Control of Suspended Wheeled Mobile Robots with Multiple Arms during Object Manipulation Tasks

Mahdy Eslamy¹  S. Ali A. Moosavian²

Advanced Robotics & Automated Systems (ARAS) Laboratory
Department of Mechanical Engineering, K. N. Toosi University of Technology
Tehran, Iran, P.O. Box 19395-1999, Fax: (+98) 21 8867-4748

Abstract—In this paper, a suspended mobile platform with two 3-DOF manipulators, is used to manipulate an object in a combined circular-straight path. The Multiple Impedance control (MIC) is used as a Model-Based algorithm which has been shown to have desirable performance in case of impact with an obstacle and system flexibility. The MIC enforces same impedance law both on the robotic system, and the manipulated object level. However, to apply model-based control laws, it is needed to extract system dynamics model. For such complicated robotic systems with suspension system, it is necessary to have a concise set of dynamic equations of motion with few mathematical calculations. Therefore, the concept of Direct Path Method is extended to derive explicit dynamics modeling for such challenging systems. Then, non-holonomic constraint of the wheeled system is derived, and the obtained dynamics model is reformatted to become a more concise one using Natural Orthogonal Complement Method. Next, the MIC law is applied to cooperative manipulation of an object by two manipulators mounted on a suspended wheeled robotic system. The obtained results reveal a smooth system performance, i.e. negligible small tracking errors in the presence of impacts with obstacles and significant disturbances.

Keywords—Mobile robots, Suspension, Dynamics Model, Control, Cooperative Object Manipulation.

I. INTRODUCTION

Exploiting the mobility of mobile robotic systems, mobile manipulators can do tasks that may be out of reach of their manipulators as opposed to fixed base robots. Mobile robotic systems are of great interest for their wide area of applications such as spatial explorations, hazardous environments, nursing, storing and many others. Including a suspension system in such mobile platforms will certainly add to the system maneuverability and performance as well as its complexity and nonlinearity. However, we will have a system with more flexibility and better performance even in undesirable situations such as moving on uneven environments. On the other hand, wheeled robotic systems are usually subjected to non-holonomic constraints, [1-3], while free-flying robots can move freely in space, [4]. To apply model-based control laws, it is needed to extract explicit system dynamic model. A systematic method for the kinematics and dynamics modelling of a two degree-of-freedom (DOF) Automated Guided Vehicle (AGV) has been presented by Saha and Angeles, [5-6]. They have employed the notion of Natural Orthogonal Complement to eliminate the Lagrange multipliers. To reduce the huge amount of mathematical calculations for complex dynamical systems, Direct path method (DPM), [7], has been utilized for the kinematics of space free flyers which requires significantly less computations. Explicit dynamics of spatial mobile robotic systems was obtained based on DPM approach, [8], while the method will be extended in this paper to derive the equations of motion for a general suspended wheeled mobile robot where the terms induced by considering a suspension system are also included.

Various control algorithms have been used for motion control of a mobile platform, [9-10], but manipulating objects by multiple arms mounted on a moving base has remained almost untouched. The Multiple Impedance Control (MIC) is an algorithm that imposes a reference impedance to all elements of a mobile system, including its base, the manipulator end-points, and the manipulated object itself. This algorithm has been used for space free-flyers and is shown to give more desirable manipulation results even in the presence of impacts due to contact with obstacles, external disturbances and flexibility in the system, versus other criteria such as object impedance control, [11-12].

The main focus of this paper is on performing object manipulation tasks by multiple manipulators of a suspended mobile robotic system traveling in a mixed circular-straight path. To do this, first, using Lagrange method the equations of motions are derived based on direct path method (DPM). Next, the system non-holonomic constraint will be derived, and using Natural Orthogonal Complement Method the independent set of equations of motion for the system is derived. Finally, the MIC law is applied to cooperative manipulation of an object by two manipulators mounted on the suspended wheeled platform. The obtained results reveal the success of the suspended mobile manipulator in its defined mission with negligible small tracking errors even in the presence of impacts due to contact with obstacles and significant disturbances.

