Equalization and Interference Cancellation in Linear Multiuser Systems Based on Second-Order Statistics

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Abstract—Potential applications of blind channel identification and equalization in data communication systems have recently been explored. For multiuser systems that are irreducible and column-reduced, second-order statistical methods normally can identify channel dynamics up to a unitary mixing matrix. Additional user separation (equalization) can rely on higher order statistics and other prior information. In this paper, we investigate the equalizability of user signals and the cancellation of unwanted interfering signals based only on second-order output statistics. We show that a user channel can be equalized if it has the longest memory. Furthermore, interfering user signals can be cancelled under a more relaxed multiuser channel condition.

Index Terms—Blind equalization, blind identification, interference cancellation, second-order statistics.

I. INTRODUCTION

BLIND equalization and identification of both single user and multiple users have been important research issues in communications and signal processing. For single-user systems, second-order statistical methods exploit the channel diversity in terms of the single input multiple output (SIMO) framework to identify and equalize the unknown channel without the use of training signals [1]–[4]. In many applications, however, multiple users are present, and the blind equalization and identification of multi-input multi-output (MIMO) systems must be dealt with.

Although the problem of blind MIMO channel identification has been studied rather extensively [6]–[11], in general, effort has been focused on identifying and equalizing unknown channel dynamics involved in the linear MIMO system. Practical considerations, however, may allow us to consider a less ambitious requirement. In practice, a receiver often faces some desired signals and unwanted co-channel interference at the same time. Rather than identifying and equalizing MIMO linear channels for all users, including the interferers, the receiver should be content if only the desired user channels are identified (for the design of minimum mean square error filters) or directly equalized while cancelling all interference sources. In this paper, we specifically show that the conditions that permit such less demanding equalization are milder than a full-rank assumption traditionally imposed on an associated channel matrix.

In addition, the use of only second-order statistics in the blind identification of MIMO system typically leaves an unresolved ambiguity in that the equalized signals are instantaneous mixtures of multiple input signals that include both the desired and interfering signals. The extraction of a given user signal often needs to rely on additional signal information such as higher order statistics or signal constellation features. The utilization of second-order statistics for blind extraction of one or more unknown signals has not been addressed very often and is thus the focus of this work. In this regard, we show that a single target user’s signal can be extracted from other interfering signals by blind means, using second-order output statistics of the channel, if the target signal’s channel has longer memory than that of the channels seen by the interferers.

In Section II, we present the basic description of the MIMO signal equalization problem. We provide a new equalizability condition in Section III. We then describe a procedure to identify and equalize individual channels with the longest memory in Section IV. Finally, we present some computer simulation results in Section V as evidence of feasibility for this procedure.

II. DESCRIPTION OF LINEAR MULTIUSER SYSTEM MODELS

Consider a multiuser system at baseband. All users transmit data signals at identical baud rate. From oversampling and multiple receiver antennae, multiple signal samples can be extracted during each baud period. Assuming that we have \( p \) received signal samples during each baud, a \( q \) user linear system can be fully described by a \( p \times q \) impulse response matrix \( \{H_k\}_0^L \). The relationship between the system input and output is

\[
\bar{y}_n = \sum_{k=0}^{L} H_k \bar{x}_{n-k} + \bar{u}_n \tag{1}
\]

where

- \( \bar{y}_n \) \( p \times 1 \) received signal vector;
- \( \bar{x}_n \) \( q \times 1 \) input signal vector;
- \( \bar{u}_n \) \( p \times 1 \) channel noise.

The goal of the receiver is to recover \( \bar{x}_n \) from the sequence of received signal vectors \( \{\bar{y}_n\} \). In order to utilize second-order statistics, some channel diversity in the form of multiple antenna and/or excess bandwidth must be present. In other words, it is necessary that \( p > q \). It is under this condition for diversity that we present our framework.

