Impact of delivery time on Optimal Production/Delivery/Maintenance Planning

Turki Sadok, Hajej Zied, and Rezg Nidhal

Abstract—this paper deals the combination between production, delivery and maintenance plan for a manufacturing system satisfying a random demand under service level. A jointly optimization is made in order to establish an optimal production, delivery planning and scheduling maintenance strategy showing the machine degradation. The key of this study is to consider the influence of the delivery time on the production/delivery/maintenance context. We prove simultaneously, with a constrained stochastic production-delivery-maintenance planning problem under hypotheses of service level, delivery time and failure rate, an optimal production, delivery plan and maintenance scheduling which minimizes the total production, inventory, delivery and maintenance costs. Numerical results are presented to highlight the application of the developed approach.

Keywords: Delivery time, failure rate, random demand, service level, minimal repair.

I. INTRODUCTION

The combination between maintenance and production plans represents an interesting work in the research area. In recent years, the development of an optimal production and maintenance plan which minimizes the total cost including production, inventory and maintenance is one of the first actions of a hierarchical decision making process. Indeed, this plan consists on finding the optimal production plan and maintenance strategy required by the company to manufacture their products which satisfy a random demand over future periods. About the production/inventory problem without maintenance, the linear decision rule developed by Holt, Modigliani, Muth and Simon (HMMS)[6] has being considered an important contribution for production planning decisions. This analytical rule is determined the optimal solution for aggregate inventory, production and workforce levels by minimizing the quadratic cost functions subject to inventory and workforce balance equations. Taking account of the maintenance, Buzacott[2] is among the first authors who studied the problem of integrated production and maintenance strategies. He studied the role of buffer stocks in increasing the system productivity. Rezg et al.[11], Chelbi, and Ait-Kadi [3] and Aghezzaf et al. [1] examined the strategy of building a safety stock to meet demands during periods of production interruption due to maintenance actions. Cheung, and Hausman [5] proposed a joint optimization of the strategic stock and the age type maintenance policy. In reality, the failure rate increases with time and the use of the equipment, is rarely considered. Indeed, the consideration of the equipment failure according to the production rate is rarely dealt in the literature. We can cite Hu et al. [7] who discussed the conditions of optimality of the hedging point policy for production systems in which the failure rate of machines depends on the production rate.

Most of the researchers focus on a perfect manufacturing system and a perfect service level, and do not present the effect of the service level and the proportion of defective items on relevant performance measures and costs. Recently Hajej et al. [8], Hajej et al.[9] studied a randomly failing manufacturing system which has to satisfy a random demand during a finite horizon given a required service level. However, Hajej et al. haven’t taken into account some important characteristics of manufacturing systems such as transport (delivery time, transported quantity). Indeed, many manufacturers are working to reduce transportation delays such as the delivery time, which is the period of time that the part takes between a manufacturing store and a purchase warehouse (customer), and which usually has great impact on performance measures. Turki et al. [13] studied the impact of delivery time on the optimal buffer level take into account to the machine failures and random demands. Dolgui and Ould-Louly [4] presented a model for supply planning under lead time uncertainty and proposed a method to determine the optimal value of the planned lead time under lead time uncertainty. Hsu-Hua Lee [10] developed a model for supporting investment strategies about inventory and preventive maintenance in an imperfect production system take into account the delivery time to the customer. In recent years, another important characteristic of transport is considered in the manufacturing system study such as transported quantity, which is the quantity of parts to be transported between the manufacturing store and the purchase warehouse Richard and Fung [12]. Indeed, in order to respect the service level, the warehouse should contain enough parts for satisfying customer demands. Thus, an optimal planning of the transported quantity between the manufacturing store and a purchase warehouse should be determined based on the relationship with the production/maintenance planning and the service level. However, systems with delivery combined with integrated production-maintenance strategies is a very recent topic, and the few existing results indicate that the problem may become challenging.

Turki. Sadok. LGIPM-Paul Verlaine University, Metz, 57045 France (e-mail: turki@univ-metz.fr).
Hajej. Zied., LGIPM-Paul Verlaine University, Metz, 57045 France. (corresponding author to provide e-mail: hajej@univ-metz.fr).
Rezg. Nidhal LGIPM-Paul Verlaine University, Metz, 57045 France (e-mail: rezg@univ-metz.fr).
The objective of this paper is to determine simultaneously the economical production planning, optimal maintenance strategy and the optimal delivery plan taken into account the delivery time, machine failures, random demand and withdrawal right (where the products returned by the customer who are still new) to minimize the sum of inventory, maintenance, production and transportation costs. The impact of delivery time and withdrawal right on optimal production/maintenance planning and transported quantity will be studied thereafter.

