A Design Criterion of Error Correcting Codes for Spectrum-Overlapped Resource Managements

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Abstract—This paper proposes a new design criterion for error correcting codes (ECC) in the spectrum-overlapped resource management (SORM) for a broadband single carrier transmission. In the SORM technique, each user can ideally obtain the maximum channel gain by allowing overlapped allocation, assuming a soft canceller with minimum mean square error (SC/MMSE) turbo equalization for multi-user detection. When the interference caused by overlapped allocation is not completely removed by SC/MMSE turbo equalization, frame error rate (FER) remarkably degrades. The reason is that conventional ECCs are inadequate in the SORM scenario. To solve this problem, we propose a new criterion which is used for design of an ECC based on the extrinsic information transfer (EXIT) analysis in order to improve convergence property of SC/MMSE turbo equalization without unnecessarily large redundancy of an ECC. In the proposed criterion, we design the ECC so that it has minimum redundancy corresponding to a channel and greatly high error correcting ability in the lower amount of input mutual information. This paper evaluates FER performance with the designed irregular low-density parity-check (LDPC) codes based on this criterion. As a result, this paper shows that the designed irregular LDPC code based on the proposed criterion for the SORM outperforms conventional codes.

I. INTRODUCTION

Uplink multiple access techniques for the broadband wireless communication systems have been widely studied for the improvement of peak data rate and spectral efficiency under limited transmission power. The third generation partnership project (3GPP) long term evolution (LTE) system employs a single carrier frequency division multiple access (SC-FDMA) in the uplink [1]. Although this access scheme has a low peak to average power ratio (PAPR) property due to continuous allocation of its spectrum in the frequency domain, its spectral efficiency is not so high because the available spectrum tends to be spot-wise rather than continuous, and aggregation of spot-wise spectra cannot be allocated to a user in the case of continuous spectrum allocation.

A challenge for efficient utilization of spot-wise spectra is the dynamic spectrum control technique [2] based multiple access e.g., the clustered discrete Fourier transform spread orthogonal frequency division multiple access (clustered DFT-S-OFDMA) [3], [4]. In this access scheme, the single carrier spectrum is divided into plural clusters and they are mapped to spot-wise available spectra. In this process, it is desirable that a base station (BS) allocate spot-wise spectra having the highest channel gain to each user. Although the FDMA-based approaches keep users’ signals orthogonal in order to avoid inter-user interference (IUI), channel gain for each user cannot be maximized.

To overcome this issue, we have proposed a spectrum-overlapped resource management (SORM) [5]. In the SORM, regardless of the number of receive antennas, each user’s spectra can be allocated to the spectrum having the highest channel gain because a BS accepts spectrum overlapping between users. Then, IUI caused by overlapping spectrum is eventually canceled out in conjunction with compensation of inter-symbol interference (ISI) using a soft canceller with minimum mean square error (SC/MMSE) turbo equalizer [6]. For user signal discrimination using the SC/MMSE turbo equalizer in the SORM, it is mandatory that the iterative process of SC/MMSE turbo equalizer should be sufficiently converged, because otherwise, orthogonality between user signals cannot be satisfied. The convergence property depends on a combination of an equalizer as the inner code and a decoder as the outer code. Therefore, one of the most important issues is how to design error correcting codes (ECC) to improve the convergence property of the turbo equalization for the SORM.

The extrinsic information transfer (EXIT) analysis is well known as a useful technique to visually predict the convergence property of iterative system based on input versus output characteristic of mutual information (MI) [7] In the EXIT analysis, the area minimization criterion is used as an approach to achieve near-channel capacity [8]. However, it is difficult to apply this design method in the multi-dimensional EXIT chart. In the SORM, since the multi-dimensional EXIT chart should be used due to being fed back from all users’ decoders, it is also difficult to utilize the criterion. As one of the methods to analyze multi-dimensional EXIT chart, the EXIT chart
projection has been proposed [9]. This method can guarantee a successful convergence by fitting an EXIT curve of channel decoder to that of the lower bound, which means no extrinsic information regarding IUI is fed back from the decoders. In such a case, the designed ECC based on the lower bound is too redundant, since this ECC is appropriate under the condition of no extrinsic information.