Consider a system in which \( q_1 \) of the channel inputs, comprised of \( \bar{x}_n^{(1)} \), are desired signals, whereas the remaining \( q_2 = p - q_1 \) are interference signals.
signals, comprised of $s_k^{(2)}$, are regarded as interferences. Without loss of generality, denote

$$\mathbf{z}_n^{(2)} \triangleq \begin{bmatrix} s_n^{(1)} \\ s_n^{(2)} \end{bmatrix}$$

and

$$\mathbf{H}_k \triangleq \begin{bmatrix} \mathbf{H}_k^{(1)} \\ \mathbf{H}_k^{(2)} \end{bmatrix}.$$  

The received signal vector can then be written as

$$\mathbf{z}_n = \sum_{k=1}^{L_1} \mathbf{H}_k^{(1)} s_{n-k}^{(1)} + \sum_{k=0}^{L_2} \mathbf{H}_k^{(2)} s_{n-k}^{(2)} + \mathbf{w}_n.$$  

Now, for $i = 1, 2$, form the $M \times (M + L_i)q_k$ block Toeplitz matrices

$$\mathcal{H}_i = \begin{bmatrix} \mathbf{H}_0^{(i)} & \mathbf{H}_1^{(i)} & \cdots & \mathbf{H}_{L_i}^{(i)} \\ 0 & \mathbf{H}_0^{(i)} & \cdots & \mathbf{H}_{L_i-1}^{(i)} \\ \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & 0 & \mathbf{H}_0^{(i)} \end{bmatrix}.$$ 

By introducing the following notations:

$$s_k[n] \triangleq \begin{bmatrix} s_n^{(i)} \\ s_{n-L_i-M+1}^{(i)} \\ \vdots \\ s_{n-M+1}^{(i)} \end{bmatrix}, \quad ((M + L_i)q_k \times 1)$$

for $i = 1, 2$, and

$$\mathbf{w}[n] \triangleq \begin{bmatrix} \mathbf{w}_n \\ \vdots \\ \mathbf{w}_{n-M+1} \end{bmatrix}, \quad (M \times 1)$$

a sampled channel output signal vector of length $M$ can be written as

$$\mathbf{x}[n] = \begin{bmatrix} \mathbf{x}_n \\ \mathbf{x}_{n-1} \\ \vdots \\ \mathbf{x}_{n-M+1} \end{bmatrix} = \mathcal{H} \mathbf{s}[n] + \mathbf{w}[n]$$

where $\mathcal{H} = \mathcal{H}_1 \mathcal{H}_2$ and $\mathbf{s}[n] = [s_1[n]^H \ s_2[n]^H]^H$.

Based only on channel output $\mathbf{x}[n]$, blind identification of such a MIMO dynamic system has been studied in [6]–[12] and various algorithms have been presented. Given the $p \times q$ MIMO transfer function

$$\mathbf{H}(z) = \begin{bmatrix} \sum_{i=0}^{L_1} \mathbf{H}_0^{(1)} z^{-i} \\ \sum_{i=0}^{L_2} \mathbf{H}_0^{(2)} z^{-i} \end{bmatrix} = [\mathbf{H}_1(z) \ \mathbf{H}_2(z)]$$

it has been established in [7] that a sufficient condition for $\mathcal{H}$ to be identifiable from second-order statistics is that $\mathcal{H}$ have full column rank or, equivalently, that

a) $\mathbf{H}(z)$ is irreducible;
b) $\mathbf{H}(z)$ be column-reduced [5], [7], [10].

Note that for single input multiple output (SIMO) systems, condition b) is implied by condition a). Thus, for $q = 1$, $\mathbf{H}(z)$ only needs to be irreducible. In this paper, we study the condition under which the desired user(s) can be blindly identified and equalized without necessarily satisfying the conditions above. An important difference between our work and previous studies reported in the literature is that if only one user is desired, then it must be identified/equalized without any matrix ambiguity.

Our study has its physical significance. Many practical receivers must deal with desired signals and co-channel interference at the same time. Thus, the objective of the receiver is not to identify and equalize all MIMO channels belonging to the desired users and all the interferers. Rather, only the desired user channels need to be identified so that the design of minimum mean square error (MMSE) filters can be carried out. Equivalently, it would be desirable to simply cancel all co-channel interference and equalize only the desired signals. Thus, a blind algorithm is not reliable if system identifiability can be altered due to the presence of an additional interferer. Our study on the identifiability and equalizability of desired users would, therefore, help understand the condition under which such an objective is achievable and will possibly lead to more robust blind equalization and identification algorithms. In particular, our work focuses only on the use of second-order statistics of channel output signals.

