INNOVATION AND SELF-ORGANIZATION IN A MULTI-AGENT MODEL

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A model is developed to study the effectiveness of innovation and its impact on structure creation on agent-based societies. The abstract model that is developed is easily adapted to any particular field. In an interacting environment, the agents receive something from the environment (the other agents) in exchange for their effort and pay the environment a certain amount of value for the fulfilling of their needs or for the very price of existence in that environment. This is coded by two bit strings and the dynamics of the exchange is based on the matching of these strings to those of the other agents. Innovation is related to the adaptation by the agents of their bit strings to improve some utility function.

Keywords: Innovation; agent-based models; artificial societies.

1. Introduction

Agent-based models are increasingly being used to model artificial societies. Some of these models fall in the field of biological sciences and a very important part of them deal with economical problems [1–6]. Economical, ecological and social environments share as a common feature the fact that the agents operating in these environments spend a large amount of their time trying to improve some kind of actual or perceived utility, related to profit, food, reproduction or comfort and power. It so happens that many times the improvement of one agent’s utility is made at the expense (or causes the decrease) of the other agents’ utilities. In cooperative circumstances, a similar situation may occur when considering intergroup...
competition. A general concept that is attached to this improvement struggle is the idea of innovation.

In the economy, innovation may be concerned with the identification of new markets \cite{7,8}, with the development of new products \cite{9–12}) to capture a higher market share or with the improvement of the production processes to increase profits. In ecology, innovation concerns better ways to achieve security or food intake or reproduction chance and, in the social realm, all of the above economical and biological drives plus a few other less survival-oriented needs. In all cases, innovation aims at finding strategies to better deal with the surrounding environment and to improve some utility function. In any system where at least some agents are trying to innovate, the perfect strategy of today may, in time, become a losing one. It is the well-known “red queen effect”: You must run as fast as you can, to stay in the same place.

It is in the economy field that innovation has been more extensively studied. Three main types of innovations were identified:

(i) **Market innovation**: the identification of new markets and finding out how they are better served or how they may become more receptive to the available products;

(ii) **Product innovation**: the identification and development of new products;

(iii) **Process innovation**: the identification of better and less expensive production ways or the improvement of internal operations.

Although these classification types were developed for economics, it is an easy exercise to find the corresponding notions in other environments. That also applies to the classification of the intensity of the innovations as radical, incremental, architectural and modular. An important point to emphasize is that the intensity of the innovation is an agent-dependent concept. An innovation that is radical for one agent might just appear as incremental or of any other type to some other agent \cite{13}. Another important concept concerning systems of innovation \cite{14} is the flow of information \cite{15,16} between the agents in the systems and its appropriation in terms of knowledge. Another important issue in the innovation field is the identification of the basic mechanisms leading the agents to innovate \cite{17–22} and its impact on social change and human evolution \cite{23}.

Several authors have approached innovation systems from an agent-based model (ABM) perspective. An exhaustive set of papers on agent-based economics can be found in The Handbook of Computational Economics \cite{24}, where an extensive survey of agent-based computational research dealing with issues of innovation and technological change is presented, particularly in the chapter by Herbert Dawid. The multi-agent work developed there is built upon the lines of biologically-inspired models of social behavior, as also is most of work in evolutionary economics, initiated by Nelson and Winter \cite{4}. Evolutionary economists interested in modeling innovation and growth \cite{25} have been searching for the emergence of macro-invariants
Innovation and Self-Organization in a Multi-Agent Model

from micro-level behavior. Studying innovation at the micro level (firm and industry level) is also the core issue of Freeman’s seminal paper [26]. There, and quoting the work of Nelson and Winter [4], neo-Schumpeterians are challenged with the following questions: *How can ordered patterns of innovation emerge despite the industrial diversity and the uncertainty inevitably associated with innovation?*, *How can structure emerge from apparently chaotic variety?* Commenting on diversity and structure generation, Freeman also points out that, by definition, innovation involves initially an increase of diversity and, at the initial stage, there are no standards and there is great uncertainty about the future of the new products.