This remainder of this paper is organized as follows: Section II formulates a general stochastic production, delivery and maintenance model. Section III presents and develops the policy and analytical expression of production/delivery and maintenance of considering the influence of the delivery time.

On the production, delivery and maintenance plans. A simple numerical example is presented in section IV. Finally, the conclusion is given in Section V.

II. PRODUCTION/Maintenance/Delivery Problem

In this section, a jointly optimal production/delivery planning and maintenance strategy problem is formulated. The proposed model for this problem is a finite-horizon, discrete-time. We consider a manufacturing system which produces one type of product and composed of a single machine \( M \) and two stores \( (S_1, S_0) \), \( S_1 \) is the manufacturing store (where the manufactured products are stored) and \( S_0 \) is the purchase warehouse (where the customer receives his demand (products)). The customer demand which is denoted by \( d \) is random and given by a Normal distribution. Hence the delivery time, denoted by \( \tau \), is considered between the store \( S_1 \) and the warehouse \( S_0 \). Indeed, the products outgoing from \( S_1 \) are transported to the customer and take a delivery time \( (\tau) \) to arrive at the customer (see figure 1). In other words, if we assume that the products leave \( S_1 \) at period \( k \), they will arrive at the period \( k+\tau \). We assume that the products are transported between \( S_1 \) and \( S_0 \) in a vehicle with capacity \( Q_v \). Also, we consider the order preparation cost. Indeed, the products which will be transported need a preparation, thus we will consider the cost of the products preparation which is called order preparation cost.

The demand \( d \) is satisfied from the warehouse \( S_0 \) with inventory service level \( \theta \). The part of returned products that are in saleable condition and packaged are collected in the warehouse, called the withdrawal right.

The machine \( M \) is subject to a random failure. The probability degradation law of machine \( M \) is described by the probability density function of time to failure \( f(t) \) and for which the failure rate \( \lambda(t) \) increases with time and according to the production rate \( u(t) \).

Our objective is to establish simultaneously an economical production, delivery plans and an optimal preventive maintenance period satisfying the randomly demand. The aim is to minimize the sum of the inventory costs at the two stores, the manufacturing and delivery costs along with the costs associated with the maintenance policy.

Fig. 1. Problem description

A. Notation

The following parameters are used in the mathematical formulation of the model:

- \( \delta \): percentage of returned product that is saleable and sent back to warehouse \( S_0 \)
- \( \tau \): backorder delay
- \( \tau \): delivery time
- \( \Delta t \): length of a production period
- \( H \): number of production periods in the planning horizon
- \( H - \Delta t \): length of the finite planning horizon
- \( u(k) \): production rate of machine \( M \) during period \( k \) \( (k=0, 1, \ldots, H-1) \)
- \( \{ u(0), u(1), \ldots, u(H-1) \} \)
- \( Q(k) \): delivery rate during period \( k \) \( (k=0, 1, \ldots, H-1) \)
- \( \{ Q(0), Q(1), \ldots, Q(H-1) \} \)
- \( \hat{d}(k) \): average demand during period \( k \) \( (k=0, 1, \ldots, H) \)
- \( V_d(k) \): variance of demand during period \( k \) \( (k=0, 1, \ldots, H) \)
- \( S_i(k) \): inventory level of \( S_i \) at the end of period \( k \) \( (k=0, 1, \ldots, H) \)
- \( \tilde{S}_i(k) \): average inventory level of \( S_i \) during period \( k \) \( (k=0, 1, \ldots, H) \)
- \( \tilde{S}_0(k) \): average inventory level of \( S_0 \) at the end of period \( k \) \( (k=0, 1, \ldots, H) \)
- \( \tilde{S}_0(k) \): average inventory level of \( S_0 \) during period \( k \) \( (k=0, 1, \ldots, H) \)
- \( \hat{C}_{pu} \): unit production cost of machine \( M \)
- \( \hat{C}_{pi} \): inventory holding cost of one product unit during one period at the first store \( S_1 \)
- \( \hat{C}_{po} \): inventory holding cost of one product unit during one period at the warehouse \( S_0 \)
- \( C_{M} \): total maintenance cost
- \( C_{pm} \): preventive maintenance action cost
- \( C_{cm} \): corrective maintenance action cost
- \( m_{u} \): monetary unit
- \( U_{max} \): maximal production rate of machine \( M \)
- \( U_{min} \): minimal production rate of machine \( M \)
\( \theta \) probability index related to customer satisfaction and expressing the service level.

\( f(t) \): probability density function associated with the time to failure of \( M \).