### III. Equalizability Condition for Target Users

Among $q$ active users, there are $q_1$ desired users and $q_2$ interfering signals. Thus, we write

$$\mathcal{H} = \begin{bmatrix} \mathcal{H}_1 \\ \mathcal{H}_2 \end{bmatrix} \text{ with } d_1=(M+L_1)p_1 \text{ and } d_2=(M+L_2)p_2.$$ 

Our goal is to recover the $q_1$ desired users while rejecting the $q_2$ interfering sources. It should be noted that such an objective may not be realizable for all MIMO systems. Thus, the first issue is to determine the condition under which such separability and equalizability is possible. The sufficient and necessary equalizability condition for the desired users is first presented here as a theorem.

**Theorem 1:** Let the columns of matrix $\mathcal{H}_1$ be reordered such that

$$\mathcal{H} = \begin{bmatrix} \mathcal{H}_1 \\ \mathcal{H}_2 \end{bmatrix}$$

in which $\mathcal{H}_e$ has full column rank $m$. There exists an $m \times M$ matrix $\mathcal{G}$ such that

$$\mathcal{G} = \begin{bmatrix} \mathcal{H}_1 \\ \mathcal{H}_2 \end{bmatrix}$$

if and only if rank($\mathcal{H}$) = rank($\mathcal{H}_1$) + rank($\mathcal{H}_2$).

**Proof:** Suppose such a $\mathcal{G}$ exists. Then, (8) shows that the rows of $[\mathcal{G} \mathcal{H}_1 \mathcal{H}_2]$ are linear combinations of the rows of $\mathcal{H}$. Hence

$$\text{rank}(\mathcal{H}) = \text{rank} \left( \begin{bmatrix} \mathcal{H}_1 \\ \mathcal{H}_2 \end{bmatrix} \right).$$

In addition, no column of $[\mathcal{H}_1 \mathcal{H}_2]$ is a linear combination of those of

$$\begin{bmatrix} \mathcal{H}_1 \\ \mathcal{H}_2 \end{bmatrix}.$$
Therefore
\[
\text{rank} \left( \begin{bmatrix} H_e & H_1 \\ I_{m \times m} & 0_{m \times (d_1 - m)} \\ 0_{m \times d_2} & 0_{m \times d_2} \end{bmatrix} \right)
= \text{rank} \left( \begin{bmatrix} H_e \\ I_{m \times m} \end{bmatrix} \right) + \text{rank} \left( \begin{bmatrix} H_1 \\ 0_{m \times (d_1 - m)} \\ 0_{m \times d_2} \end{bmatrix} \right)
= m + \text{rank} \left( \begin{bmatrix} H_1 \\ H_2 \end{bmatrix} \right).
\]
\[
(10)
\]
From (9) and (10), one obtains
\[
\text{rank}(H) = m + \text{rank} \left( \begin{bmatrix} H_1 \\ H_2 \end{bmatrix} \right).
\]
\[
(11)
\]
Now, assume that (11) holds. Let \( r_1, r_2 \) be the ranks of \( H_e \) and \([H_1, H_2]\), respectively, and consider singular value decompositions (SVDs) of these matrices:
\[
H_e = U_1 \Sigma_1 V_1^H, \quad [H_1, H_2] = U_2 \Sigma_2 V_2^H
\]
\[
(12)
\]
where for \( i = 1, 2 \), \( U_i \) has size \( M_p \times r_i \), and \( \Sigma_i \) is a \( r_i \times r_i \) full-rank diagonal matrix of singular values. Note that \( r_1 = m \) since \( H_e \) is assumed full column rank. The matrices \( V_1, V_2 \) have sizes \( r_1 \times r_1 \) and \( (d_1 + d_2 - r_1) \times r_2 \), respectively. We can rewrite
\[
H = U \begin{bmatrix} \Sigma_1 & 0 \\ 0 & \Sigma_2 \end{bmatrix} \begin{bmatrix} V_1^H & 0 \\ 0 & V_2^H \end{bmatrix}
\]
\[
(13)
\]
in which the \( M_p \times (r_1 + r_2) \) matrix \( U = [U_1 \quad U_2] \) has full column rank according to our condition \( \text{rank}(H) = r_1 + r_2 \). Thus, \( U \) has a unique pseudo-inverse matrix \( U^\# \) such that
\[
U^\# U = I_{(r_1 + r_2) \times (r_1 + r_2)}.
\]
As a result
\[
[V_1 \Sigma_1^{-1} \quad 0_{r_1 \times r_2}] U^\# H = [V_1 \Sigma_1^{-1} \quad 0_{r_1 \times (d_1 + d_2 - r_1)}] \quad \text{and} \quad [I_{r_1 \times r_1} \quad 0_{r_1 \times (d_1 + d_2 - r_1)}].
\]