The application of ABM ideas — originally inspired by the study of population dynamics — has also been adopted in the development of opinion formation [27,28] and voter models [29]. The genetic algorithm approach has a wide range of applications in economics. Similarly to the work presented here, all these models have in common a bit-string description and, frequently, a recognition (matching) process, where principles of similarity and imitation play the fundamental role. Axelrod was one of the first authors to work on the mechanisms through which similarity leads to interaction and interaction leads to even more similarity.

Agent-based computational models of innovation [30], in their most general form, describe evolutionary processes as being characterized by three main stages: generation of variety, selection based on some measure of success and reduction of variety due to adaptation. Dawid also emphasizes that there are few models of innovation that consider the evolution of consumer preferences. One of the papers that considers this issue [31] studies the influence of supply on consumer preferences. The model is developed to capture the example of the computer industry’s transition from transistor to micro-processor technology and the associated emergence of the market for PCs.

One of the pioneer researchers on innovation comments on the limitations of neo-classical formulations of innovation and production of technology: “they make exogenous the production of technology and innovation” [32]. Exogeneity is also criticized by Freeman [26] when commenting on the black-box explanation found in books on invention and innovation: the idea that technical change is outside the specialized competence of most economists and the convenient assumption that science and technology could be treated as exogenous and that, for most purposes, need not be deeply examined. Another issue related to the exogenous perspective is the difficulty to explain how innovation occurs. According to Fagerberg [33], “one of the reasons innovation was ignored in the mainstream social science for so long was that it was seen as impossible to do. The best one could do was to look at innovation as a random phenomenon.” Schumpeter in his early works was one of the first to object to this practice.

The association of innovation with success appears quite often in the literature. Schumpeter, in his early works, considered innovation processes as having three main aspects: the fundamental uncertainty inherent in all innovation projects, the need to move quickly before somebody else did (and reap the potential economic
reward) and the inertia of the society. Such inertia in Schumpeter’s view, was to some extent endogenous, since it reflected the embedded character of existing knowledge and habit, which through an energy-saving bias tended to decide against any new ways of doing things. Technological competition (competition through innovation) was considered as the driving force of economic development. If one firm in a given industry or sector successfully introduces an important innovation, the argument goes, it will be amply rewarded by a higher rate of profit. This functions as a signal to other firms (the imitators) which, if entry conditions allow, will swarm the industry or sector with the hope of sharing the benefits [33].

We have presented here this short overview of some of the innovation literature to emphasize that the innovation problem has been addressed in many different ways, with many different tools and, each time, with a specific context in mind. The fact that innovation covers so many different fields and particular settings was our motivation to develop an abstract model that might be adapted to a wide range of different fields. The dynamical structure of the model should also be sufficiently general to provide general insights on the mechanisms leading to emergent collective structures.

In this paper, innovation is represented as a mechanism to improve similarity between agents or adaptation to a prescribed goal. We also follow a biological inspiration, where innovation is related to mutation mechanisms which, by allowing for the emergence of new features, may (or may not) improve the agent success. In general, in an interacting environment, the agents receive something from the environment (the other agents) in exchange for their effort and pay the environment a certain amount of value for the fulfilling of their needs or for the very price of existence in that environment. We will code the two types of exchanges by two bit strings which conventionally will be called the products string and the needs string. In an economy environment, products and needs might be actual market products and operation or supplies needs, but in a biology environment they might stand, for example, for hunting success and predation by other species, in the political setting for “slogans and promises” and voters desires, etc. In our models the dynamics of the exchange is always based on the matching of the products string of each agent with the needs strings of the other agents.

Two types of models will be studied. In Sec. 2, separating the products and needs functionalities, we study a model of producers and consumers, the main aim being to characterize the conditions for innovation success. Different possibilities, concerning both the nature of the environment and the number of innovation agents, are considered. Once the quantitative assumptions of the model are specified, our main aim is to use the model as a laboratory to find out how and what environment conditions affect innovation efficiency (in the model of Sec. 2) and how innovation activities affect structure formation (in the model of Sec. 3). This goal is achieved by computing the correlation functions of all relevant quantities. We present several different scenarios and, in each case, we have tried to modify only one parameter at a time to isolate its specific effect. For some of the correlations that are obtained,
intuitive interpretations come easily, others are not so simple. We have attempted
some interpretation of the less obvious correlations, keeping in mind however, that
our main “laboratory” results are the correlations, not the interpretation. Of course,
a deeper interpretation might only come from analytical solutions of the models,
which we plan to attempt in the future.