¹- Graduate student, m.eslamy@gmail.com
²- Associate Professor, moosavian@kntu.ac.ir

978-1-4244-2789-5/09/$25.00 ©2009 IEEE 3730
II. SYSTEM DYNAMICS

A. Basic Definitions and Calculations:

To derive a concise set of dynamic equations with less computations, the concept of Direct path method (DPM), [7], is used and extended for such highly nonlinear system as depicted in Fig. 1. Using Lagrange approach, one can write:

\[
\frac{d}{dt} \left( \frac{\partial T}{\partial \dot{q}_i} \right) - \frac{\partial T}{\partial q_i} + \frac{\partial U}{\partial q_i} + \frac{\partial U_F}{\partial q_i} = Q_i \quad i = 1, \ldots, N
\]  

(1)

where \( T \) is the total system kinetic energy, \( U \) describes the total system potential energy, \( U_F \) is the total dissipated energy of the system due to damping equipments, \( N \) describes the system degrees-of-freedom (DOF), \( q_i \), \( \dot{q}_i \)

and \( Q_i \) are the \( i-th \) element of the vector of generalized coordinates, generalized speeds, and generalized forces, respectively, as defined below:

\[
q = [\theta^T, \dot{\theta}^T, \ddot{\theta}^T, \ldots, \theta^{T_h}]^T
\]

where \( \theta \) and \( \dot{\theta} \) describe position vector of the base center of mass (CM) and the base Euler angles and \( \theta_1, \theta_2, \ldots, \theta_m \) are the vectors of joint angles for the first, second and \( m-th \) manipulator respectively, as described below:

\[
\theta_1 = (x_G, y_G, z_G)^T \quad (3a)
\]

\[
\theta_2 = (\phi, \theta, \psi)^T \quad (3b)
\]

\[
\theta_m = (\theta_{1(m)}, \ldots, \theta_{k(m)})^T \quad (3c)
\]

Where \( x_G, y_G, z_G \) are the base CM position with respect to the inertial XYZ axes coordinates and \( \phi, \theta, \psi \) are the base yaw, roll and pitch angles respectively. Finally \( \theta_{k(m)} \)

denotes the \( k-th \) joint variable of the \( m-th \) manipulator. The terms of the total system kinetic energy \( T \) are explicitly detailed in [8] for an unconstrained space free flying robotic system.

The system total potential energy, \( U \), contains both gravitational potential energy and the potential energy due to springs tensions or compressions:

\[
U = \frac{1}{2} (K_1 \ddot{\delta}_1 + K_2 \ddot{\delta}_2 + \ldots + K_m \ddot{\delta}_m + \ddot{\theta} + K_4 \delta_4)
\]

\[+ m_b \ddot{R}_b + \sum_{k=1}^{N} \left( \sum_{i=1}^{N} \left( m_k^{(m)} \ddot{r}_{k(i)} + \ddot{R}_k + \ddot{r}_{k(i)}^{(m)} \right) \right) \]

(4)

where \( m_b \) is the base mass, \( g \) is the gravity acceleration vector, \( \ddot{r}_{k(i)}^{(m)} \) and \( m_k^{(m)} \) are the position vector of CM for the \( k-th \) link of the \( m-th \) manipulator with respect to

the base CM and the mass of the \( k-th \) link of the \( m-th \) manipulator respectively, \( K_1, \ldots, K_4, C_1, \ldots, C_4 \) are the stiffness coefficient for the springs according to the base four corners, i.e. the front left, rear left, rear right and front right corners respectively, and \( \delta_1, \ldots, \delta_4 \) are the corresponding displacements for each spring.

Next the system total dissipated energy, \( U_F \), is obtained as:

\[
U_F = \frac{1}{2} \left( C_1 \ddot{\delta}_1 + C_2 \ddot{\delta}_2 + \ldots + C_m \ddot{\delta}_m + \ddot{\theta} - \ddot{\delta}_4 \right)
\]

(5)

where \( C_1, \ldots, C_4 \) and \( \ddot{\delta}_1, \ldots, \ddot{\delta}_4 \) are damping coefficients and time rate of corresponding displacements in each suspension equipment in the corners respectively.

B. Extracting Dynamics Equations:

Exploiting Lagrange equations, Eq. (1), and substituting the system kinetic, potential and dissipated energies, the dynamics model is obtained as:

\[
H(q) \ddot{q} + C(q, \dot{q}) + G(q) = Q
\]