\( F(t) \): probability distribution function associated with the time to failure of \( M \).

\( R(t) \): reliability function, equal to \( 1 - F(t) \).

\( \lambda_h(t) \): nominal failure rate corresponding to the maximal production rate.

\( \lambda_0(t) \): machine failure rate function during period \( k \) for \( k=0, 1, \ldots, H \).

\( S_0 \): initial inventory level of \( S_t \).

\( S_0^o \): initial inventory level of \( S_0 \).

### B. Problem formulation

The idea is to minimize the expected production, inventory and delivery costs over a finite time horizon \([0, H]\). It’s assumed that the horizon is portioned equally into \( H \) periods. The demand is satisfied at the end of each period. The stochastic problem as follows:

\( f_s(.) \) denotes functions that represent the expected value of production, inventory and delivery costs. The maintenance cost \( C_M(.) \) is characterized by the preventive and corrective maintenance costs and the expected number of failures.

\[
\max_{U \in [U_{min}, U_{max}]} \left\{ \sum_{k=0}^{H-1} \left[ f_p(S_k(H), S_k(H)) + f_q(k, S_0^o, u(k), Q(k)) \right] \right\}
\]

Subject to

The inventory level of the store \( S_t \) at the period \( k+1 \) equals to the inventory level of \( S_t \) at the period \( k \) plus the production rate of machine \( M \) during period \( k \), minus the delivery rate during period \( k \). Therefore, the inventory level of \( S_t \) at the period \( k+1 \) is given by the following equation:

\[
S_k(k+1) = S_k(k) + u(k) - Q(k)
\]

The products quantity that arrives to the warehouse \( S_0 \) at the period \( k \) is the products quantity which has left the store \( S_t \) at the period \( k-\tau \). That means, that the rate of products that arrives to \( S_0 \) equals to the rate of products which has left the store \( S_t \) at the period \( k-\tau \) (i.e. \( Q(k-\tau) \)). The inventory level of the warehouse \( S_0 \) at the period \( k+1 \) equals to the inventory level of \( S_0 \) at the period \( k \) plus the rate of products that arrives to \( S_0 \) (i.e. \( Q(k-\tau) \)) minus the customer demand at the period \( k \) plus the product quantity returned by the customer. Therefore, the inventory level of \( S_0 \) at the period \( k+1 \) is given by the following equation:

\[
S_0(k+1) = S_0(k) + Q(k-\tau) - d(k) + r(k)
\]

The product quantity returned by the customer, denoted by \( r(k) \), is a part of the demand returned by the customer after the specific deadline \( \tau \). The value of \( r(k) \) can be described as follows:

\[
r(k) = \delta \cdot d(k - \tau)
\]

The service level requirement constraint for each period is expressed by the following constraint:

\[
\text{Prob}[S_0(k+1) \geq 0] \geq \theta
\]

The following constraint defines an upper and lower bounds on the production level during each period \( k \).

\[
U_{min} \leq u(k) \leq U_{max}
\]

### C. Production/Delivery/Maintenance policies

In this section, a constrained stochastic optimal problem is formulated. It is used represent a constrained production/delivery/Maintenance problem under service level, delivery time, random demand, failure rate and withdrawal right (product returned). The proposed model for this problem used the HMMS model.

The classical HMMS model provides a decision rule that allows the determination of an aggregate inventory, production and workforce policy. This rule is derived from the minimization of a quadratic production cost subject to linear equation that represents the balance among inventory, production and work-force components.

In our problem, we adapted this HMMS model to establish an inventory, production, delivery and maintenance policies.

#### 1) Production/Delivery Policy

The principle characteristic of HMMS model is the use of a quadratic cost function allows penalizing both excess and shortage in the inventory level.

The expected cost including production and holding costs for the period \( k \) is given by:

\[
f_p(S_t(k), S_0(k), u(k), Q(k)) = f_{u(k)}(u(k)) + f_{Q(k)}(S_t(k), S_0(k)) + f_{Q(k)}(Q(k))
\]

Where the expected production cost for period \( k \)

\[
f_{u(k)}(u(k)) = C_p \times E\left[u(k)^2\right]
\]

the expected holding costs of period \( k \)

\[
f_{Q(k)}(S_t(k), S_0(k)) = C_h \times E\left[S_t(k)^2\right] + C_h \times E\left[S_0(k)^2\right]
\]

The expected transported cost for period \( k \)
The number of preventive maintenance actions is a function of the production plan defined by the vector \( A \). An approach that transforms the stochastic problem into a deterministic equivalent is necessary. This deterministic equivalent problem maintains the main properties of the original problem.

