Spectrum Sensing over SIMO Multi-Path Fading Channels based on Energy Detection

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Abstract—One of the techniques for spectrum sensing is the energy detection of signals from primary transmitters. However, when detecting in a certain frequency band, this technique suffers from the random fading introduced by the channel. This problem can be alleviated by the use of multiple antennas introducing spatial diversity. The performance of the energy detector with multiple antennas has already been analytically analyzed for flat fading channels.

In this paper, we propose analytical expressions for the performance of multi-antenna energy detector with multi-paths fading channels. These expressions will enable to compare for different channel power profiles the gains brought by the two sources of diversity: spatial (or antenna) diversity and multi-path diversity. This analytical performance is compared to simulations to show the validity of the used approximations.

I. INTRODUCTION

Static spectrum assignment has been lately described as an inefficient way to distribute the spectrum resources [1], [2]. In order to reuse the under-utilized spectrum bands, spectrum sensing and cognitive radio have been proposed as solutions to this problem [3]. Sensing of primary transmitters in the band of interest is the most common approach and can be carried out by several type of detectors. Among them, the energy detectors is often chosen due to its simplicity and to the low knowledge it requires about the signal to detect [4]. Unfortunately, its performance rapidly decreases in case of random fading which can be alleviated by the use of multiple antennas at the cognitive radio terminal.

Energy detection with multiple antennas has already been proposed and its performance analytically studied but only in the case of flat fading channels [5], [6], [7], [8].

In this paper, we provide the expression of the multi-antenna energy detector with multi-path fading channel and derive analytical expressions of it’s performance. Multiple antennas are only considered at the receiver side, meaning that we target a Single Input Multiple Output (SIMO) channel. These expressions enable to compare for different channel power profiles the gains brought by the spatial (or antenna) diversity from the one hand and by multi-path diversity from the other hand. Because they rely on approximations, their validity is also validated by simulation results.

The outline of this paper is as follows. In section II, we will describe the system model including the binary hypothesis testing formulation and the channel fading model. In section III, we will introduce the detector model and calculate the distributions of its statistic test for AWGN and Multi-tap random fading channels. Detector analytical performance will be derived in section IV. Simulations results will be presented in section V to validate the proposed analytical expressions. Finally, section VI will conclude our discussion.

II. SYSTEM MODEL

The system considered in this paper is a base-band equivalent model for signal detection. In particular, the detection of the unknown signal presence is carried out by binary hypothesis testing as explained in the next section II.A. Later on, in section II.B, we will present the considered channel fading models.

A. Binary Hypothesis Testing

After filtering, down-conversion and signal sampling in the front-end of every antenna, the received signal can be modeled as a low-pass baseband complex digital signal. These received samples are represented, by $y_r[n]$ with $n$ the sample index, given $0 \leq n \leq N-1$ and $r$ the antenna index, with $1 \leq r \leq R$. Also, $N$ is the number of samples and $R$ the number of antennas at the cognitive radio terminal.

Hence, the signal detection problem can be described as a binary hypothesis testing over these received samples:

\begin{align*}
H0 &: y_r[n] = w_r[n] \\
H1 & : y_r[n] = \sum_{l=0}^{L-1} h_r[l]\theta[n-l] + w_r[n]
\end{align*}

where $H1$ and $H0$ refer to the hypotheses of signal presence and signal absence respectively.

When $H0$ is true, the resulting signal is only noise $w_r[n]$. When $H1$ is true, with the noise, we receive the same modulated symbols $\theta[n]$, on all antennas. At antennas, $\theta[n]$ is convoluted with the channel $h_r[l]$, where $0 \leq l \leq L-1$ and $L$ is the number of taps. The next subsection will be dedicated to explain the model chosen for $h_r[l]$ in further detail.

The noise samples $w_r[n]$ are independent and identically distributed (i.i.d.) random processes following the same complex Gaussian distribution on each antenna. This distribution...
is zero-mean and of variance $\sigma^2$. (Both of them are assumed to be known by the receiver) $w_r[n] \sim N_c(0, \sigma^2)$

The modulated symbols $\theta[n]$ are also (i.i.d.) on a normalized constellation.

