Analysis of Cooperative Spatial Multiplexing for Ad Hoc Networks with Adaptive Hybrid ARQ

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Abstract—Existing cooperative diversity techniques for wireless ad hoc networks mostly consider space-time block codes for cooperation. In this paper we propose a cross-layer design of ad hoc wireless networks based on spatial multiplexing (SM). Each node is equipped with multiple antennas and the spatial dimensions of the channel are exploited to support multiple simultaneous transmissions. Diversity is then provided only for failed transmissions by means of selective cooperation among nodes that still use SM for transmission. Moreover, to increase the efficiency of the system, a hybrid automatic repeat request (ARQ) protocol integrated with packet coding is used at the medium access control layer. We analyze the proposed network architecture with a Markov chain description of the decoding process and we derive a closed form expression for the achieved throughput in Rayleigh fading channels.

I. INTRODUCTION

Sensor and ad hoc networks are characterized by fading channels and interference among nodes that can be overcome by letting nodes cooperate, each acting as an element of a distributed multiantenna system. Most cooperation approaches use space-time codes (STC) to allow simultaneous cooperative transmission of more nodes [1], [2]. However, STCs i) are oriented to diversity rather than multiplexing, and may not be the optimal solution to maximize the network throughput, and ii) require symbol synchronization among cooperating nodes, which may be hard to achieve in non-infrastructured networks.

In this paper we design a cooperating network using spatial multiplexing (SM), which does not require symbol synchronization among nodes, and has a simple signal processing for both the multicast and the cooperative phase, aiming at maximizing the throughput with a limited exchange of signalling among nodes. Each node of the network has several antennas that are used for a layered space-time multuser (LASTMUD) transmission [3]. Diversity is provided only when the initial transmission fails, by a selective cooperative protocol, and cooperating nodes transmit simultaneously, still using LASTMUD. The efficiency of the cooperative phase is further enhanced by the cross-layer design of a hybrid ARQ mechanism based on linear erasure codes [4] applied to packets rather than symbols. We denote the resulting system as layered packet coded cooperative system (LPCCS).

We analyze the proposed network architecture with a Markov chain description of the decoding process, and we derive a closed form expression for the achieved throughput in Rayleigh fading channels. Numerical results are then provided and compared with those for a network architecture using truncated ARQ and cooperation [5].

II. PHYSICAL LAYER

Linear erasure packet codes (LEPC) make it possible to implement forward error correction (FEC) on entire packets of bits rather than single symbols [4].

For LEPC encoding, the data packet of $M$ symbols is first split into the $N_p$ sub-packets, $p_q$, $q = 1, 2, \ldots, N_p$, and from these data sub-packets $N_C > N_p$ coded sub-packets, $c_i$, $i = 1, 2, \ldots, N_C$, are obtained, each with $M = M'/N_p$ symbols. A systematic LEPC provides the data sub-packets as the first $N_p$ coded sub-packets, i.e. $c_i = p_i$ for $i = 1, 2, \ldots, N_p$, while coded sub-packets $c_i$ for $i = k + 1, k + 2, \ldots, N_C$ are the parity sub-packets.

An interesting property of any LEPC having a full-rank generating matrix is that if any $N_p$ distinct coded sub-packets are correctly detected at the receiver, then the entire packet can be decoded, [4]. In LPCCS, each coded sub-packet has also a cyclic redundancy code (CRC) that allows the receiver to understand which sub-packets have been correctly detected. Hence, if the receiver has correctly received any $N_p$ distinct coded sub-packets, it can retrieve the entire data packet by the properties of LEPC.

Each node is equipped with $N$ antennas and can operate either as a transmitter or as a receiver (half duplex). Let us suppose that node $n$ is required to transmit $P$ sub-packets. In order to reduce the interference among nodes, symbols of each packet may also be spread with a node-specific spreading code [3] to obtain the vector $a_i^{(n)}$ of the spread symbols relative to the $i$th sub-packet.