In all cases we find general conditions for innovation success, which are related
both to the agent strategies and to the environment conditions. Translated to actual
environments (for example the economy), these conditions might already provide
useful lessons on how and when to carry out a successful innovation.

In the second model, Sec. 3, each agent is equipped with two strings represent-
ing either products and needs or, more generally, what an agent profits from the
environment and what the environment profits from him. The main result aris-
ing from the study of this model is a clear evidence on how (starting from a
uniform distribution of fitness, randomly chosen strings and identical dynamical
laws) structure formation is obtained in the agents’ society, both with and without
innovation.

2. A Model of Producers and Consumers

At the start, there are $2N$ agents in the model: $N$ producers and the same number
of consumers. Each consumer has a set of needs coded by a string of $k$ bits and each
producer has a product coded by a string of $k$ bits. The bit string of a consumer
represents what the consumer agent needs to receive from the environment and the
bit string of a producer is a code for the products that she is able to supply. No
passive actors are assumed in the environment and the environment for each agent
is just the set of all the other agents.

In addition to the two bit strings that code for needs and products, each agent
has a scalar variable $S$ or $C$, depending on the agent type (consumer or producer,
respectively). The variable $S$ represents the degree of satisfaction of the needs and
$C$ represents the amount of some commodity (or cash) that may be exchanged for
the products that are supplied. In the economy this role is played by money, but
in other contexts it might be protection capacity, power or status.

The dynamics of the model is characterized by exchange, evolution and adap-
tation. The basic driver of the exchange dynamics of the model is the matching
between needs and products. At each time step, the matching of the bit strings
representing needs and products is computed and each consumer chooses at ran-
dom one among the products that better matches her needs. The producer that
has this product is a potential supplier. The dynamical evolution is

$$S_i(t+1) = S_i(t) - ac + \frac{q^*_i}{k},$$

and

$$C_j(t+1) = C_j(t) - ap + \sum_{j(i)} \frac{q^*_{ij}}{k}.$$  


The index $j(i)$ runs over all the consumers $j$ that are supplied by the $i$ producer. Of course, in each particular application the terms in these equations should be normalized by appropriately chosen units in order to represent commensurable or dimensionless quantities.

On receiving a product from the producer $i$ the consumer $j$ increases her satisfaction (or energy $S$) by $\frac{q_{ij}^*}{T} - ac$. The variable $q_{ij}^*$ stands for the matching of the producer $i$ that supplies the consumer $j$. At the same time, the producer $j$ increases her commodity $C$ by $\sum_{j(i)} \frac{q_{ij}^*}{T} - ap$, where $ac$ and $ap$ stand for two constant costs of living that are subtracted at each time step from the consumers’ satisfaction and the suppliers’ cash. The fact that, independently of the value of the transactions, a cost of living is subtracted at each time step from both $S$ and $C$ makes unnecessary the inclusion of the satiation of consumption parameter that is included in some equilibrium models.

At the initial time all producers and consumers are assigned a fixed value $S = S_0$ and $C = C_0$. Then, at each time step needs and products are compared. The producer that supplies each consumer is chosen at random among those with the larger matching. In some of the runs of the model we have also included a degree of uncertainty, giving some small probability $p_b$ to non-maximizing choices of the matching. As long as $p_b$ is not very large, the qualitative results are not changed.

When $C_i < 0$ this producer $i$ either disappears and is not replaced (Sec. 2.1) or it is replaced by a new random producer (Sec. 2.2). When $S_j < 0$ this consumer $j$ is replaced by a new one with random needs string and $S_i = S_0$. As such, a consumer only remains in the field as long as her energy $S$ is positive. If it becomes negative, she dies and is replaced by a new random consumer. Initially all agents and the replacement (consumer) agents are endowed with the same initial $C_0$ and $S_0$.