Therefore, this paper proposes a new design criterion for the ECC in the SC/MMSE turbo equalizer. In the proposed criterion, the ECC is designed by minimizing the redundancy of the channel decoder corresponding to the channel, resulting in good convergence property. To clarify the validity of the proposed criterion, we employ an irregular low-density parity-check (LDPC) code [10] which has more flexibility to control a decoder property than turbo codes [11]. We show the construction of the irregular LDPC code designed by our proposed criterion. Computer simulation shows that the designed irregular LDPC code can improve the convergence property of SC/MMSE turbo equalizer for the SORM. And then, comparing the frame error rate (FER) performance, we present that the designed irregular LDPC code based on the proposed criterion outperforms conventional codes.

II. SPECTRUM-OVERLAPPED RESOURCE MANAGEMENT

A. Concept of SORM

Fig. 1 (a) and (b) show the concepts of the clustered DFT-S-OFDMA and the SORM where two users simultaneously communicate with a BS, respectively. In the conventional allocation for the uplink in cellular systems, the BS allocates the frequency resource having the highest channel gain to each user while satisfying orthogonality between users. An example is the case of the clustered DFT-S-OFDMA shown in Fig. 1 (a). In this case, because orthogonality between users is prioritized over the maximization of channel gain for each user, frequency selection diversity gain cannot be fully obtained.

On the other hand, in the SORM case shown in Fig. 1 (b), since the BS accepts overlapped spectrum allocation, a cluster of the frequency resources having the highest channel gain for each user is allocated, while IUI caused by the overlapped allocation can be cancelled by the SC/MMSE turbo equalizer. Thus, the SORM has the potential to satisfy both requirements: maximization of each user’s channel gain and guaranty of orthogonality between users’ signals at the same time.

However, there is one important condition for guaranteeing orthogonality in the SORM, i.e., the iterative process in the receiver should be sufficiently converged. In the iterative process of the SC/MMSE turbo equalizer, the convergence property is determined by the relationship between the property of the channel decoder, the channel gain for the selected spectrum, and IUI in the spectrum. Specifically, as will be explained in Section III, when the EXIT property of the decoder is designed to suit the EXIT property of the equalizer in the SORM, we can improve the convergence property of the turbo equalizer in the SORM, and thereby we can guarantee the orthogonality between users.

B. Multuser Detection for SORM

Fig. 2 shows transmitter configuration for each user in the SORM. At the transmitter, each user generates an independent transmit signal. In the spectrum mapper, the user’s spectrum is divided into plural clusters, and the clusters are mapped to the available spectra with higher channel gains without any care about IUI due to spectrum overlapping.

At the receiver shown in Fig. 3, the received signal is converted to the frequency domain signal by the fast Fourier transform (FFT). Then, the soft canceller subtracts interference using the soft replicas generated by the extrinsic log-likelihood ratio (LLR) from the channel decoder, where no cancelation is conducted in the first iteration since no extrinsic LLR $LLR_{dec_out}$ is generated. The spectrum demapper then shifts back the spectrum to the original position. The extrinsic LLR of the coded bit sequence is calculated from the MMSE filter output by the demodulator. In this stage, the LLR can be approximated as a Gaussian random process, and the extrinsic LLR sequence from each user’s decoder is fed back to the soft canceller. After this process is repeated by a preliminarily determined number of times, the transmitted data sequence is detected by hard decision of the a posteriori LLR sequence from the decoder.