In general, by using SM, each node may transmit simultaneously $N$ data packets on the $N$ antennas. For ease of analysis and to gain a fundamental insight into the characteristics of the proposed network architecture, we consider the transmission of a single packet per node. Hence, each node uses only one antenna for transmission, selected randomly in the set of the $N$ available antennas. SM transmission is simply implemented transmitting the spread symbol vector $a_i^{(n)}$, $i = 1, 2, \ldots, P$ in sequence on the selected antenna. For reception instead, nodes use all the $N$ antennas and, since more nodes may be transmitting at the same time, the receive node see a multiple input-multiple output (MIMO) system.

As transmission scenario we consider a flat fading channel and we indicate with $h_i^{(n, m)}(k)$ the complex channel gain between the transmit antenna of node $n$ and receive antenna $j$ of node $m$ at the discrete chip-time $k$. Nodes are in fixed positions and the channel gain is factorized into two terms,
one accounting for the path-loss between nodes $n$ and $m$, and the other accounting for fading between two specific antennas, i.e.,
\[ h_j^{(n,m)}(k) = a^{(n,m)}_j \phi_j^{(n,m)}(k). \] (1)

When $U$ nodes are transmitting simultaneously, the signal at the output of a receiving antenna is the superposition of signals coming from the antennas of all transmitting nodes. In formula, letting $N$ be the set of $U$ transmitting nodes, the received signal at time $k$ on the antenna $j$ of a generic node $m \notin N$ can be written as
\[ r_j^{(m)}(k) = \sum_{n \in N} h_j^{(n,m)}(k)a^{(n)}(k) + w_j^{(m)}(k), \] (2)
where $j = 1, 2, \ldots, N$ and $w_j^{(m)}(k)$ is an additive Gaussian noise term with zero mean and variance $\sigma^2$. Note that we dropped the sub-packet index $i$ from $a^{(n)}(k)$ in (2) for ease of notation.

We consider the layered space-time multiuser (LASTMUD) receiver of [3] that performs detection of signals coming from different transmit antennas in sequence. After being detected, each signal is remodulated in order to obtain, for a correct detection, an interference- and noise-free replica of the received signal. This replica is subtracted from the received signal before detection of the next stream, in order to reduce interference. A receiving node not only detects streams intended to itself, but also streams transmitted to other nodes but generating strong interference. Let $\mathcal{N}_U(m)$ be the set of detected streams by node $m$. In general, if many interfering streams are correctly decoded and canceled, the detection of data is more reliable.

In formula, define the column vector collecting the samples received at all the antennas of node $m$ at time $k$ as
\[ r^{(m)}(k) = [r_1^{(m)}(k), r_2^{(m)}(k), \ldots, r_N^{(m)}(k)]^T. \] (3)
Let $d^{(m)}(k)$ be the column vector collecting the chips of the streams in $\mathcal{N}_U(m)$ at time $k$. Also, let $H^{(m)}(k)$ be the matrix having as $n$th row the channel gains at the $n$th antenna of node $m$, relative to all the streams $\mathcal{N}_U(m)$ at time $k$. Lastly, let $\hat{r}^{(m)}(k)$ be the $N$-size column vector collecting both the noise and the interference seen by the receive antennas of node $m$ due to transmitting nodes not in $\mathcal{N}_U(m)$. Then the received vector can be written as
\[ r^{(m)}(k) = H^{(m)}(k)d^{(m)}(k) + \hat{r}^{(m)}(k). \] (4)

In order to extract sufficient statistics for decoding, the receive node applies a matrix filter matched to the channel [3], [6] to the received vector, i.e.,
\[ \tilde{r}^{(m)}(k) = H^{(m)}(k)H^T(k)r^{(m)}(k). \] (5)
Then, streams of nodes in $\mathcal{N}_U(m)$ are detected in sequence by first combining the elements of $\tilde{r}^{(m)}(k)$ with suitable weights that set to zero interferers, and then performing despreading of the signal. The interference due to the detected stream is cancelled from $\tilde{r}^{(m)}(k)$ so that detection of the following streams is more reliable. For further details on the LASTMUD receiver, the interested reader can refer to [3]. Here we only mention that the order of their detection has an impact on the network performance [6].