The replacement mechanism of the agents means that, when applied to real world situations, each replacement agent in the model represents a new or an already existing consumer trend or product. In biology, this should preferably refer not to an individual species but to an ecological niche.

Several possible evolution mechanisms may be implemented in the model:

- **Innovation by the producers**
  - Market-oriented innovation: the innovating producers find the consumers that have a matching above a certain threshold and flips the worse bit. Corresponds to adaptation of a particular product to expand an existing market. This is really a proxy for market innovation in the sense that improving the producer-consumer matching better serves an existing market. As far as intensity is concerned, this would fall under the heading of incremental innovation.
  - Process innovation: process innovation corresponding to a decrease in production costs may be simulated in the model by adding a certain amount (a half
point for example) to the matching results of this producer. This increase of matching favors the chance of this producer to be chosen by the consumers. However, the value of the products remaining the same, it does not affect the amount that is added to $S_i(t + 1)$ and $C_i(t + 1)$.

— **Product innovation**: the innovating producer finds a set of consumers that among themselves have a matching above a certain threshold and develops a new product string according to their need bits. From the intensity point of view this might be considered a *radical* innovation.

- **Evolution and Adaptation of the consumers**: after the exchanges, the less satisfied *consumers* find the products that have a matching above a certain threshold and flip the (need) bits with the worst scores when compared with the same position bits in the products.

There are of course some important features of real world environments that are not explicitly included in our abstract coding of the products offered by each agent. For example, products sometimes have some core features that are fixed and some others that are adjustable. Then the agent may supply the same core product to different customers as different offerings. This *market segmentation* technique is particularly important in the services industry [7, 8]. The choice preference in the model being achieved by maximization (with or without uncertainty) of the partial matching between products and needs, one may take the point of view that one is dealing only with the core features of products. An explicit coding of core versus adjustable features might be included by keeping some product bits fixed and fuzzying a few others. However, we believe that the qualitative dynamical features of the model would not be very much affected by this change.

In the next subsections, the model is tested in several different scenarios, which are characterized by different combinations of the parameter values, namely:

- Changing the consumer *cost of living* parameter $ac$, providing either stable or volatile environments, that is, environments where there are high rates of consumer replacement.
- Changing the producer *cost of living* parameter $ap$, providing environments with either low or high producer replacement rates.
- Changing the innovation mechanism that is adopted: either market-oriented innovation (MOI) or adaptation to available products (CAP).
- Changing the quantity of agents that are allowed to perform the innovation mechanisms. Two possibilities are considered: just one agent or a random number of agents.

In all our simulations we have chosen (the initial number of producers and consumers) $N = 100$, (the bit-string length) $k = 10$ and (the initial endowments) $S_0 = C_0 = 50$. 
2.1. Market-oriented innovation

As stated before, in these innovation mechanisms the innovating producers find the consumers that have a matching above a certain threshold and flip the worse bit, that is, the bit that has the smallest matching with this set of consumers. An important result is already implicit in the choice of consumers with matching above a non-negligible threshold. In fact, we found out that without this threshold this innovation mechanism is not efficient at all. It means that there are no universal products or alternatively that by incremental innovations addressed to the universal consumer, one cannot expand her own market.

In the first two scenarios, the innovation mechanism is market-oriented innovation (MOI) by one innovating producer. Scenarios 1 and 2 differ on the \( ac \) value, representing either stable \( (ac = 0.5) \) or volatile \( (ac = 1) \) consumer environments. Stable here is not an absolute concept. It only means that there will be a very small rate of replacement of consumers in the first case. In both cases producers with \( C_i < 0 \) are not replaced.