(6)

where the mass matrix \( H \), non-linear velocity vector \( C \), and gravity vector \( G \), are obtained as below:

\[
H = M \ddot{R}_b + \sum_{k=1}^{N} \left( \sum_{i=1}^{N} \left( m_k^{(m)} \ddot{r}_{k(i)} + \ddot{R}_k + \ddot{r}_{k(i)}^{(m)} \right) \right)
\]

\[+ \sum_{k=1}^{N} \sum_{i=1}^{N} \left( m_k^{(m)} \frac{\partial \ddot{r}_{k(i)}^{(m)}}{\partial q_j} \frac{\partial q_j}{\partial q_i} + \frac{\partial \ddot{R}_k}{\partial q_j} \frac{\partial q_j}{\partial q_i} \right) \]

\[+ \sum_{k=1}^{N} \sum_{i=1}^{N} \left( m_k^{(m)} \frac{\partial \ddot{r}_{k(i)}^{(m)}}{\partial q_j} \frac{\partial q_j}{\partial q_i} + \frac{\partial \ddot{R}_k}{\partial q_j} \frac{\partial q_j}{\partial q_i} \right) \]

(7)

where \( M \) is the total mass of the robotic system, \( N \) is the total number of manipulators mounted on base and \( N_m \) denotes the number of the links of the \( m-th \) manipulator. Vector \( C \) can be written as:

\[
C = C_1 \dot{q} + C_2
\]

(8a)

where:

\[
\dot{q} = \dot{\theta} + \dot{\delta}_1 + \dot{\delta}_2 + \dot{\delta}_3 + \dot{\delta}_4
\]

\[
\dot{\theta} = \dot{\phi} + \dot{\theta} + \dot{\psi}
\]

\[
\ddot{\theta} = \ddot{\phi} + \ddot{\theta} + \ddot{\psi}
\]
where $V_o$ is the velocity of the midpoint of the rear axle and $\hat{f}_j$ is the unit vector along $y'$ as shown in Fig. 2. The velocity of the base CM, i.e. point $G$, can be written as:

$$\vec{V}_G = \vec{V}_O + \vec{\omega}_b \times \vec{L}_{G/O}$$

(11)

in which $O$ is the point on base above $O'$ and $\vec{L}_{G/O}$ is the vector from point $O$ to point $G$. Using Eq. (10) and (11) the system non-holonomic constraint will be obtained as:

$$\dot{x}_G \sin(\phi) - \dot{y}_G \cos(\phi) + l \dot{\phi} \cos(\theta_p) = 0$$

(12)

Considering Eq. (12) the DOF of the base at the speed level is reduced to five, as discussed in [8-9]. The angular velocities of the right and left wheels, i.e. $\dot{\theta}_r$ and $\dot{\theta}_l$, can be chosen as new components of general speeds vector of the base. Then, it can be written:

$$\dot{q} = S(q) \cdot \dot{\nu}$$

(13a)

where $S(q)$ is a Jacobian matrix which relates the new general speeds, i.e. $\dot{\nu}$, to the previous ones, i.e. $\dot{q}$. $V$ is the same as $q$ except for the first three elements in $q$ replaced by left/right rear wheels angles in $\nu$. For the base we have:

$$\begin{bmatrix} \dot{x}_G \\ \dot{y}_G \\ \dot{\nu} \end{bmatrix} = J_{ac12} \begin{bmatrix} \dot{\theta}_r \\ \dot{\phi} \end{bmatrix}$$

(13b)

where $J_{ac12}$ is a non-square Jacobian matrix which relates the new general speeds of the base to the prior ones as follows:

$$J_{ac12} = \begin{bmatrix} \frac{r}{2} \cos(\phi) + \frac{r}{b} \sin(\phi) \cos(\theta_p) \\ \frac{r}{2} \cos(\phi) - \frac{r}{b} \sin(\phi) \cos(\theta_p) \\ \frac{r}{2} \cos(\phi) + \frac{r}{b} \sin(\phi) \cos(\theta_p) \\ \frac{r}{2} \cos(\phi) - \frac{r}{b} \sin(\phi) \cos(\theta_p) \\ \frac{r}{2} \cos(\phi) + \frac{r}{b} \sin(\phi) \cos(\theta_p) \\ \frac{r}{2} \cos(\phi) - \frac{r}{b} \sin(\phi) \cos(\theta_p) \end{bmatrix}$$

(14)

In which $r$ is the wheels radius and $l_g$ and $b$ are depicted in Fig. 2. Next, in order to simplify the matrices forms to be used in programming, one can rearrange the generalized coordinates elements order, (i.e. put the $x_G, y_G, \Phi$ at first in $q$ ) so that the constraint equation (12) can be expressed as:

$$A(q) \cdot \dot{q} = 0$$

(15)

where $A(q)$ is

$$A(q) = [\sin(\phi) - \cos(\phi) \ l_\phi \cos(\theta_p)]_0 \cdot \nu_{|\nu=0}$$

