resultant subchannels is given by:

\[ E_{\hat{i}} \geq E_{\hat{2}} \geq \cdots \geq E_{\hat{J}} = \text{sort}(E_1, E_2, \ldots, E_J). \]  

We assume the energy of the received signal to be a random variable with Gaussian distribution. The mean and variance of this variable differ for an occupied and an unoccupied subchannel, as expressed by [11]:

\[ E_{\hat{i}} \sim \begin{cases} 
N(\mu_i, \sigma_i^2), & i < J \\
N(\mu_J, \sigma_J^2), & i > J,
\end{cases} \]  

where \(N(\mu, \sigma)\) and \(N(\mu, \sigma)\) denote Gaussian distributions of the received signal energy for occupied and unoccupied subchannels, respectively. These means and variances are given by:

\[ \mu_i = 2T\Delta f \left( \sigma_n^2 + \frac{T_i^2}{2} \right), \]
\[ \mu_J = 2T\Delta f \sigma_n^2, \]
\[ \sigma_i = 4T\Delta f \sigma_n^2 \left( \sigma_n^2 + \frac{T_i^2}{2} \right), \]
\[ \sigma_J = 4T\Delta f \sigma_n^4, \]

where \(T\) and \(\sigma_n^2\) represent sensing time and AWGN variance.

\[ P_{fa}^k = \prod_{j=1}^{J} p_E \left( E_{\hat{j}} - E_{\hat{j}} < 0 \right) \prod_{j=J+1}^{J+i} p_E \left( E_{\hat{j}} - E_{\hat{j}} < 0 \right) \times \prod_{j=J+i}^{J+1} p_E \left( E_{\hat{j}} - E_{\hat{j}} < 0 \right), \]  

(16)

where,

\[ \left( E_{\hat{J}p} - E_{\hat{J}q} \right) \sim \begin{cases} 
N(\mu_1 - \mu_2, \sqrt{\sigma_1^2 + \sigma_2^2}), & p < J, q > J \\
N(0, 4\sqrt{2}\sigma_\text{J}), & p, q > J
\end{cases} \]  

(17)

After some manipulations, the false alarm probability is given by:

\[ P_{fa} = \left( Q \left( \frac{\mu_1 - \mu_2}{\sqrt{\sigma_1^2 + \sigma_2^2}} \right) \right)^j \left( \frac{1}{2} \right)^{N-1}. \]  

(18)

As can be seen, here, the false alarm probability is independent of the index of subchannels. By substituting the mean and variance values in (18), the false alarm probability is achieved by (10).

REFERENCES