### B. Channel Fading Models

In the previous binary hypothesis testing, we represented the signal to detect by the linear model $\sum_{l=0}^{L-1} h_r[l] \theta[n - l]$, which is formerly called a Bayesian linear model, due to the use of probability distributions on the parameters [9].

As already mentioned $h_r[l]$ ($0 \leq l \leq L - 1; 1 \leq r \leq R$) are the taps of the SIMO fading channel. We assume here a Rayleigh fading, i.e. $h_r[l]$ are random variables following a complex Gaussian distribution: $h_r[l] \sim N_c(0, \sigma^2_l[l])$

In this article we will use two channel cases, the Additive White Gaussian Noise (AWGN) for which there is no Multi-Path nor random fading, and the Multi-Path with Rayleigh fading. The first, formerly implies $l = 1$ and $h_r[l] = 1$ deterministic. The second will be further divided in to equal power taps and different power taps. Equal power taps is a particular case of different power taps, but it will be useful in illustrating the equivalency between antenna and tap diversity on the performance of the detector. For the simple case of equal power taps, each tap variance has the same value: $\sigma^2_h[l] = \sigma^2$. For different power taps, the dependency of $\sigma^2_h[l]$ with respect to index $l$ remains.

In order to enable the comparison of different power profiles for the taps, the average power introduced at each antenna will be normalized as:

$$
E \left[ \sum_{l=0}^{L-1} |h_r[l]|^2 \right] = \sum_{l=0}^{L-1} E [ |h_r[l]|^2 ] = \sum_{l=0}^{L-1} \sigma^2_h[l] = 1.
$$

### III. ENERGY DETECTOR FOR SIMO MULTI-PATH FADING CHANNELS

In this section we will first provide the expression of the energy detector statistic test for multi-path fading channels. Then, we will calculate the distribution of the statistic test for the AWGN channel and finally, that for the multi-tap fading channel.

#### A. Expression of the statistic test

The detection is a processing operation $T(y[n])$ on the received samples. In it’s ultimate stage, it decides which of the two hypothesis $H_0$ or $H_1$ is true, comparing the result with a threshold $\gamma$.

$$
T(y[n]) \xleftrightarrow{H_0 \rightarrow H_1} \gamma.
$$

From the Neyman-Pearson criterion we can derive the single antenna energy detector that maximizes the Probability of detection $P_D$ when the Probability of False Alarm $P_{FA}$ is fixed [9]. However, to simplify later expressions we will use instead the equivalent Power Detector or averaged Energy Detector over samples, also in [9]. Further, the Equal Gain Combining Energy Detector is again the average over the different antenna branches.

$$
T(y[n]) = \frac{1}{R} \sum_{r=1}^{R} \frac{1}{N} \sum_{n=1}^{N} |y_r[n]|^2 \overset{H_1}{\sim} \gamma. \quad (1)
$$

For multi-antenna energy detectors antennas having the same total power the correlation between samples or between antennas has no effect on the final performance.

In order to isolate the effect of diversity and the fact of having more averaged samples when using multiple antennas, we are going to define a constant number of total samples $K$. Then, we replace $N = K/R$, which divides the number of samples by the number of antennas.

To derive the performance of the detector, formerly the $P_D$, we will need to find the distribution function (PDF) of the detector introduced by (1).

#### B. Distribution of the statistic test for AWGN Channel

First we look at the case when the modulated symbols $\theta[n]$ are not affected by a Multi-tap Fading channel, that is if $|h_r|^2 = 1$ and $L = 1$.

The distribution function of the $T(y)$ can be described accurately in terms of the $\chi^2$ distribution knowing the signal variance of $\theta[n]$, $\sigma^2_\theta$ and the noise variance $\sigma^2$.

However, the number of samples is always very high for the necessary performances of realistic applications. In addition, we know that correlated samples or correlated diversity branches will have no influence on an Energy Detector. As long as the total energy remains the same, the performance can be calculated assuming independent samples and diversity branches.