### III. MEDIUM ACCESS CONTROL

With LPCCS we exploit the presence of multiple antennas on nodes in order to provide spatial multiplexing rather than diversity. This allows a more intensive spatial reuse with a consequent increase of the overall throughput since nodes are transmitting simultaneously on the same band. Interference at the receiving nodes is limited both by spatial separation and by the spreading that characterizes each node. Hence, no special coordination is required among nodes to avoid collisions. Still, channel conditions are variable and not predictable, since interference is changing according to the transmission schedules of other nodes. Hence, we resort to an error control mechanism based on an adaptive automatic repeat request (A-ARQ) that can be seen as a coding strategy with an adaptive coding gain according to the network conditions. Moreover, cooperation among nodes further provides diversity to the transmission, and makes A-ARQ more efficient.

With A-ARQ, when transmission of a packet fails, the source node will retransmit part of the packet information so that the destination node attempts a new packet decoding, using both the information received in the first transmission and the further redundancy obtained from retransmission.

Contrary to conventional hybrid ARQ which transmits different punctured versions of the symbol-coded data packet [7], in the A-ARQ scheme implemented by LEPCC different parity sub-packets are transmitted. The number of retransmitted sub-packets is changed according to the number of sub-packets correctly decoded by the destination at the first transmission.

However, when transmission failure is due to the combination of fading and path-loss between the source and the destination, A-ARQ can be greatly improved by cooperative diversity. In fact, cooperative diversity provides spatial diversity through the cooperating node, which operates as a relay. In LPCCS if any node $C$ is idle, i.e. it is neither transmitting nor receiving any packet, it cooperates by first decoding the initial transmission between a source node $S$ and a destination node $D$. Then, if $C$ was able to decode the packet but $D$ was not, $C$ can cooperate with $S$ and help the retransmission. This is achieved by transmitting redundancy sub-packets, using the same A-ARQ scheme as the source. Note that since LASTMUD is used, no synchronization is required among nodes and cooperation is implemented by simply transmitting the redundancy on one antenna. Note also that if the packet is correctly decoded by $D$ at the first transmission, cooperation is not activated (selective cooperation).

We now describe in more detail the adaptive ARQ and cooperation mechanisms.

**Adaptive ARQ.** When node $S$ has a packet to transmit to node $D$, it sends a signalling packet of request to send (RTS) to $D$, using LASTMUD. If $D$ is not involved in other transmissions, it replies with a clear to send (CTS) signalling packet. We assume that both RTSs and CTSs are transmitted with additional FEC, in order to ensure their decoding with high probability. Upon reception of CTS, node $D$ transmits the data sub-packets to $D$. At the end of the transmission, $D$ replies with an acknowledgment (ACK) signaling packet containing the number and the identifiers of un-decoded sub-packets.
If $N_F$ sub-packets failed to be decoded, $S$ transmits $N_F$ sub-packets randomly chosen in the set of the parity sub-packets of the LEPD. Node $D$ then replies with a new ACK relative to the new sub-packets. The process is iterated until either $k$ sub-packets of the data packet have been decoded or a maximum number $N_{max}$ of retransmissions has been reached. The number of retransmitted sub-packets can be optimized for the network scenario, but this optimization has not been considered in this paper for lack of space.

**Cooperative diversity.** A node $C$ cooperates in the transmission between node $S$ and $D$ when it is idle and it is in a better position than the source. In order to evaluate the position of the node, we assume that in the CTS packet, the destination node includes an estimate of the signal gain from the source and it is used to ensure that cooperating nodes are closer to the destination than the source. At the first transmission attempt from $S$ to $D$, node $C$ listens to the transmission. If it is able to decode the entire packet, it is ready to cooperate. By listening to the ACK from node $D$, the cooperating node understands whether A-ARQ is activated. In this case, node $C$ transmits some redundancy sub-packets, acting as an additional source for the same data packet. The destination detects the transmission of the cooperating node and identifies it as a cooperation from a suitable preamble. The ACK from $D$ includes also information about correctly decoded sub-packets coming from $C$. Note that there is no additional signalling or synchronization needed between the cooperative node and the other two nodes. Hence this MAC cooperation strategy is very simple to implement. There are various options for the combining at node $D$ of the signals coming from $S$ and $C$. Here, in order to have a very simple receiver, we assume that the streams coming from $S$ and $C$ are decoded separately and the decoded sub-packets are then used for LEPC decoding. We also assume that $C$ and $S$ select randomly sub-packets from the $n$ available for retransmission and the probability of transmitting the same sub-packet is negligible. Lastly, note that the cooperative procedure is completely distributed and due to the use of spatial multiplexing more than one node can simultaneously cooperate for the same transmission.