III. EXTRINSIC INFORMATION TRANSFER ANALYSIS

A. EXIT analysis of SORM

The EXIT analysis proposed in [7] is a useful method to visually analyze convergence property in the iterative process of the turbo algorithm. This method uses input versus output property of MI calculated from the extrinsic LLR and can be utilized to design the iterative system. In the additive white Gaussian noise (AWGN) channel, the inner code and the outer code are designed so that the area between EXIT curves for both codes is minimized. This design criterion is called the area minimization [8]. Contrary to the AWGN channel case, there are two challenges for the design of a channel decoder in the introduced SORM systems. The first one is that it is quite difficult to design a decoder based on the area minimization.
criterion for the SORM under multi-user environments, because the EXIT chart is expressed by more than two dimensions. To solve this issue, we will employ the projected EXIT curves that project the multi-dimensional EXIT trajectory onto as a set of two-dimensional EXIT charts [9] for simplifying the decoder design. The other challenge is that it is more probable to get into a stack in which is the unsuccessful convergence of the turbo algorithm, because the equalizer’s output MI at the starting region of the EXIT chart, which means low input MI from own outer code, is low due to IUI by the spectrum overlapping. In such a case, the channel decoder (outer code) included in the SC/MMSE turbo equalizer is required to have the ability to sufficiently increase the output (extrinsic) MI at the first iteration even with very low input MI. The irregular LDPC code is generally known to have the ability low input MI from own outer code, is low due to IUI by the spectrum overlapping. In such a case, the channel decoder (outer code) included in the SC/MMSE turbo equalizer is required to have the ability to sufficiently increase the output (extrinsic) MI at the first iteration even with very low input MI. The irregular LDPC code is generally known to have the ability to flexibly control the decoder property. Thus, this paper will focus on the irregular LDPC code design for enhancement of the convergence property of the turbo equalization in the case of the introduced SORM systems.

B. EXIT chart for SORM with irregular LDPC

Reference [12] explains the EXIT analysis based code design method for the irregular LDPC code with SC/MMSE turbo equalizer in the multiple-input multiple-output (MIMO) case. Fig. 4 shows the EXIT analysis model for the SORM under a two-user case. In this model, a combination of an equalizer and a variable node decoder (VND) is regarded as the inner code, whereas a check node decoder (CND) is regarded as the outer code. In this structure, the behavior can be analyzed by the EXIT curves of these two codes and can be controlled by the constitution of the VND. The output MI for the combination of the equalizer with the VND can be calculated as

\[
i_{\text{eq,in}1} = \sum_{i=1}^{D} b_i J\left(\sqrt{(w_{v,i} - 1)\left[J^{-1}(I_{\text{eq,in}1})\right]^2 + \left[J^{-1}(I_{\text{eq,out}1})\right]^2}\right)
\]

where \(J()\) is the J-function [12] that can be approximately converted from the Gaussian distributed LLR to the MI, \(D\) is the number of different variable node degrees, \(w_{v,i}\) is the \(i\)-th variable node degree (1 \(\leq i \leq D\)), and \(I_{\text{eq,in}1}\) is calculated from the input MI fed from each user’s decoder, and \(E_b/N_0, I_{\text{eq,ini}}, I_{\text{eq,in}2}\) are the input MI concerning ISI and IUI, respectively. \(b_i\) is defined by

\[
b_i = a_i w_{v,i} / (1 - r) w_c,
\]

where \(r\) is coding rate, \(a_i\) is the fraction of nodes having \(w_{v,i}\), and \(w_c\) is degree of the check node.

The EXIT property of the CND is expressed by

\[
i_{\text{cnd,in}1} = \left[1 - J^{-1} \left(1 - I_{\text{cnd,in}1}\right) / \sqrt{w_c - 1}\right],
\]

where \(I_{\text{cnd,in}1}\) denotes the output MI from the VND, and the input MI to the CND.

Fig. 5 provides the EXIT properties for the SORM under two users. In this case, the actual EXIT property is expressed by a three-dimensional EXIT chart consisting of the input MI fed from all users’ decoder and the output MI fed to the outer decoder. In Fig. 5, this paper employs the projected EXIT chart constructed by the inner code’s output MI and the outer code’s one. The weight distribution of the irregular LDPC code in the chart is an example of the design based on area minimization criterion in the AWGN channel.

In Fig. 5, the red lines show the upper and the lower bound of the projected EXIT curve for the inner code (combination of the equalizer and VND). These curves are obtained by computer simulation in the \(E_b/N_0 = 5\) dB case. In this case, FER of less than 0.01 is achieved. The other simulation parameters are described in TABLE I. The upper bound in this figure corresponds to the performance in no IUI case, which is identical to the single user case. On the other hand, the lower bound corresponds to the performance with IUI case. The actual inner code’s EXIT curve is located between the upper and the lower bound curves. The black line plots the EXIT trajectory which shows actual output MI from the inner and the outer codes. The blue line shows the EXIT curve for the outer code. For convergence of the iterative detection for the SORM, the EXIT curve for the inner code should always be located upper than the EXIT curve for the outer code. In this figure, however, the trajectory shows unsuccessful convergence (stack at the starting region), because the input MI fed from the outer code at the first iteration is too low to improve MI by the outer code. This suggests importance of increasing the MI in the outer code even if input MI is greatly low.