Thus, if we consider a big number of samples and they are independent and identically distributed, by the Central Limit Theorem, the distribution of the detector is Gaussian. That is, for the two original hypothesis we will have the following distributions:

$$
H_0 : T(y) \sim N \left( \sigma^2, \frac{\sigma^4}{RN} \right) \quad (2)
$$

$$
H_1 : T(y) \sim N \left( \sigma^2 + \sigma^2_0, \frac{(\sigma^2 + \sigma^2_0)^2}{RN} \right) \quad (3)
$$

Extending the distribution of the energy detection test from one antenna to multi-antenna is straight-forward. In fact, we are averaging $R$ times more samples. It follows that this distribution of the EGC-ED is the EGC version of the Edell’s Model [10].

We must also note, that the distribution is not dependent on the number of antennas $R$ when we use the total number of averaged samples $K = RN$.

#### C. Distribution of the statistic test for Multi-tap Rayleigh Fading Channel

The distribution of $T(y)$ under hypothesis $H_0$ remains the same (2) as the random fading is only applied when there is signal.
Under hypothesis $H1$, for every antenna the power of the signal to detect can be considered as a random variable $S_r$. Each realization of this variable will depend on the symbols and the channel but as long as the symbols are i.i.d. and normalized $S_r$ will only depend on the channel taps:

$$S_r = E_y \left[ \sum_{l=0}^{L-1} h_r[l] \theta[n-l] \right]^2$$

$$S_r \sim \sum_{l=0}^{L-1} |h_r[l]|^2. \quad (4)$$

Additionally, the distribution of the detector for a channel joint realization of the antennas will have the following mean and variance:

$$E[T(y)] = \sigma^2 + \frac{1}{R} \sum_{r=1}^{R} S_r \quad (5)$$

$$Var[T(y)] = \frac{1}{(RN)^2} \sum_{r=1}^{R} N \cdot (\sigma^2 + S_r)^2$$

$$= \frac{1}{R^2N} \sum_{r=1}^{R} (\sigma^2 + S_r)^2. \quad (6)$$

Thus the final Gaussian distribution can be written as:

$$H1 : T(y) \mid S_1, ..., S_R \sim N \left( \sigma^2 + \frac{1}{R} \sum_{r=1}^{R} S_r, \frac{1}{R^2N} \sum_{r=1}^{R} (\sigma^2 + S_r)^2 \right). \quad (7)$$

**IV. PERFORMANCE ANALYSIS OF THE PROPOSED ENERGY DETECTION**

In this section, we will first derive the $P_{FA}$. Then we will derive the $P_{MD}$ for the two channel cases presented previously in section II.B.

**A. Calculation of the $P_{FA}$**

The $P_{FA}$ is calculated integrating the distribution of $T(y)$ under $H0$ (2). This distribution does not depend on what we set for the channel, so the $P_{FA}$ will be in both channel cases:

$$P_{FA} = p(T(y) \mid H0 > \gamma) = \int_{\gamma}^{\infty} N \left( \sigma^2, \frac{\sigma^4}{RN} \right) dx$$

$$P_{FA} = Q \left( \frac{\gamma - \sigma^2}{\sqrt{\frac{\sigma^4}{RN}}} \right). \quad (8)$$

**B. Calculation of the $P_{MD}$**

Without random fading, NF (No Fading), the $P_{MD,NF}$ is calculated as the complementary of the Probability of Detection $P_{D,NF}$. The signal variance is represented by $\sigma^2$:

$$P_{MD,NF} = 1 - P_{D,NF}$$

$$P_{D,NF} = p(T(y) \mid H1 > \gamma)$$

$$P_{D,NF} = \int_{\gamma}^{\infty} N \left( \sigma^2 + \sigma^2, \frac{\sigma^4 + \sigma^4}{RN} \right) dx$$

$$P_{D,NF} = Q \left( \frac{\gamma - (\sigma^2 + \sigma^2)}{(\sigma^2 + \sigma^2)/\sqrt{RN}} \right). \quad (9)$$

Like the distribution of $T(y)$, if $K = RN$ the performance becomes independent of the number of antennas.