The matrix \( G \) is thus \( G = [V_1 \Sigma_1^{-1} \quad 0_{r_1 \times r_2}] U^\# \).

Theorem 1 establishes the matrix condition under which a desired (group of) signal(s) may be fully equalized with single or multiple delays while annihilating all interference.

For simplicity, we will now consider a special case that allows blind estimation of equalizer parameters. Specifically, we consider full-rank \( H_1 \).

Theorem 2: Suppose

A1) \( H_1(z) \) is irreducible and column-reduced.

Then, for all \( \delta = 0, 1, \ldots, d_1 - 1 \), there exists a vector (equalizer) \( \hat{g}_\delta \) such that
\[
g_\delta^H H = c_{\delta+1}^H = \begin{bmatrix} 0 & \cdots & 0 & 1 \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \cdots & 0 & 1 \end{bmatrix}
\]
if and only if \( \text{rank}(H_1) + \text{rank}(H_2) \).

Proof: Assumption A1 implies that \( H_1 \) is full column rank. Thus, by Theorem 1, A1 and A2 are equivalent to the existence of a matrix \( G \) satisfying (8). The corresponding interference canceling equalizer vector is
\[
\hat{g}_\delta^H = c_{\delta+1}^H G, \quad \delta = 0, 1, \ldots, d_1 - 1
\]
which has associated delay \( \delta \).

Observe that the traditionally assumed full rank \( H \) is a special case of conditions A1 and A2), which, while requiring user channels \( H_1(z) \) to be irreducible and column-reduced, no longer place such requirements on \( H(z) \). These equalizability conditions do not describe how the desired equalizer can be found blindly from the output signal statistics. That depends on the input signal characteristics and what output information is exploited. The next theorem helps determine how such an equalizer may be extracted blindly from second-order statistics of the channel output.

Theorem 3: Consider the partition (7). Under A1 and A2 in Theorem 2
\[
H^H (\hat{H} H^H)^\# H = \begin{bmatrix} I_{d_1 \times d_1} & 0 \\ 0 & B \end{bmatrix}
\]
\[
(14)
\]
for some matrix \( B \), where \( \{ \}^\# \) denotes the pseudo-inverse.

Proof: Consider the SVDs (12), where now, \( H_e = H_1 \) and \( H_1 \) is absent. Then, the matrix \( H \) can be written as in (13), with \( U = [U_1 \quad U_2] \) having full column rank. \( U \) has a unique pseudo-inverse matrix \( U^\# \) such that \( U^\# U = I \). Observe that for any matrix \( H \), it holds that \( H^H (\hat{H} H^H)^\# H = \hat{H} \).

IV. EQUALIZATION AND IDENTIFICATION PROCEDURE

A. Basic Definitions

We define \( J \) as a matrix whose first subdiagonal entries below the main diagonal are unity while all remaining entries are zero. It can be seen that \( J A \) shifts all elements in matrix \( A \) down by one row, whereas \( A J \) shifts all elements in \( A \) to the right by one column. Thus, \( J A J \) shifts \( A \) downward by one row and to the right by one column.

Denote the FIR equalizer parameter as \( \hat{g} \) with output \( \hat{y}_n = \hat{g}^H x[n] \). The minimum MSE filter to estimate \( y_\delta(n-\delta) \) is the solution to the Wiener–Hopf equation
\[
E \{ x[n] x[n]^H \} \hat{g}_\delta = E \{ x[n] (y_\delta(n-\delta))^H \}.
\]
\[
(15)
\]
\[ E\{x[n]x[n]^H\} \] is the autocorrelation matrix of the channel output and can be estimated from the observed data. Thus, to obtain the MMSE filter, \( E\{x[n](x[n-\delta])^H\} \) must be found.