In each scenario, one looks for correlations between the nature of the environment and the efficiency of the innovation process. The rate of MOI efficiency \( (g) \) of an innovating producer \( (IP) \) is defined by

\[
g_{ip} = \frac{C_{ip}(t_{end}) - C_{ip}(t_{innov})}{t_{end} - t_{innov}},
\]

where \( C_{ip}(t_{end}) \) and \( C_{ip}(t_{innov}) \) represent, respectively, the amount of cash of the innovating producer at the end of the simulation \( (t_{end}) \) and when innovation starts \( (t_{innov}) \).

2.1.1. Stable and volatile environments with one innovating agent

With \( t_{innov} = 250 \) and \( t_{end} = 1000 \), some results are shown in Figs. 1 and 2 for \( ap = 5, ac = 0.5 \) and \( ac = 1 \), respectively. At time \( t_{innov} \) the surviving producer with the lowest cash starts the market-oriented innovation process as defined above. The two upper plots in the figures show examples of the producers’ cash evolution and mean consumer satisfaction. The bold line refers to the innovating producer. The lower plots are obtained from a large number of runs. Notice that the upper left plot in Fig. 1 refers to one of the most successful innovation runs. The histograms in the lower right plots show that this type of innovation is much more efficient on a highly volatile environment \( (ac = 1) \) than in a stable consumer environment \( (ac = 0.5) \).

We also found inverse correlation between MOI efficiency and the distance of the innovating producer \( (IP) \) to the nearest consumers (clients). But the correlation is strong only in a volatile environment, as the lower left plot in Fig. 2 shows. Simulations have also shown a negative correlation of the innovation efficiency \( (g) \) with the rate of gain before innovation and with the distance to the nearest competitor.
Fig. 1. Stable environment and one innovating producer.

Fig. 2. Volatile environment and one innovating producer.
Some of these correlations have a simple interpretation, namely that it is not easy to increase one’s market by incremental innovations in a stable market. Therefore success is more probable if the environment changes. In an economic market, that is the role of publicity when it aims at changing or polarizing the consumers’ tastes. Less obvious are the inverse correlations with the distance to the nearest consumers, the rate of gain before innovation and the distance to the nearest competitor. On hindsight however, two of these correlations might be interpreted as meaning that success also requires some pre-existing affinity to market segments not yet covered.

2.1.2. Stable and volatile environments with many innovating agents

Here the system was tested for different numbers of innovating producers (IP). When more than one producer is allowed to innovate, the innovation efficiency rate \((g)\) is computed as the average value obtained for the set of innovating producers. In each simulation, the number of \(IP’s\) is determined at random, being the innovating producers chosen among the poorest ones.

The results in the histograms of Figs. 3 and 4 show that market-oriented innovation by more than one \(IP\) is also much more efficient in a volatile environment.

**Fig. 3.** Stable environment and a random number of innovating producers.
than in a stable one. The relation between innovation efficiency and the structure of the environment is consistent with the results obtained for the case of one innovating producer. In fact, one finds inverse correlation between MOI efficiency and the distance among consumer needs in the stable environments. The efficiency of this type of innovation is also inversely correlated to the distance among the innovating producers and their nearest consumers, but the correlation is strong only in volatile environments as the lower left plots in Fig. 4 show.

2.2. Evolution of needs and adaptation to available products

In this scenario, the model contains a mechanism for the evolution of the needs. This mechanism is one of partial adaptation or conformity with the available products (CAP). In this scenario producers and consumers with negative cash or satisfaction are replaced by new random ones.

2.2.1. One adapting consumer

When just one consumer is allowed to adapt, the model is implemented as follows: at each time step (after the exchanges) the less satisfied consumer finds the products
with which she has a matching above a certain threshold and flips the (need) bit with the worst score in the comparison with the corresponding bit in the products. This mechanism aims at mimicking the adaptation of a consumer to the products that are available in the market or the adaptation of the survival needs of a species to its environment.

The system was tested for different values of \( ap \), allowing to simulate environments with different rates of replacement of producers. In this scenario, a producer only remains in the field as long as her capital \( (C) \) is positive; if it becomes negative, the producer is replaced by a new random producer.