**IV. Analysis of LPCCS**

For ease of description and in order to first gain fundamental insight, as commonly done in the literature [1], [8]–[10] we limit the analysis to a network with a source node $S$, a destination node $D$ and a possible cooperating node $C$.

The analysis of the cooperating protocol with LASTMUD and A-ARQ is based on a Markov chain, whose states identify both how many sub-packets have been correctly detected and whether or not node $C$ is cooperating. For the considered network, the Markov chain is shown in Fig. 1. The initial state of the network is state $S$. Upper states in Fig. 1, denoted as $(C, \nu)$, correspond to a situation where node $C$ collaborates and $\nu$ sub-packets have been correctly detected at the destination. State $(C, s)$ in the lower part of Fig. 1 refers to the case where no collaboration is active and $\nu$ sub-packets have been correctly detected at the destination.

We indicate with $z_\nu$ the probability that node $D$ has correctly detected $\nu$ sub-packets after the multicast phase. As for node $C$, it can have either decoded the entire packet or not. In the first case, cooperation starts while in the second case only node $S$ transmits redundancy sub-packets to $D$. We indicate with $p_C$ the probability that node $C$ decodes the entire packet. For flat uncorrelated fading, transmission channels seen by nodes $C$ and $D$ are independent. In case of cooperation, the transition probability to the state where $\nu$ sub-packets are correctly decoded at $D$ is

$$p_\nu = z_\nu p_C, \quad \nu = 0, 1, \ldots, N_F,$$

while for non-cooperation and $\nu$ sub-packets correctly decoded is

$$p_\nu = z_\nu (1 - p_C), \quad \nu = 0, 1, \ldots, N_F.$$

For a cooperative transmission, destination $D$ attempts to detect sub-packets coming from both $S$ and $C$. At each retransmission, the destination may not be able to correctly decode any new packet, and in this case the chain remains in the current state. Otherwise, the state is updated accordingly. In the Markov chain of Fig. 1 $q_{s,\nu}$ indicates the transition probability from state $(C, s)$ to state $(C, s + \nu)$, i.e. from the case of $s$ correctly detected sub-packets at the destination, to the case of $s + \nu$ correctly detected sub-packets.

Correspondingly, for a non-cooperative transmission $q_{s,\nu}$ in Fig. 1 indicates the transition probability from state $(C, s)$ to state $(C, s + \nu)$ where only node $S$ is transmitting to $D$. In general, $q_{s,\nu}$ and $q_{s,\nu}$ are different, since the first is obtained with two nodes transmitting to $D$, while the second is obtained with only one node transmitting to $D$. Also, we note that the statistical description of $q_{s,\nu}$ is different from that of $p_\nu$, since in the multicast phase all transmitting antennas belong to the same node and thus have a common path-loss, while in the cooperation phase transmitting antennas belong to different nodes, having different path-loss. This provides a further degree of diversity to our scheme.

Fig. 1. Example of Markov chain for LPCCS.
From the Markov chain of Fig. 1 we can capture two distinctive features of LPCCS, namely a) the cooperative diversity, which is reflected into the two distinct sub-chains, and b) the adaptive ARQ, which is reflected into the \( N_p \) states of each sub-chain which represents the adaptive behavior of ARQ.