Assume that the channel input signals (i.e., the entries of \( x[n] \)) are white and mutually uncorrelated with zero mean and unit variance. In addition, assume that the channel noise \( \eta[n] \) is white with zero mean and variance \( \sigma^2_{\eta} \) and independent of \( x[n] \). As a result

\[ E\{x[n](x[n-\delta])^H\} = \mathbf{H}^{(1)}(\delta) \] (16)

where \( \mathbf{H}^{(1)}(\delta) \) denotes the \((\delta+1)\)st block column of \( \mathbf{H} \) defined in (5), i.e.,

\[ \mathbf{H}^{(1)}(\delta) = \begin{bmatrix} 0_{\delta \times q_1} & \mathbf{I}_{q_1 \times q_1} \\ \mathbf{I}_{(q_1,+,1) \times q_1} \end{bmatrix}. \] (17)

Identifying \( \mathbf{H}^{(1)}(\delta) \) allows the computation of the MMSE filter parameter \( \mathbf{g}_0 \) specifically by

\[ \mathbf{R}_M(0) = \mathbf{H}^{H}(\delta) \mathbf{H}(\delta) \] (18)

Thus, from Theorem 3 and (17)

\[ \mathbf{g}_0^Hx[n] = \mathbf{H}^{(1)}(\delta)^H\mathbf{R}_M(0)^{\#}\mathbf{H}(\delta)x[n] = \mathbf{T}^H\mathbf{s}_{n-\delta} \] (19)

i.e., the signals of the desired users are resolved to within this ambiguity. This contrasts with the traditional results that resolve the signals of all the users including the interferers to within a memoryless ambiguity matrix. Note, in particular, that if \( q_1 = 1 \), i.e., there is only one desired user, then (19) yields the delayed signal of this user to within a scaling constant.

We now show how this vector \( \mathbf{g}_0 \) can be estimated blindly without training data.

### B. Estimation of Cross-Correlation Vector

For stationary white noise and uncorrelated input signals, the autocorrelation matrices of \( \mathbf{s}[n] \) and \( \mathbf{w}[n] \) are

\[ \mathbf{R}_s(\delta) = E\{x[n+\delta]x[n]^H\} = \begin{bmatrix} \mathbf{J}^{\delta}_{q_1} & 0 \\ 0 & \mathbf{J}^{\delta}_{q_2} \end{bmatrix} \] (20)

\[ \mathbf{R}_w(\delta) = E\{w[n+\delta]w[n]^H\} = \sigma^2_{\eta}\mathbf{I}^p. \] (21)

We exploit output second order statistics contained in

\[ \mathbf{R}_M(\delta) = \mathbf{H}\mathbf{H}^H + \sigma^2_{\eta}\mathbf{I}^p. \] (22)

In particular

\[ \mathbf{R}_M(0) = \mathbf{H}\mathbf{H}^H + \sigma^2_{\eta}\mathbf{I} \] (23)

First, we define

\[ \mathbf{I}_{\delta q_1} \overset{\Delta}{=} \begin{bmatrix} 0_{\delta \times q_1} & 0 \\ 0 & \mathbf{I}_{(M+q_1) \times (M+q_1)} \end{bmatrix} \]

and

\[ \Delta\mathbf{I}_{\delta q_1} \overset{\Delta}{=} \begin{bmatrix} 0_{\delta \times q_1} & \mathbf{I}_{q_1 \times (M+q_1)} \\ \mathbf{I}_{(M+q_1) \times q_1} & 0_{(M+q_1) \times (M+q_1)} \end{bmatrix} \]

i.e., \( \mathbf{I}_{\delta q_1} \) is an identity matrix, except for its first \( \delta q_1 \) zero diagonal entries, and \( \Delta\mathbf{I}_{\delta q_1} \) is all zero except for unit entries on its \((\delta+1)\)th diagonal element. It can be shown that

\[ \mathbf{I}_{\delta q_1} = \mathbf{J}^{\delta q_1}(\mathbf{J}^{H})^{\delta q_1}, \quad \Delta\mathbf{I}_{\delta q_1} = \mathbf{I}_{\delta q_1} - \mathbf{I}_{(\delta+1)q_1} \]