A new efficiency rate \( (s) \) is then defined in order to compute the difference between the satisfaction level of the innovating consumer \( (IC) \) after and before innovation,

\[
s_{ic} = \frac{S_{ic}(t_{end}) - S_{ic}(t_{innov})}{t_{end} - t_{innov}},
\]

where \( S_{ic}(t_{end}) \) and \( S_{ic}(t_{innov}) \) represent, respectively, the amount of satisfaction (or energy) of the innovating consumer at the end of the simulation and at the start of the innovation process.

As seen from the histograms in Figs. 5 and 6, the main result is that adaptation to the available products is equally efficient for a small or a large rate of replacement of producers \( (ap = 4 \) or \( ap = 6 \), respectively). It means in fact that this type of consumer adaptation to the environment is easier than producer adaptation. All the other types of correlations that were found in the case of the innovating producers are either very small or non-existing. This is understandable because the situations are not symmetrical, because many consumers may share a producer, but there is only one producer supplying each consumer.

2.2.2. Many adapting consumers

Here, the model is tested with different numbers of innovating consumers. When more than one consumer is allowed to adapt, the efficiency rate \( s \) is computed as the average of the rate for the set of innovating consumers.

The results presented in the histograms of Figs. 7 and 8 seem to imply that adaptation to the available products by many innovating consumers is slightly more efficient in a market with a high rate of substitution of producers \( (ap = 6) \) than in the case where the rate of replacement is low \( (ap = 4) \).

There are also some weak (not very significant) inverse correlations between the efficiency rate and the distance between the \( IC \)'s and the set of producers, in environments with a low rate of substitution of producers, and, for volatile environments, a weak negative correlation of the satisfaction rate with the distance between the \( IC \)'s and the producers.
Fig. 5. One innovating consumer in an environment with a low rate of replacement of products.

Fig. 6. One innovating consumer in an environment with a high rate of replacement of products.
Fig. 7. Several innovating consumers in an environment with low rate of replacement of products.

Fig. 8. Several innovating consumers in an environment with high rate of replacement of products.
3. A Self-Organizing Agent Model: Innovation and Emergent Structures

Here we study a model where all agents have two strings, which, as before, we denote as the $P$ string and the $N$ string. Here however, rather than products and needs, as in the producers and consumers model, it is more appropriate to interpret the $P$ string as the code for the benefits (or energy) that the agent can extract from the environment (the other agents) and the $N$ string as a code for what the other agents may extract from her.

As before, the dynamical evolution is based on the matching of the $P$ and $N$ strings. Each agent has a fitness function $F$ which evolves as follows:

\[ F_i(t+1) = F_i(t) + \sum_{j(i)} \frac{q_{ij}^*}{k} - \frac{q_l(i)}{k}. \tag{5} \]

The $P$ strings of the agents are matched with the $N$ strings of the other agents. Then, among the agents with maximum overlap with a particular $N$ string, one is chosen at random. $\sum_{j(i)}$ denotes a sum over all the agents $j$ for which the $P$ string of $i$ not only had maximal overlap $q_{ij}^*$, but also was chosen in the random selection process if she was not the only one with maximal matching. $q_l(i)$ denotes the maximal matching of the $N$ string of agent $i$ with the other agents. At time zero the $P$ and $N$ strings of all agents are chosen at random and the fitness is initialized to some fixed value $F(0)$. Whenever, during the time evolution, the fitness of one agent becomes negative, this agent is replaced by another random agent with the initial fitness.

One of the purposes of the study of this model was to show how, starting from a set of agents in identical conditions, the time evolution spontaneously creates fitness inequalities among them. How the structures are affected by innovation will also be studied. An important issue in structure creation, in agent societies, is also how evolution affects diversity.