(16)

Considering (16), and defining corresponding Lagrangian multiplier as $\lambda$, [13], Eq. (6) can be written as:

$$H(q) \dot{q} + C(q, \dot{q}) + G(q) + A(q)^T \cdot \lambda = Q$$

(17)

Then, using Natural Orthogonal Complement Method, [5-6], the equations of motion can be derived as a set of unconstrained equations which is detailed next. The relationship between constrained general forces and unconstrained ones, can be written as:

$$Q_{NS} = E(q) \tau_{(N\times(N-1))}$$

(18a)

in which $E(q)$ is an $N \times (N-1)$ matrix, and
\[ \tau = [t_1, t_2, 0, 0, 0, \ldots, t_1^{(m)}, t_2^{(m)}, \ldots, t_k^{(m)}]^T \]  

(18b)

where \( t_1 \) and \( t_2 \) and \( t_1^{(m)} \) are the torques to be exerted on the left and right rear wheels and the \( k-th \) link of the \( m-th \) manipulator, respectively. To obtain \( E(q) \), based on principle of virtual work, one can write

\[ Q^T \cdot dq = \tau^T \cdot d\nu \]  

(18c)

Substituting \( dq \) from Eq. (13a), it is obtained:

\[ \tau = S^T \cdot Q \]  

(18d)

Finally, substituting \( Q \) from Eq. (18a), we will have

\[ S^T \cdot H(q)\ddot{q} + C_1 \dot{q} + C_2 \dot{q} + G(q) - \tau \cdot E = 0 \]  

(20a)

Noting the fact that \( A(q) \) is in the null space of \( S(q) \), the second part will vanish, and (20a) reduces to:

\[ S^T \cdot H(q)\ddot{q} + S^T \cdot C_1 \dot{q} + S^T \cdot C_2 \dot{q} + S^T \cdot G(q) = \tau \]  

(20b)

Now, based on Eq. (13a) we have:

\[ \ddot{q} = \dot{S} \dot{q} + \ddot{S} \dot{q} \]  

(21a)

where

\[ \ddot{S}(q, \dot{q}) = \begin{bmatrix} \frac{\partial \text{Jac}_{22}}{\partial \dot{q}} & 0_{3x(N-3)} \\ 0_{(N-3)x2} & \frac{\partial \text{Jac}_{22}}{\partial \ddot{q}} \end{bmatrix} \]  

(21b)

Finally, based on (21a) and necessary substitutions in (20b), the equations of motion for the constrained system will reduce into an independent set in the following form:

\[ \ddot{H} \cdot \dot{q} + \ddot{C} \cdot \dot{q} + \ddot{C} \cdot \dot{q} + \ddot{G} = \tau \]  

(22a)

in which

\[ \ddot{H} = S^T \cdot H \cdot \dot{S} \]  

(22b)

\[ \ddot{C}_1 = S^T \cdot (C_1 \cdot \dot{S} + H \cdot \ddot{S}) \]  

(22c)

\[ \ddot{C}_2 = S^T \cdot C_2 \]  

(22d)

\[ \ddot{G} = S^T \cdot G \]  

(22e)

These matrices are symbolically calculated in Maple to obtain a concise dynamics model.

IV. DIFFERENTIAL KINEMATICS

The jacobian matrix for the considered robotic system in case of no constraint is defined as:

\[ \dot{X}_{\text{task}} = \text{Jac} \cdot \dot{q} \]  

(23)

where \( \dot{X}_{\text{task}} \) is the vector of task space speeds that can be chosen as:

\[ \dot{X}_{\text{task}} = \begin{bmatrix} x_{\text{e}}^\prime \phi (x_{\text{e}}^\prime)^T \end{bmatrix} \]  

(24)

where \( x_{\text{e}}^\prime \) describes the \( m-th \) end-effector linear position in the inertial frame, and its orientation using Euler angles. Therefore, the jacobian matrix can be written as:

\[ \text{Jac} = \begin{bmatrix} 1_{3x3} & 0_{3x(N-3)} \\ \text{Jac}_{21}^{(N-3)x3} & \text{Jac}_{22}^{(N-3)x(N-3)} \end{bmatrix} \]  

(25)

As described in (13a), if we use \( \dot{\nu} \) instead of \( \dot{q} \), it is obtained

\[ \dot{X}_{\text{task}} = \text{Jac} \cdot \dot{\nu} \]  

(26)