Let \( p (\bar{p}) \) be the \( (N_p + 1) \times 1 \) column vector containing the probabilities \( p_i \) (\( \bar{p}_i \)). Let us define the transition probability matrix

\[
P = \begin{bmatrix}
0_{1 \times 2N_p+3} & Q & 0_{N_p+1 \times N_p+1} \\
0_{1 \times N_p+1} & 0_{N_p+1 \times N_p+1} & 0_{1 \times N_p+1}
\end{bmatrix}.
\]

(8)

where \( Q \) is an \( (N_p + 1) \times (N_p + 1) \) matrix with entry \( Q(\ell_1, \ell_2) = q_{\ell_2, \ell_1} \) (\( \ell_1 \neq \ell_2 \)) for \( \ell_1, \ell_2 = 0, 1, \ldots, N_p \), \( \ell_1 \leq N_p - \ell_2 \) and zero otherwise. Let also

\[
s(u) = [s_S(u) s_C(u) s_C^T(u)]^T,
\]

(9)

be the \( 2N_p + 3 \) column vector of state probabilities at the \( u \)th transmission, with \( s_S(u) \) the state probability of the state \( S \), \( s_C(u) \) the \( (N_p + 1) \) column vector containing the probabilities of states \( (C, 0), (C, 1), \ldots, (C, N_p) \) and \( s_C^T(u) \) the \( (N_p + 1) \) column vector containing the probabilities of states \( (C, 0), (C, 1), \ldots, (C, N_p) \). At the first transmission we have \( s(0) = [1, 0, \ldots, 0]^T \), since we start from state \( S \). The state probabilities are obtained as

\[
s(u + 1) = Ps(u), \quad u = 0, 1, \ldots, N_p.
\]

(10)

The probability of a complete packet decoding in no more than \( u \) steps is obtained as

\[
p_{\text{dec}}(u) = s(0)[s(u)]_{2N_p+3} + [s(u)]_{N_p+2}.
\]

(11)

Since at each retransmission node \( S \) and \( C \) retransmits in sequence a number of sub-packets equivalent to those that have not been decoded, the average length of the transmission after \( u \) retransmissions is

\[
L(u) = \sum_{u=1}^{u} \sum_{\ell=0}^{N_p} (N_p - \ell) \{s(u)\}_{\ell+2} + s(u)\}_{N_p+\ell+3}.
\]

(12)

Hence, the average throughput after \( u \) retransmissions is

\[
T(u) = \frac{p_{\text{dec}}(u)}{L(u)} \quad \text{[bit/s/Hz]}.
\]

(13)

A. Transition probabilities computation

We derive the transition probabilities of the LPCCS Markov chain under the following assumptions: a) error propagation is neglected, \( b \) each sub-packet retransmission is characterized by independent fading, \( c \) all nodes transmit with one antenna.

We indicate with \( \alpha(S,D) \) the path loss between the source and the destination and with \( \alpha(C,D) \) the path loss between the cooperative node and the destination. We indicate with \( \alpha(S,C) \) the path loss between the source and the cooperative node. We also assume that when both \( S \) and \( C \) are active, the LASTMUD receiver first detects streams of the cooperative node and then that of the source.

Let us consider sub-packets with \( L \) BPSK symbols which are decoded at stage \( q \) in the LASTMUD receiver. Transmission is performed on a Rayleigh fading channel and we assume that the channel does not change over one sub-packet (block fading). Considering nodes with one transmit and \( N \) receive antennas the probability of correct detection of the entire packet is

\[
Q(q, \alpha(S,D)) = \int_{x=0}^{\infty} \left[ 1 - Q\left( \frac{x\sqrt{\alpha(S,D)^2}}{\sigma^2} \right) \right]^M p_H(x) dx
\]

(14)

where \( p_H(x) \) is the probability distribution of the power gain. It has been shown in [11] that \( x \) is chi-square distributed with \( 2(N - 1 + q) \) degrees of freedom and

\[
p_H(x) = \left[ 2^{N-1+q} \Gamma(N-1+q) \right]^{-1} x^{N-1+q-1} e^{-x/2}, \quad x \geq 0.
\]

(15)

Multicast phase. Let us suppose that \( N_p \) data sub-packets are transmitted by the source. In this case the transition probability from state \( S \) are

\[
z_{\nu} = \left( \frac{N_p}{\nu} \right) Q(1, \alpha(S,D))^{\nu} [1 - Q(1, \alpha(S,D))]^{N_p-\nu}
\]

(16)

for \( \nu = 0, 1, \ldots, N_p \).