**IV. PROPOSED DESIGN CRITERION**

In the introduced SORM systems, the EXIT curve for the inner code lies between the upper and lower bounds (red curves in Fig. 5) when a priori MI fed back from the other user’s decoder is non zero. Thus, the EXIT trajectory climbs up from the location closer to the lower bound (that corresponds to a priori MI of 0 from the other user’s decoder) to that closer to the upper bound (that corresponds to a priori MI of 1 from the other
user’s decoder) with the increase of iteration. A simple way to guarantee the convergence of this turbo equalizer is to design the inner code’s EXIT curve with lower bound to move upward so as to make the stack less probable. However, it is not suitable, because its design strategy is to reserve SINR margin to cope with unknown IUI from the other user, thereby requiring quite a large amount of redundancy in terms of throughput.

Therefore, we have introduced the other way that may reduce such a redundancy; to design the inner code’s EXIT curve with upper bound to minimize the redundancy corresponding to the channel. This way can guarantee the convergence at the ending region, where input MI ($I_{eq\_in1}$) is high, since the upper bound closely coincides with the practical inner code’s property. Of course, it cannot guarantee the stack near the starting point in the iterative process. Thus, we will propose a criterion which enhances the EXIT property at the starting and ending regions separately. At the starting region, the area between the inner and outer code’s EXIT curves is enlarged so that a sufficient width of the EXIT tunnel can be obtained. At the ending region, the inner code’s EXIT curve asymptotically approaches the outer code’s one.

Fig. 6 illustrates the output MI versus the VND degree in the case of $D = 1$. The red line plots the output MI for the $I_{eq\_in1} = 0.1$. According to this figure, the output MI increases with the increase of a VND degree at the starting region, which means that we can heighten the output MI at the starting region even if the a priori MI fed back from the other user is fairly low. Thus, we have applied a high number of degree to a part of VND in the proposed scheme. In such a case, the slope of the inner code’s EXIT curve becomes quite sharp at the starting region, and thereby the stack near the starting point is less probable to occur.

The EXIT chart for the irregular LDPC code based on our proposed criterion is presented in Fig. 7. In this case, $w_v,3$ is set to be more than 40 because a wider tunnel between the EXIT curves of the inner code with the upper bound and the outer code can be obtained with increase of the degree, especially at lower MI feedback from the other user’s decoder. In this figure, the blue line provides the EXIT curve of the outer code (CND) having check node degree $w_c = 12$. The red lines show the upper bound and lower bound of the inner codes’ EXIT curves, where $E_b/N_0$ is set to 5 dB. The black line shows the EXIT trajectory. When the overlap rate for the SORM is defined as the fraction of mutually overlapped region in the spectrum, its average value is set to 50%. The other simulation parameters are given by TABLE I, and the degree distribution of the outer code is shown in TABLE II. In the Fig. 7, although the lower bound property lies below the CND property, the EXIT trajectory of the code based on the proposed criterion converges successfully, because MI increment at the starting region is very sharp so that the actual trajectory always is located higher than the CND property. Consequently, the code based on proposed criterion has better property than the code in Fig. 5.

![EXIT Chart](image)

**A. Performance of proposed code**

Fig. 8 demonstrates the PER performance as a function of the average transmitted $E_b/N_0$. The solid and dashed lines show the performances for the introduced SORM in the two-user case and single user case, respectively. Each construction of the irregular LDPC codes is described in TABLE II and III. The coding rate of 3/4 is employed. For comparison, the performance for the case of turbo code adopted in 3GPP is also shown with red lines.