For simplicity, we first assume that the channel noise is absent so that

\[ \mathbf{R}_M(\delta) = \mathbf{H}\mathbf{R}_s(\delta)^{\#}\mathbf{H}^{H} \quad \text{and} \quad \mathbf{R}_M(0) = \mathbf{H}\mathbf{H}^{H}. \]

For white noisy cases, the noise power \( \sigma^2_{\eta} \) must be estimated and subtracted from the covariance matrices.

The critical identity that enables us to estimate the blind MMSE equalizer follows Theorem 3 and (14). Recall that \( \mathbf{J}^{(H)} \) will shift the elements in \( \mathbf{A} \) downward by \( i \) rows and to the right by \( j \) columns. This equality leads to the following result.

**Theorem 4:** For channels satisfying conditions \( A1 \) and \( A2 \), assume further that

- \( A3 \) \( L_1 > L_2 \) (desired user channels are longer than interference channels).
- \( A4 \) All inputs are i.i.d.

Then, an MMSE equalizer vector \( \mathbf{g}_0 \) can be extracted blindly from channel output second-order statistics.

**Proof:** From the expression of \( \mathbf{R}_M(\delta) \) and (14)

\[ \begin{align*}
\mathbf{D}_s & \overset{\Delta}{=} \mathbf{R}_M(\delta)\mathbf{R}_M(0)^{\#}\mathbf{R}_M(\delta)^{H} \\
& = \mathbf{H}\mathbf{R}_s(\delta)^{H}(\mathbf{H}\mathbf{H}^{H})^{\#}\mathbf{R}_s(\delta)^{H}\mathbf{H}^{H} \\
& = \mathbf{H}\begin{bmatrix} \mathbf{J}^{\delta q_1} & 0 \\ 0 & \mathbf{J}^{\delta q_2} \end{bmatrix}\begin{bmatrix} \mathbf{I}_{(q_1+1) \times q_1} & \mathbf{B} \end{bmatrix}\begin{bmatrix} \mathbf{J}^{\delta q_1} & 0 \\ 0 & \mathbf{J}^{\delta q_2} \end{bmatrix}^{H} \\
& = \mathbf{H}\begin{bmatrix} \mathbf{J}^{\delta (q_1)}(\mathbf{J}^{H})^{\delta q_1} & 0 \\ 0 & \mathbf{J}^{\delta (q_2)}(\mathbf{J}^{H})^{\delta q_2} \end{bmatrix}^{H}. 
\end{align*} \]
If $L_1 > L_2$, then for $M + L_1 > \delta > M + L_2$

$$J^{\delta_2} B (J^H)^{\delta_2} = 0 \quad \text{and} \quad J^{\delta_n} (J^H)^{\delta_n} = I_{\delta_n}.$$ 

Thus, $D_{\delta} = H_1 I_{\delta_1} H_1^H$. Similarly, for $M + L_1 \geq \delta + 1 \geq M + L_2$

$$D_{\delta+1} \triangleq R_M (\delta + 1)^n R_M (\delta + 1)^n = H_1 I_{\delta+1} H_1^H.$$ 

Then, for $(M + L_1 - 1) \geq \delta \geq (M + L_2)$, we can form the Hermitian matrices

$$\Delta D_{\delta} \triangleq D_{\delta} - D_{\delta+1} = H_1 I_{\delta_1} H_1^H = H_1 \Delta I_{\delta_1} H_1^H = H_1^{(1)} (\delta) H_1^{(1)} (\delta)^H.$$ (24)

Since the matrix $\Delta D_{\delta}$ has rank $q_1$ (recall the full-rank assumption on $H_1$), an eigendecomposition will allow us to identify

$$\tilde{H} (\delta) = H_1^{(1)} (\delta) T$$ (25)

in which $T$ is a $q_1 \times q_1$ unitary matrix. Thus, the arguments surrounding (19) provide $\hat{g}_k$.