With random fading, the distribution of $T(y)$ is affected by the power of the particular realization of the channel. The signal power per realization for every antenna is $S_r$:

$$\begin{align*}
P_{MD} & | S_1, ..., S_R \equiv 1 - P_{D} | S_1, ..., S_R \\
P_{D} & | S_1, ..., S_R = p(T(y) \mid H1, S_1, ..., S_R > \gamma) \\
P_{D} & | S_1, ..., S_R = Q \left( \frac{\gamma - (\sigma^2 + \frac{1}{R} \sum_{r=1}^{R} S_r)}{\sqrt{\frac{\sigma^4 + \sigma^4}{RN}}} \right). \quad (10)
\end{align*}$$

Finally, the averaged performance is the expectancy of this $P_{D}$ per realization of the antenna channels $S_1, ..., S_R$:

$$\begin{align*}
\overline{P_{D}} & = E_{S_1, ... , S_R} [P_{D} | S_1, ..., S_R] \\
\overline{P_{D}} & = \int_{S_1 = 0}^{\infty} ... \int_{S_R = 0}^{\infty} Q \left( \frac{\gamma - \left( \sigma^2 + \frac{1}{R} \sum_{r=1}^{R} S_r \right)}{\sqrt{\frac{\sigma^4 + \sigma^4}{RN}}} \right) \cdot \ldots \cdot \prod_{r=1}^{R} p_{S_r} (S_1, ..., S_R) dS_1 \ldots dS_R. \quad (11)
\end{align*}$$

However, if the channels for each antenna are independent the joint distribution $p_{S_r} (S_1, ..., S_R)$ can be replaced by the product of the individual distributions:

$$\begin{align*}
\overline{P_{D}} & = \int_{S_1 = 0}^{\infty} ... \int_{S_R = 0}^{\infty} Q \left( \frac{\gamma - \left( \sigma^2 + \frac{1}{R} \sum_{r=1}^{R} S_r \right)}{\sqrt{\frac{\sigma^4 + \sigma^4}{RN}}} \right) \cdot \ldots \cdot \prod_{r=1}^{R} p_{S_r} (S_r) dS_1 \ldots dS_R. \quad (12)
\end{align*}$$

The distribution of the signal power per antenna $p_{S_r} (S_r)$ will be dependent on the power profile of the channel and the number of taps $L$. This leads us to the following expressions. For equal power taps:

$$p_{S_r} (S_r) = \frac{1}{\sigma^2 \Gamma(L)} S_r^{L-1} \exp \left( -\frac{S_r}{\sigma^2} \right). \quad (13)$$

For different power taps:

$$p_{S_r} (S_r) = \sum_{l=0}^{L-1} \frac{B_l}{\sigma^2^l} \exp \left( -\frac{S_r}{\sigma^2^l} \right). \quad (14)$$

The definition of the $B_l$ coefficients and the development of both expressions can be found in the Appendix, in the case $R = 1$ and with $S_r$ instead of $S$.

However the multiple integral of (12) is not really convenient for numerical analysis. Additionally to the exact
expression, we can have a useful approximation under very low SNRs:

$$(\sigma^2 + S_r)^2 \approx \sigma^4 + 2\sigma^2 S_r.$$  \hfill (15)

If we define the random variable $S$,

$$S = \frac{1}{R} \sum_{r=0}^{R-1} S_r$$  \hfill (16)

and we use the total number of samples $K = RN$, the $P_D$ expression can be rewritten without the multiple integral:

$$P_{D,APRX} = \int_{S=0}^{\infty} Q \left( \frac{\gamma - (\sigma^2 + S)}{\sqrt{\sigma^2(\sigma^2 + 2S)}} \right) \cdot p_S(S) dS.$$  \hfill (17)

At this point, we have found a simple analytical expression of the performance that will be very useful for further analysis of this type of detectors.