Before proceeding, the following notation is introduced:

- Mean variables:
  \[
  E \{ S_i(k) \} = \hat{S}_i(k), \quad E \{ S_u(k) \} = \hat{S}_u(k), \quad E \{ u(k) \} = u(k), \quad E \{ Q(k) \} = Q(k)
  \]

- Variance variables:
  \[
  \nu_{u(k)} = \nu_{Q(k)} = 0.
  \]

(Note that this reflects the fact that the control variables \( u(k) \) and \( Q(k) \) are deterministic.)

- The production, delivery and inventory costs simplified as:
  \[
  f(\{S(k),S_u(k),u(k)\},Q(k)) = \sum_{k=0}^{H} c_{s0} \hat{S}_i(k)^2 + c_{s0} \hat{S}_u(k)^2 + \sum_{k=0}^{H} c_n \hat{S}_u(k)^2 + c_n \hat{S}_u(k)^2 + c_n \hat{S}_u(k)^2 + c_n \hat{S}_u(k)^2
  \]

- The inventory balance equation (2) can be reformulated as:
  \[
  \hat{S}_i(k+1) = \hat{S}_i(k) + u(k) - Q(k) \quad k = 0,1,\ldots,H-1
  \]

Likewise, the inventory balance equation (3) can be reformulated as:

- The service level constraint:

  Another important transformation changes the service level constraint into equivalent, but deterministic, inequalities by specifying through the following lemma a minimum cumulative transported quantity depending on the service level requirements.

**Lemma 2**

For \( k=0,1,\ldots,H-1 \) we have:

\[
\text{Prob}[S_i(k+1) \geq \theta] \geq \left[ \frac{Q(k-\tau)}{\theta} \right] \times \phi^2(\theta - \hat{S}_i(k) + \hat{d}_i(k) - \tau \hat{d}(k-\tau))
\]

\[
k = 0,1,\ldots,H-1
\]
\( \Phi \): Cumulative Gaussian distribution function with mean 
\[
\left( \frac{1}{\sqrt{2\pi\tau_\epsilon}} \times \hat{d}(k) - \frac{\delta}{\sqrt{\tau}} \times \hat{d}(k - \tau) \right) \\
\left( \frac{1}{\sqrt{2\pi\tau_{\epsilon\tau}}} \times \hat{d}(k) - \frac{\delta}{\sqrt{\tau_{\epsilon\tau}}} \times \hat{d}(k - \tau) \geq 0 \right)
\]

\( \varphi^{-1} \): Inverse distribution function

**B. Maintenance cost**

For the maintenance policy, we seek to determine the optimal maintenance strategy characterized by the optimal number \( N^* \) of preventive maintenance actions and the time between them \( T^* \), as given by Eq. (15).

\[
T^* = \frac{H}{N^*}
\]

The analytic expression of the total maintenance cost is as follows, with \( N \in \{1, 2, 3, \ldots\} \).

\[
C_M(U, N) = (N - 1) \times C_{pm} + C_{cm} \times \varphi_M(U, N)
\]

Where \( \varphi_M(U, N) \) corresponds to the expected number of failures that occur during the horizon \( H \), considering the production rate in each production period \( \Delta t \).

If we assume that \( \lambda(t) \) represents the linear failure rate function at production period \( t \) is expressed as following:

\[
\lambda_s(t) = \lambda_{s-1}(\Delta t) + \frac{u(k)}{U_{max}} \times \lambda_v(t) \quad \forall \ t \in [0, \Delta t]
\]

\( \lambda_s(t) \): failure rate for nominal conditions which is equivalent to the failure rate with maximal production.

The average failure number over the horizon \( H \times \Delta t \) is:

\[
\varphi_L(U, N) = \sum_{k=0}^{N} \sum_{t=0}^{\tau_{\epsilon t}} \sum_{d=0}^{\tau_{\epsilon d}} \left( \varphi(t, \frac{T}{\Delta t}) \times \lambda(0) + \int_{0}^{T/\Delta t} \lambda(t) dt \right)
\]

\[
+ \int_{0}^{T/\Delta t} U_{max} \times \lambda'(t) dt
\]

\( \lambda(0) + \int_{0}^{T/\Delta t} \lambda(t) dt \)

\( \lambda'(t) = \frac{\tau}{\beta} \left[ \left( \frac{t}{\beta} \right)^{\gamma-1} \right] \)