Thus, if $q_1 = 1$, i.e., a single user has the longest channel span, then direct equalization and interference cancellation are achieved by the above approach. The constructive proof of Theorem 4 yields the requisite equalizer.

Once the desired user signals have been equalized (up to the mixing matrix), the equalizer output in the noiseless case is $z_n = T^H s_n$ [see (19)]. This can be used as reference to identify the desired user channels up to the mixing matrix $T$ via cross-correlation:

$$\hat{H}_j^{(1)} = \mathbb{E} \{ x[n] z_{n-\delta+j}^* \} = H_j^{(1)} T.$$ (26)

Again, when $q_1 = 1$, $T$ is a scalar.

C. Remarks

The physics behind requiring $L_1$ to be the longest for ambiguity removal can be clearly demonstrated with a simple explanation. Recall that with full rank $H$, the unknown channel response can be identified up to a unitary matrix $T$ [11]. In other words, we have $\tilde{H}_i = H_i T$. However, if $L_1 > L_2$; then, $H_{L_1} = [H_{L_1}^{(1)} 0]$. Through parameter constraints, we can force the estimated channel response to satisfy the condition $\tilde{H}_{L_1} = [H_{L_1}^{(1)} 0]$. Thus

$$\begin{pmatrix} H_{L_1}^{(1)} & 0 \end{pmatrix} = \begin{pmatrix} H_{L_1}^{(1)} & 0 \end{pmatrix} T = \begin{pmatrix} T_{11} & T_{12} \end{pmatrix}^{H}.$$ (27)

This leads to $H_{L_1}^{(1)} T_{12} = 0$. Since $H_{L_1}^{(1)}$ is full rank according to A1) and A2), $T_{12} = 0$. The unitary nature of $T$ also implies that

$$T_{11} T_{11}^{H} = I \quad \text{and} \quad T_{11} T_{21}^{H} = 0.$$ (28)

It is therefore evident that $T_{21} = 0$ and $\hat{H}_j^{(1)} = H_j^{(1)} T_{11}$ is the only (unitary) ambiguity among users sharing the longest channel length. If $q_1 = 1$, then $H_j^{(1)}$ is a vector and is identified up to a constant.

Some related results can be found in the literature. In [13], blind equalization of nonlinear channels was investigated. By viewing the nonlinear terms as co-channel interference, the problem of Volterra multichannel blind equalization is equivalent to the problem considered here with $q_1 = 1$ (one desired user). In fact, it was shown in [13] that the desired signal can be equalized by a deterministic method under the following conditions.

a) Matrix $H$ has full column rank.

b) $H$ is square.

c) $L_1$ is the longest.

Here, our result requires more stringent i.i.d. inputs but fewer restrictive channel conditions

i) rank-deficient $H$ satisfying A1) and A2);

ii) nonsquare $H$;

For multiple users sharing $L_1$, i.e., $q_1 > 1$, they may be separated using additional information such as the input constellation or higher order statistics as in [13].

Certainly, signal separation can be attempted by utilizing higher order statistics (HOS). However, HOS does not provide any distinction among different user signals: Signal contents must be exploited to determine their destination. In this paper, we show that users with the longest channel memory can be separated using second-order statistics. Thus, subsequent signal separation of the remaining users becomes an easier problem.

In practical applications, we may apply prefiltering of the desired signal at the transmitter. As shown in Fig. 1, the desired signal can be prefiltered by a partial response filter $P (z)$ to increase the memory length artificially. Consequently, $s_1 [k]$ can be extracted apart from mixtures of other user signals by the proposed method.

V. SIMULATION RESULTS

We present the results obtained by the proposed algorithm in a number of numerical examples. The data input signals are i.i.d. BPSK, and the number of subchannels is $p = 3$. The number of signals present is $q = 3$, of which only one is of interest; the other two are regarded as interference. The noise is zero-mean white Gaussian.

In the actual implementation of the equalization algorithm, we did not use denoising, and noise contribution is maintained in

$$R_M (0) = \mathbb{H} H^H + \sigma_u^2 I.$$ (29)

There are two reasons against denoising. First, ill-conditioning of $\mathbb{H} H^H$ can cause serious numerical errors. Second, accurate
estimation of the noise variance $\sigma_n^2$ is not a simple process. As a result, we took the simpler approach without denoising by applying $R(0)$ wherever $\mathcal{H} \mathcal{H}^H$ is needed in the algorithm.