Of course, the random variable $S$ which is the average signal power received for all the antennas will have a distribution resulting from the power introduced by the different channels.

The distribution of $S$ for equal power taps is on the Appendix (18).

And the distribution of $S$ for different power taps is on the Appendix (20), (21) and (22) and can be easily derived for any number of $L$ and $R$ following the same reasoning.

V. SIMULATION RESULTS

In this section, we wanted to verify the obtained analytical performance expressions by simulating the detector for several situations of interest. Usually, the detection performance is illustrated by a Receiver Operating Characterististics (ROC) curve ($P_D$ vs $P_{FA}$). Instead, we use a complementary-ROC ($P_{MD}$ vs $P_{FA}$). This complementary-ROC, has the advantage of showing how close it’s the detector from the no error operation.

The data symbol of the primary user signal to be detected belongs to a QPSK constellation an the operating $SNR(= 10 \log(1/\sigma^2))$ is equal to -20dB. Parameter $K$ has been set to $2 \times 10^5$.

Two different channel power profiles are examined: an equal power tap profile and an exponentially decaying power tap profile. Both of them concern a multi-path Rayleigh fading SIMO channel. In any case, the channel total averaged power is normalized to one as explained in Section II.

A. Validation of the proposed analytical expressions

Fig.1 compares the performance obtained with the energy detector obtained by simulations (solid curves) and using analytical expressions (17) (dashed curves) with varying number of receive antennas.

We clearly see that both set of curves overlap despite the approximation used to establish the distribution of the statistic test and the low SNR approximation (15). This validates the analytical expression proposed in this paper. In this figure and the figures hereafter the following curves are also represented:

- the curve corresponding to the single tap, single antenna Rayleigh fading case (named ‘1 Antenna 1 Tap’) which serves as an upper bound and the curve corresponding to the non-fading AWGN channel case (named ‘AWGN Channel’) which serves as a lower bound.

B. Comparison of gains brought by antenna diversity and tap diversity

First we consider the channel model with equal power taps. From (18), we can easily see that $R$ an $L$ are in this case interchangeable in the pdf of $S$, which means that only the product of these two variables counts. This result clearly appears in Fig.2 which gives the performance of the detector ($P_{MD}$ vs $P_{FA}$) for the three following cases for which $RL = 4$:

- $R = 1, L = 4$
- $R = 2, L = 2$
- $R = 4, L = 1$

Indeed the curves corresponding to those cases overlap. Therefore, in the case of equal power taps, the antenna diversity and the multi-path diversity are equivalent in terms of performance improvement.

Secondly, we consider a channel model with different power taps. Fig.3 illustrates this for the same three following cases leading to $RL = 4$ as in Fig.2. Actually, we consider here an exponentially decaying power profile for the taps given by: $\sigma^2_l = \exp(-Al)$ where $A$ is here set to $A = 3$.

Then we notice that the antenna diversity is no longer equivalent to the tap diversity since the curves corresponding to the three cases do not overlap. This could be expected from the expression of the pdf of $S$ given in the appendix for the different power tap case. Variables $R$ an $L$ are then not interchangeable in the pdf expression. For the case where $L=1$ and $R=4$ we use $p_S(S)$ from (20). For $L=2$ and $R=2$ we use $p_S(S)$ from (21). If $L=4$ and $R=1$ we use $p_S(S)$ from (22). Actually, it appears then that increasing the number of antennas brings

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more improvement in the detector performance than increasing the number of taps. This is all the more valid as coefficient $A$ is large (result not reported here).

C. Influence of the SNR

Now, we want to illustrate the performance of the energy detector when varying the SNR. Fig4 illustrates the $P_{MD}$ vs SNR (ranging from -35 to -10dB) for a given $P_{FA} = 0.05$ and with the exponentially decay power profile. The different represented curves are for the same number of antennas and taps as in the preview subsection. We notice in Fig4 that the conclusion of Fig3 (the larger increase in performance due to antenna diversity compared to tap diversity) obtained for a given SNR is actually also valid for other SNR values.