Finally, the time derivative of jacobian matrix is obtained as

\[ \ddot{J}_{\dot{q}} = \begin{bmatrix} \text{Jac}_{21}^{dot} \cdot \text{Jac}_{22}^{dot} + \text{Jac}_{21}^{dot} \cdot \text{Jac}_{22}^{dot} \end{bmatrix} \]  

(28)

V. THE MIC LAW

The MIC law enforces an impedance relationship at the object level, as well as the manipulators-base level, and yields proper results even in the presence of object flexibility, and impacts due to contact with the environment. This strategy allows coordinated motion and force control of wheeled mobile robots to perform a desirable manipulation task. To apply the MIC law, a desired impedance relationship at the object level is written as:

\[ M_{\text{des}} \ddot{\hat{e}} + k_d \dot{\hat{e}} + k_p \varphi + F_c = 0 \]  

(29)

where \( M_{\text{des}}, k_d, k_p \) are the desired mass, damping and stiffness matrices, and \( F_c \) is the contact force (in contact phase), and \( e = x_{\text{des}} - x \) is the object tracking error. On the other hand, as described in [10-11], the object equation of motion can be obtained as:

\[ M \ddot{x} + \dddot{F} = F_c + F_c + G \dddot{F} \]  

(30)

where \( M, F_c, F_c, G_c \) represent mass matrix, vector of nonlinear velocity terms, external forces/torques, grasp matrix and finally forces/torques exerted by the manipulators and end effectors, respectively. As mentioned before, the MIC law enforces the same impedance on various parts of the system. Therefore, we can write the same impedance law for the system as:

\[ M_{\text{des}} \ddot{\hat{e}} + k_d \dot{\hat{e}} + k_p \varphi + F_c = 0 \]  

(31)

where \( \ddot{\hat{e}} = x_{\text{des}} - \hat{x} \) is the tracking error in the system based on the following feedback linearization approach as follows, [10-11]:

\[ \tau_{\text{mot}} = \hat{H} M_{\text{des}}^{-1} \left( \hat{M}_{\text{des}} \ddot{x} + k_d \dot{\hat{e}} + k_p \varphi + U_j \cdot F_c \right) + \hat{C} \]  

(33a)
\[
\begin{align*}
\dot{H} &= \dot{J}acH \dot{J}ac^{-1} \\
\dot{C} &= \dot{J}ac^{-1} \dot{C} - \dot{H} \dot{J} \dot{\nu} \\
\ddot{U} &= \left[ \dot{J}ac_{32}^T \begin{bmatrix} 1 & 0 & 0 & 0 & 1 \end{bmatrix} \right]^T
\end{align*}
\] (33b, 33c, 33d)

To obtain \( \tau_f \), based on the object dynamics as described in Eq. (29), the required controlling force \( GF_{req} \) could be written:

\[
GF_{req} = MM_{des}^{-1} \left( M_{des} \ddot{x}_{des} + k_a e + k_p e + F_c - F_o \right)
\] (34)

The required desired force obtained in (34) could be used to determine \( \tau_f \) for both manipulators, [10-11]:

\[
\tau_f = \begin{bmatrix} \begin{bmatrix} 0_{2 \times 1} \\ GF_{req} \end{bmatrix} \end{bmatrix}^{2} \] (35)

It can be shown that by application of the MIC law, all participating manipulators, the moving base and the manipulated object behave with the same desired impedance behavior, [11].

It should be mentioned that the effects of the disturbances that are applied on the base front right side will be considered in the simulation study. The resultant generalized disturbance forces/moments can be described as following:

\[
Q_{dis} = \sum_{a=1}^{b} J_{(a)}^T \dot{Q}_{dis,a}
\] (36)

where the Jacobian matrix \( J_{(a)} \) is a \( 6 \times N \) matrix defined as:

\[
\begin{bmatrix} \dot{\bar{R}}_a \\ \omega_a \end{bmatrix} = J_{(a)} \dot{q}
\] (37)

which relates the generalized velocities \( \dot{q} \) (or in our case \( \dot{\nu} \)) to the linear velocity \( \dot{\bar{R}}_a \) and angular velocity \( \omega_a \) of the exerted body. Finally Eq. (36) will be added to the generalized forces in Eq. (1).