The signal subchannels are shown in Fig. 2; the interference subchannels, which are the same for both interfering sources are also shown. Because of this, the channel matrix $\mathcal{H}$ is necessarily rank deficient. Nevertheless, the conditions in Theorem 2 are satisfied so that linear FIR equalizers for the signal of interest can still be found.

The channel lengths for the signal and the interference are $L_1 = 9$ and $L_2 = 3$, respectively. We consider the use of an equalizer with $M = 16$ taps. Fig. 3 shows the theoretical minimum MSE together with the MSE obtained with the proposed algorithm as a function of the delay. Clearly, for delays under $M + L_2$, the performance is far from optimal, as expected, since the algorithm can only provide equalizers of delays between $M + L_2$ and $M + L_1 - 1$. Therefore, we consider an equalization delay $\delta = M + L_2 = 19$ in the following experiments in which the sign ambiguity inherent to the algorithm has been removed (in a practical system, this ambiguity can be eliminated by using differential encoding).

1 In this example, the channel and the constellation are real, and therefore, the simulations were run with real arithmetic. Nevertheless, the algorithm is still valid in the complex case.

Fig. 2. Signal and interference subchannels

Fig. 3. Minimum MSE and MSE obtained by the algorithm as a function of delay. SNR $= 15$ dB, SIR $= 10$ dB, 2500 data samples, averaged over 100 runs.

Fig. 4. Bit error rate at the equalizer output versus input SNR. Equalization delay $\delta = 19$, $K = 2500$ data samples, averaged over 100 runs.

Fig. 4 shows the bit error rate as a function of the SNR for three different values of the signal-to-interference ratio (SIR). The number of data samples used for the estimation of the autocorrelation matrices is $K = 2500$. The BER is averaged over 100 independent runs. The SNR and SIR are defined as follows:

$$\text{SNR} = 10 \log \frac{\text{trace } E[\langle \mathcal{H}_1 s_1[k] \rangle (\mathcal{H}_1 s_1[k])^H]}{\text{trace } E[w[k]w[k]^H]}$$

$$\text{SIR} = 10 \log \frac{\text{trace } E[\langle \mathcal{H}_1 s_1[k] \rangle (\mathcal{H}_1 s_1[k])^H]}{\text{trace } E[\langle \mathcal{H}_2 s_2[k] \rangle (\mathcal{H}_2 s_2[k])^H]}$$

Fig. 5 shows the BER as a function of the SIR for three different values of the SNR. The number of data samples used for the estimation of the autocorrelation matrices is $K = 2500$. The BER is averaged over 100 independent runs of $10^5$ bits each. It is observed that for high SNR, the algorithm performance is good for a wide range of values of interference power.

In the next experiment, the SNR is fixed at 15 dB, and the BER is obtained as a function of $K$, which is the number of data samples. The results are shown in Fig. 6 for three different values of SIR. Performance improves as the number of samples is increased, as expected.
Finally, the normalized root-mean-square error (NRMSE) of the estimate of the desired user channel is presented in Fig. 7 as a function of SNR for three values of SIR, using $K = 2500$ data samples. The NRMSE is defined as

$$\text{NRMSE} = \frac{1}{\|\hat{h}\|} \sqrt{\frac{1}{R} \sum_{r=1}^{R} \| \hat{h}(\bar{x}) - h \|^2}$$

where $h$ is the vector of channel coefficients, and $R$ is the number of Monte Carlo trials (100 for our experiment).

VI. CONCLUSIONS

In this paper, a necessary and sufficient equalizability condition for a subset of desired users in an MIMO system is presented. Under a more relaxed multiuser channel condition, desired user(s) can be equalized, and interfering co-channel signals can be cancelled. Given second-order output statistics with channel diversity, a blind equalization and identification algorithm is provided for users whose channels have the longest memory span. Our result clarifies the blind equalizability and interference cancellability condition for multiuser systems in which only some users are of interest.

REFERENCES


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