In the multicast phase, the cooperative node detects correctly the packet with probability

\[
p_C = [1 - Q(1, \alpha(S,C))]^{N_p}
\]

(17)

while it is not able to correctly decode the packet with probability

\[
p_{\bar{C}} = 1 - [1 - Q(1, \alpha(S,C))]^{N_p}.
\]

(18)

Non-cooperative retransmissions. If the cooperative node is not able to decode the packet in the multicast phase, no cooperation begins and the transition probability is

\[
q_{s,\nu} = \left( \frac{N_p - s}{\nu} \right) Q(1, \alpha(S,D))^{\nu} [1 - Q(1, \alpha(S,D))]^{(N_p-s-\nu)},
\]

(19)

for \( \nu = 0, 1, \ldots, N_p - s - 1 \) and

\[
q_{s,\nu} = 1 - \sum_{\nu=0}^{N_p-s-1} q_{s,\nu}.
\]

(20)

Cooperative retransmission. If cooperation is active, the first node to be decoded is the cooperative and then the source. Hence, the transition probabilities can be written as

\[
q_{s,\nu} = \sum_{\nu_1=0}^{\nu} \left( \frac{N_p - s}{\nu} \right) \left( \frac{N_p - s}{\nu - \nu_1} \right) Q(1, \alpha(C,D))^{\nu_1} \left[ 1 - Q(1, \alpha(C,D))^{(N_p-s-\nu_1)} \right] Q(2, \alpha(S,D))^{\nu_1} \left[ 1 - Q(2, \alpha(S,D))^{(N_p-s-\nu_1)} \right]
\]

(21)

for \( \nu = 0, 1, \ldots, N_p - s - 1 \) and

\[
q_{s,\nu} = 1 - \sum_{\nu=0}^{N_p-s-1} q_{s,\nu}.
\]

(22)
V. NUMERICAL RESULTS

We show here preliminary results for a network with three nodes, a source, a destination and a possible cooperative node. Source and cooperative nodes use one antenna for transmission, while the destination is equipped with two antennas for reception.

The first network scenario is the one described in [5]. In this scenario, the link between the source and the collaborative node has an average SNR of 20 dB, and the both the source and the collaborative node are at the same distance from the destination. Hence, the average SNR between $S$ and $D$ is the same as that between $C$ and $D$. All links are characterized by symbol-fading, i.e., the channel changes at each transmitted symbol. In this case $Q(q, \alpha(S,D))$ becomes

$$Q(q, \alpha(S,D)) = \left\{ \begin{array}{ll} 1 - Q \left( \sqrt{\frac{\alpha(S,D)^2}{\sigma^2}} \right) p_H(x)dx \end{array} \right\}^M.$$  

The packet length is 60 symbols, while $N_P = 20$.

The average throughput of LPCCS as a function of the average SNR between the source node and the destination node is shown in Fig. 2. We also report the performance of LPCCS scheme when $p_C = 0$, i.e., when no cooperative node is available. In this case we observe the impact of A-ARQ on the throughput. We also report the throughput of the selective cooperative diversity with ARQ (SCA) network of [5]. SCA implements ARQ where the entire packet is retransmitted upon failure of the first transmission, and a third node can cooperate. Cooperation is performed by the Alamouti scheme [12], using both antennas of the destination. From the figure we observe that LPCCS outperforms SCA even without cooperation, while the presence of a possible cooperative node further increases the throughput.

A more realistic scenario for wireless ad hoc network comprises block fading as from (14) and longer data packet, and we consider a transmission with $M' = 3600$ and $N_P = 20$. Also in this case we compare LPCCS with A-ARQ and with the SCA network. The same conclusions as for Fig. 2 can be derived.

REFERENCES