Innovation in the model of this section is, as before, an adaptation of the strings to the environment (the other agents). Two kinds of innovation are considered. In first (called $P$-innovation) each agent compares her $P$ string to the $N$ strings of the other agents having matchings above a threshold ($\text{thrs}$) and flips her worst scoring bit. Therefore $P$-innovation means that the agent tries to maximize what she receives from the other agents. In the second kind of innovation (called $N$-innovation) each agent tries to minimize the matching of her $N$ string with the $P$ strings of the other agents. At each time step, this is also done by flipping a bit, this time the bit that has the better matching. The meaning of $N$-innovation is that the agent tries to give the other agents as little as possible or, in a sense, that is trying to protect herself from the wearing out effects of the environment (predation, consumption costs, etc.). In this model, whenever innovation is implemented, all agents are allowed to innovate, in line with the equal opportunity point of view of the model.
In Figs. 9 to 13 we compare different scenarios. The two upper plots compare the histograms of the initial fitness and the fitness after $T = 5000$ time steps. The value of the entropy ($H$) associated with each distribution of fitness is also computed. The middle histograms compare the diversity of the $P$ strings at the initial time and at $T = 5000$. Diversity of the strings is characterized by the histogram of their Hamming distances. The lower histograms contain a similar comparison for the $N$ strings. In all cases where innovation is implemented, $thrs = 1$.

Figure 9 is the situation without innovation. Although all agents start with similar conditions, a large stratification of fitness emerges as a result of the time evolution. The model shows how a well-defined structure emerges from the dynamical evolution. The entropy values ($H$) obtained for the fitness distribution show how dynamics and random events generate inequality.

In Fig. 10 $P$-innovation for all agents is implemented. One sees that the stratification effect is even stronger when this type of innovation is turned on. For the dynamics without innovation the string diversities at the initial time and at $T = 5000$ are similar. However, for the $P$-innovation case one sees a concentration of the $P$ strings around a dominant type. Inequality stratification is enhanced and diversity decreases.

In Fig. 11 all agents perform $N$-innovation. With this innovation process, exchanges are minimized, and the most relevant structure that develops is a drastic reduction of the $N$ strings diversity.

Fig. 9. Fitnesses and diversity of strings without innovation.
Fig. 10. Fitnesses and diversity of strings with P-innovation.

Fig. 11. Fitnesses and diversity of strings with N-innovation.
When both *P*- and *N*-innovation are implemented (Fig. 12), both the diversity of the strings and the stratification of the fitness are restored. It is interesting to notice that in terms of the global parameters of the agent’s society, the two types of innovation seem to cancel out and the results are similar to the situation without innovations.

The network structure gives an interesting insight on the effect of each one of the innovation mechanisms. The plots in Fig. 13 show four different graphs with the agents placed on a ring (or one-dimensional lattice), a pair connection between two agents being drawn whenever the *P*-string of one agent matches the *N*-string of the other agent, at the end of the evolution process.

Four typical one-dimensional lattices are drawn corresponding to the four different scenarios: without innovation, *P*-innovation, *N*-innovation and both *P* - and *N*-innovation. From the plots one sees that the most relevant structure that develops is a drastic reduction of diversity in terms of the winning agents when *P*-innovation is implemented (upper right plot). A similar situation occurs in the *N*-innovation scenario. However, as in this scenario, each agent is trying to protect itself from the effects of the environment, diversity is not so drastically reduced.
4. Conclusions

A very general feature in real world complex systems is the fact that each agent can receive something from the environment (the other passive or active agents) and the other agents may also receive something from it. This is the basic fact behind our $P$ and $N$ coding strings and their matching. This abstract coding allows one to study general effects, independently of the particularities of each actual complex adaptive system. In addition to the dynamics of interaction, ruled by the matching of the strings, one also sees that the actions of the agents in their adaptation to the environment may be coded by the evolution of the bit strings.

In the first model (consumers and producers), by separating the functionalities associated to the $P$ and $N$ strings, we were able to obtain general conclusions about the effectiveness of the innovation mechanisms and how this effectiveness relates to the overall structure of the agents’ environment and their relation to it.

In the second model, the agents being equipped with both types of interactions with the environment, we obtained a clear manifestation of the fact that even simple interaction dynamics may create strong structures in agent societies. On the
other hand, active actions by the agents to improve their fitness create further structure and, in particular, have a strong effect on diversity. Therefore, the onset of these (innovation) actions at a particular time may be the driving mechanism for structural changes in actual real world situations.

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