Finally in Fig5, we want to show the impact of the low SNR approximation (15) on the analytical expressions. Indeed, this figure compares the $P_{MD}$ obtained with (12) to that obtained by (17) in the two following cases:

- $R = 1, L = 1$
- $R = 2, L = 2$

In each case, the two curves overlap meaning that even for higher SNRs (at least up to 0dB) the low SNR approximation (15) remains valid.

VI. Conclusion

In this article, we have presented analytical expressions describing the performance of a multi-antenna energy detector under the presence of multi-path. The derivations have been
proven correct by comparison with the simulations for some cases of interest. Finally, these analytical expressions have helped us in the comparison of the gains brought by spatial and multi-path diversity against the problem of random fading.

For equal power taps, antenna and path diversity have been shown to perform in the same way. On the contrary, exponentially decaying power taps tend to degrade the performance compared to an equivalent number of antennas.

These expressions we presented are useful for the prediction of the performance of the energy detector in SIMO multi-path channels, which are very common in modern broadband standards.

VII. APPENDIX

In order to find the distribution of antenna averaged signal power \( S = 1/R \sum_{r=0}^{R-1} S_r \), we must make use characteristic functions \( \Psi_S(jw) \) in order to derive the pdf \( p_S(S) \) of each case:

\[
S_r \simeq \sum_{l=0}^{L-1} |h_r[l]|^2 \quad h_r[l] \sim \mathcal{N}_C(0, \sigma_h[l]^2).
\]

All the following expressions involving Gaussian distributions are taken from the references \([11]\) and \([12]\).

For equal power taps \( \sigma_h^2[i] = \sigma_h^2[j] = \sigma_h^2 \) for all \( i, j \)

\[
p_S(S) = \frac{1}{\sigma_h^{2RL} \Gamma(RL)} S^{RL-1} \exp \left( \frac{-S}{\sigma_h^2} \right).
\]

For different power taps: \( \sigma_h^2[i] \neq \sigma_h^2[j] \) for all \( i, j \)

\[
\Psi_S(jw) = \prod_{l=0}^{L-1} \frac{1}{(1 - jw \sigma_h^2[l])^R}
\]

L=1, any R

\[
\Psi_S(jw) = \frac{1}{(1 - jw \sigma_h^2[0])^R}
\]

\[
p_S(S) = 1 - \sum_{i=0}^{R-1} \frac{1}{i!} \left( \frac{S}{\sigma_h^2[0]} \right)^i \exp \left( \frac{-S}{\sigma_h^2[0]} \right)
\]

L=2, any R

\[
\Psi_S(jw) = \frac{1}{\left( 1 - j2w \frac{\sigma_h^2[0]}{2} \right) \left( 1 - j2w \frac{\sigma_h^2[1]}{2} \right)}^R
\]

\[
p_S(S) = \frac{1}{\sigma_h^2[0]} \exp \left( \frac{-S}{\sigma_h^2[0]} \right) \frac{1}{(R-1)!} \left( \frac{\sigma_h^2[0]}{\sigma_h^2[0] - \sigma_h^2[1]} \right)^R \ldots
\]

\[
\sum_{r=1}^{R} \frac{2(R-1) - r}{r!(R-1-r)!} \left( \frac{\sigma_h^2[1]}{\sigma_h^2[1] - \sigma_h^2[0]} \right)^r \left( \frac{S}{\sigma_h^2[0]} \right)^r + \ldots
\]

\[
\frac{1}{\sigma_h^2[1]} \exp \left( \frac{-S}{\sigma_h^2[1]} \right) \frac{1}{(R-1)!} \left( \frac{\sigma_h^2[1]}{\sigma_h^2[1] - \sigma_h^2[0]} \right)^R \ldots
\]

\[
\sum_{r=1}^{R} \frac{2(R-1) - r}{r!(R-1-r)!} \left( \frac{\sigma_h^2[0]}{\sigma_h^2[0] - \sigma_h^2[1]} \right)^r
\]

\[
(21)
\]

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