As can be seen in this figure, the single user case in which the designed irregular LDPC code based on the area minimization criterion in the AWGN channel shows almost the same performance with that for the turbo code. This implies that this criterion is a suitable method for AWGN channel. The irregular LDPC code designed with the proposed criterion achieves 2 dB better performances in the case of SORM, compared to the irregular LDPC code based on area minimization criterion in the AWGN channel and the turbo code. On the other hand, the designed code based on the proposed criterion has worse performance than other codes in the single user case. These results imply that the proposed criterion leads to degrading the error correcting ability at the ending region, and conversely, the error correcting ability is heightened at the starting region. Consequently, the code based on our proposed criterion obtains good performance for the SORM.
B. Comparison with Turbo Code

Fig. 8 and 9 show the FER performance, where the coding rates are 3/4 and 1/2, respectively. For $r = 1/2$, the construction of irregular LDPC code is described in TABLE IV. Other simulation parameters are shown in TABLE I. For comparison, we also show the performance of turbo code in this figure.

As shown in Fig. 9, the proposed code has better FER performance than turbo code. The improvement attained by the proposed code is 0.3 dB at FER = 10^{-2}. This is because the decoder is hardly affected by the IUI at the starting region, due to its error correcting ability being sufficiently in a low coding rate. In Fig. 8, the performance of the proposed code is also 2.5 dB better than that of the turbo code. In the high coding rate, this means that the error correcting ability is especially important at the starting region.

VI. Conclusion

This paper proposed a new design criterion for EXIT analysis in the SORM. The irregular LDPC code designed by our proposed criterion improves the convergence property. We showed that the constituted code based on our proposed criterion is valid by computer simulation results in the SORM.

![Fig. 8. FER for LDPC code based on proposed criterion ($r = 3/4$).](image)

![Fig. 9. FER for LDPC code and turbo code in the SORM ($r = 1/2$).](image)

### TABLE I
Simulation parameters

<table>
<thead>
<tr>
<th>Num. of FFT points</th>
<th>$N_{fft} = 3048$ num. of subcarriers in the system band</th>
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</thead>
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<tr>
<td>Num. of clusters</td>
<td>$N = 1024$, cluster size is 1 subcarrier</td>
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<tr>
<td>Num. of coded bits</td>
<td>16384 information bits per frame</td>
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<td>Modulation</td>
<td>QPSK</td>
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<tr>
<td>FEC</td>
<td>Irregular LDPC code ($r = 1/2$, 3/4)</td>
</tr>
<tr>
<td>Turbo code</td>
<td>($K = 4$, $r = 1/2$, 3/4)</td>
</tr>
<tr>
<td>Decoder</td>
<td>Irregular LDPC code : Sum-product</td>
</tr>
<tr>
<td>Other codes</td>
<td>Max log-MAP w/ Jacobian logarithm</td>
</tr>
<tr>
<td>Num. of iteration</td>
<td>Irregular LDPC code : Turbo Eq. and Dec. 100</td>
</tr>
<tr>
<td>Turbo code</td>
<td>Turbo Eq. and Dec. 8 (respectively)</td>
</tr>
<tr>
<td>Channel model</td>
<td>24-path Rayleigh with equal average powers</td>
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<tr>
<td>Antenna config.</td>
<td>Tx: 1, Rx: 1</td>
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<tr>
<td>Num. of users</td>
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<tr>
<td>Channel estimation</td>
<td>Ideal</td>
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<td>Ave. overlap rate</td>
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### TABLE II
Degree distribution ($r = 3/4$)

<table>
<thead>
<tr>
<th>$w_1 / a_1$</th>
<th>$w_2 / a_2$</th>
<th>$w_3 / a_3$</th>
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<tr>
<td>2 / 0.9570</td>
<td>4 / 0.0195</td>
<td>43 / 0.0234</td>
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### TABLE III
Degree distribution ($r = 3/4$)

Area minimization criterion for single user case

<table>
<thead>
<tr>
<th>$w_1 / a_1$</th>
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<th>$w_3 / a_3$</th>
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<tbody>
<tr>
<td>2 / 0.5102</td>
<td>4 / 0.4829</td>
<td>7 / 0.0068</td>
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### TABLE IV
Degree distribution ($r = 1/2$)

<table>
<thead>
<tr>
<th>$w_1 / a_1$</th>
<th>$w_2 / a_2$</th>
<th>$w_3 / a_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 / 0.6708</td>
<td>5 / 0.2993</td>
<td>39 / 0.0297</td>
</tr>
</tbody>
</table>

### REFERENCES