**C. Optimization**

Using the problem formulation established in the precedent sections we can formulate the problem of integrated optimization of production, inventory, delivery and maintenance costs as following:

\[
\begin{align*}
\min_{(U, N)} \quad & C_{pm} \times E[S_1(H)]^2 + C_{cm} \times E[S_2(H)]^2 + \\
& \sum_{k=0}^{H-1} C_{pm} \times E[u(k)^2] + C_{cm} \times E\left[\left( \frac{Q(k)}{Q} \right)^2 \right] \\
& + (N-1) \times C_{pm} + C_{cm} \times \varphi_M(U, N)
\end{align*}
\]

\[
S_1(k+1) = \dot{S}_1(k+1) + u(k) - Q(k) \\
S_0(k+1) = \dot{S}_0(k+1) + Q(k) - \hat{d}(k - \tau_1) - \hat{d}(k) \\
Q(k) \geq (V_{\alpha} \times V_{\alpha t}) \times \varphi^{-1}(\theta) - S_0(k) - \hat{d}(k) - \hat{d}(k - \tau_1) \quad \forall \ k \in \{0, 1, \ldots, H-1\} \\
U_{min} \leq u(k) \leq U_{max}
\]

**IV. NUMERICAL EXAMPLE (RESULTS)**

The following arbitrarily chosen input data are considered as an example to illustrate our approach:

\( H=36 \) months.

(i) Lower and upper boundaries of production capacities: \( U_{min}=5 \) and \( U_{max}=17 \).

(ii) The demand is assumed Gaussian with a standard deviation \( \sigma_d=1.2 \), mean=15.

(iii) The customer satisfaction degree, associated with the stock constraint, is equal to 90% (\( \theta=0.9 \)).

Point of view reliability, we suppose that the failure time of machine \( M \) has a degradation law characterized by a Weibull distribution. The Weibull scale and shape parameters are respectively \( \beta=100 \) and \( \gamma=2 \). The cost associated with a corrective and preventive maintenance action are respectively \( C_{cm} = 3000 \) \( \text{mu} \) and \( C_{pm} = 500 \) \( \text{mu} \) (monetary unit).

We recall that in the case of Weibull distribution \( W(\tau, \beta) \) we have:

\[
\lambda(t) = \frac{\tau}{\beta} \left( \frac{t}{\beta} \right)^{\gamma-1}
\]

The average demand is presented in table 1 below:

<table>
<thead>
<tr>
<th>Table 1</th>
<th>( d_0 )</th>
<th>( d_1 )</th>
<th>( d_2 )</th>
<th>( d_3 )</th>
<th>( d_4 )</th>
<th>( d_5 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>15</td>
<td>15</td>
<td>15</td>
<td>15</td>
<td>15</td>
<td>15</td>
<td>14</td>
</tr>
<tr>
<td>14</td>
<td>14</td>
<td>14</td>
<td>14</td>
<td>14</td>
<td>14</td>
<td>13</td>
</tr>
<tr>
<td>13</td>
<td>13</td>
<td>13</td>
<td>13</td>
<td>13</td>
<td>13</td>
<td>12</td>
</tr>
<tr>
<td>12</td>
<td>12</td>
<td>12</td>
<td>12</td>
<td>12</td>
<td>12</td>
<td>11</td>
</tr>
<tr>
<td>11</td>
<td>11</td>
<td>11</td>
<td>11</td>
<td>11</td>
<td>11</td>
<td>10</td>
</tr>
<tr>
<td>10</td>
<td>10</td>
<td>10</td>
<td>10</td>
<td>10</td>
<td>10</td>
<td>9</td>
</tr>
</tbody>
</table>

Average demand

We used the Exact Global Optimization method with MATHEMATICA, in order to realize this optimization. The economically production and delivery plans and the optimal maintenance scheduling are presented respectively in table 2.
In what follows, we interest to find the values of the delivery time $\tau$ and number of PM actions $N$. We can see that the lowest total cost value corresponds to $\tau=2$ and $N=2$. Thus, the optimal delivery time denoted by $\tau^*=2$ and the optimal number of preventive maintenance actions denoted by $N^*=2$. Therefore, over the finite horizon $H$ of 36 months, Two preventive maintenance actions should be done, i.e. for every period equals to $T^*=H/N^*= 18t_u$ a preventive maintenance action should be done.

V. CONCLUSION

This paper dealt with a constrained stochastic production/delivery/maintenance planning problem considering a delivery time, a random demand $d$, a service level and a randomly failing production system. In order to reduce the failure rate, preventive maintenance actions are planned every $T$ time units. Given the necessary service level, we have formulated and solved the related stochastic production/delivery/maintenance problem. A joint optimization has been performed obtaining an optimal production and delivery plans as well as the corresponding preventive maintenance periods taking into account the influence of delivery time on the problem optimization. A numerical example has been presented illustrating the proposed approach.

REFERENCES