VI. SIMULATION RESULTS AND DISCUSSIONS

The simulated system consists of two 3-DOF manipulators mounted on a suspended wheeled mobile platform such as shown in Fig. 1, while the moving base is driven with two rear differentially-driven wheels. All geometric and mass properties of the mobile base, and each of the two identical manipulators are given in Tables (1)-(2). The first manipulator is equipped with a remote center compliance (RCC) which is initially free of tension or compression, and its stiffness and damping properties are chosen as, [12]:

\[
k_c = 2.4 \times 10^4 \text{ kg/s}^2 \quad \text{and} \quad b_c = 5.5 \times 10^2 \text{ kg/s}
\]

and the rigid object parameters are

\[
m_{obj} = 3 \text{ kg, } r^{(1)}_v = -r^{(2)}_v = (-0.3, 0, 0)
\]

where as defined before, \( r^{(m)}_v \) is the position of the \( m \)-th end effector with respect to the object center of mass. The springs stiffness and damping coefficients are \( k=1500 \text{ N/m}, c=200 \text{ Ns/m} \) for the front wheels and \( k=1000 \text{ N/m}, c=100 \text{ Ns/m} \) for the rear ones, respectively. The MIC algorithm is used to successfully move an object in a mixed circular-straight path as depicted in Fig.3. The object should move on a 5-meter-radius circular path which continues through a straight part at the end of this maneuver. To examine the capabilities of the MIC law, the object will deliberately face a contact with an obstacle on its path (along straight part) in both of these cases, where a smooth stop at the obstacle will be desirable. The obstacle is at \( x_o = 4.55 m \), and it is taken as a spring with \( k_s = 1e5 \text{ N/m} \).

Figure 3. The object and end-effectors real path, consisting circular-straight portions

<table>
<thead>
<tr>
<th>TABLE (1): PROPERTIES OF BASE</th>
</tr>
</thead>
<tbody>
<tr>
<td>mass ( \left( \text{kg/m}^2 \right) )</td>
</tr>
<tr>
<td>-------------------------------</td>
</tr>
<tr>
<td>50</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>TABLE (2): PROPERTIES OF EACH MANIPULATOR LINKS</th>
</tr>
</thead>
<tbody>
<tr>
<td>( i )-th body</td>
</tr>
<tr>
<td>-----------------------------------------------</td>
</tr>
<tr>
<td>1</td>
</tr>
<tr>
<td>2</td>
</tr>
<tr>
<td>3</td>
</tr>
</tbody>
</table>

To examine the capabilities of the MIC law, a substantial disturbance force equal to \( [-350N; -350N; 90N] \) of step type is applied on the base on the front right side when it reaches \( x = 1 m \) and continues until \( x = -1 m \) on its circular path which will be compared to the results without exerting such disturbance.

Fig. 4 shows the position error of the object, and the first and second end effector for the two cases of with and without exerting disturbance force. As it can be seen there are three points of disruption; which correspond to the time of the disturbance exertion at \( t \approx 2.5 \text{ sec} \), switching from the circular part to the straight part of the path at \( t \approx 7 \text{ sec} \), and finally the impact due to contact with the obstacle at \( t \approx 18 \text{ sec} \), respectively. After the contact, the object tracking error will undergo a steady error which implies that the MIC law will not allow the object to experience excessive force. In other words, the cause of the steady error lies in the fact that the MIC law concentrates on the object dynamical behavior by applying defined impedances both at the object level and the robotic system level, rather than explicit force/position tracking control. So as it is seen the object will come into smooth
stop at the obstacle, which shows the success of applying the desired impedance. In fact, the contact force is zero during no contact phase, but during contact phase it gradually increases. These results reveal the merits of the MIC algorithm in terms of smooth performance, i.e. negligible small tracking errors in the presence of impacts due to contact with obstacles and significant disturbances. The existence of flexibility in the system due to RCC and suspension system does not have any permanent undesired effect on the object control which reveals the high capabilities of the MIC law in the presence of flexibility in the system.

![Figure 4](image)

**Figure 4.** Position tracking errors without (left) and with (right) exerting disturbance for the: (a) object, (b) 1st end effector, (c) 2nd end effector

VII. CONCLUSIONS

In this paper, using Lagrange method the equations of motions were derived based on direct path method (DPM). Next, the system non-holonomic constraint was derived, and using Natural Orthogonal Complement Method the independent set of equations of motion for the system was derived. Finally, the MIC law was applied to manipulate an object by two 3 DOF cooperating manipulators, one of them equipped with a remote center compliance (RCC), mounted on a suspended wheeled platform while the moving base is driven with two differentially driven wheels. The obtained results reveal a coordinated smooth motion of the object, manipulators and the moving base, even in the presence of system flexibility, and impacts due to contact with an obstacle, and significant disturbances.

REFERENCES


